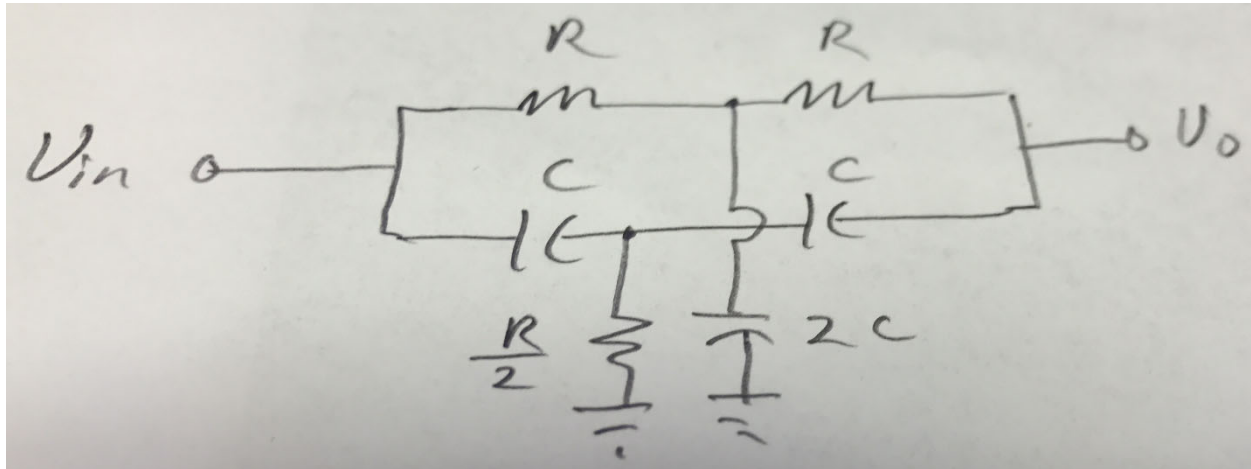


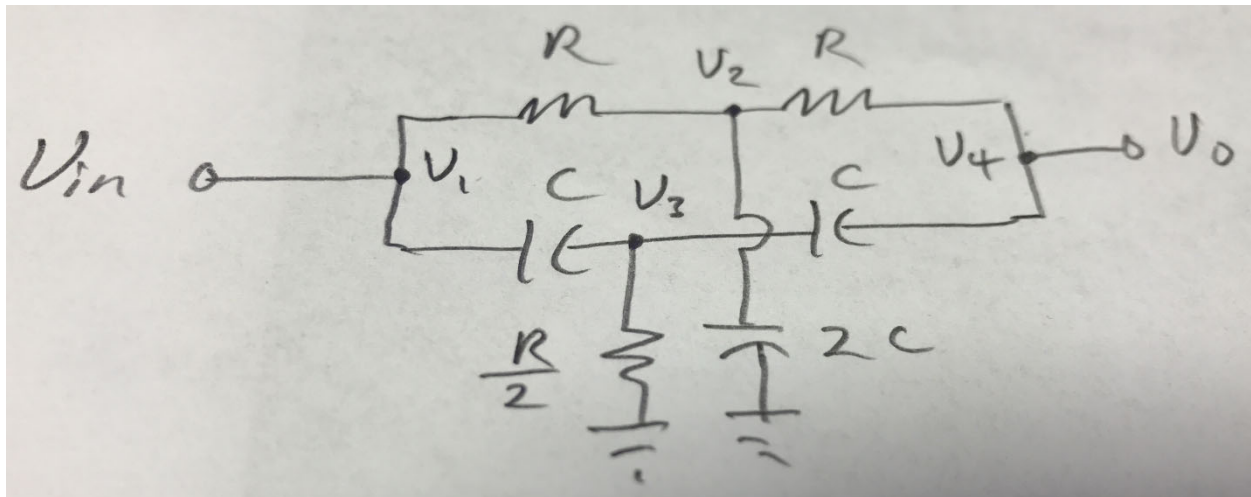
Tuesday 2/7/23

3) Twin T Oscillator

Consider the twin T network below:



To analyze, assign nodes and perform nodal analysis:



$$V_1 = V_{in} \quad (1)$$

$$\frac{V_2 - V_1}{R} + \frac{V_2 - V_4}{R} + V_2 2sC = 0$$

$$\text{Therefore: } V_2 - V_1 + V_2 - V_4 + V_2 2sCR = 0$$

$$\text{And: } V_2(2 + 2sCR) = V_1 + V_4$$

$$\text{Leading to: } V_2 = \frac{V_1 + V_4}{2 + 2sCR} \quad (2)$$

$$(V_3 - V_1)sC + (V_3 - V_4)sC + \frac{2V_3}{R} = 0$$

$$\text{Therefore: } (V_3 - V_1)sCR + (V_3 - V_4)sCR + 2V_3 = 0$$

$$\text{And: } V_3(2 + 2sCR) = (V_1 + V_4)sCR$$

$$\text{Leading to: } V_3 = \frac{(V_1 + V_4)sCR}{2 + 2sCR} \quad (3)$$

$$\frac{V_4 - V_2}{R} + (V_4 - V_3)sC = 0$$

$$\text{Therefore: } V_4 - V_2 + (V_4 - V_3)sCR = 0$$

$$\text{And: } V_4(1 + sCR) = V_2 + V_3sCR \quad (4)$$

(2) into (3) into (4):

$$V_4(1 + sCR) = \frac{V_1 + V_4}{2 + 2sCR} + \frac{(V_1 + V_4)sCR}{2 + 2sCR} sCR$$

$$V_4(1 + sCR)(2 + 2sCR) = V_1 + V_4 + (V_1 + V_4)(sCR)^2$$

$$V_4(2 + 2sCR + 2sCR + 2(sCR)^2 - 1 - (sCR)^2) = V_1(1 + (sCR)^2)$$

$$V_4(1 + 4sCR + (sCR)^2) = V_1(1 + (sCR)^2)$$

$$V_4 = V_o$$

And

$$V_1 = V_{in}$$

Therefore:

$$\frac{V_o}{V_{in}} = \frac{1 + (sRC)^2}{1 + 4sCR + (sRC)^2}$$

Leading to:

$$\frac{V_o}{V_{in}}(j\omega) = \frac{1 - (\omega RC)^2}{1 - (\omega RC)^2 + 4j\omega CR}$$

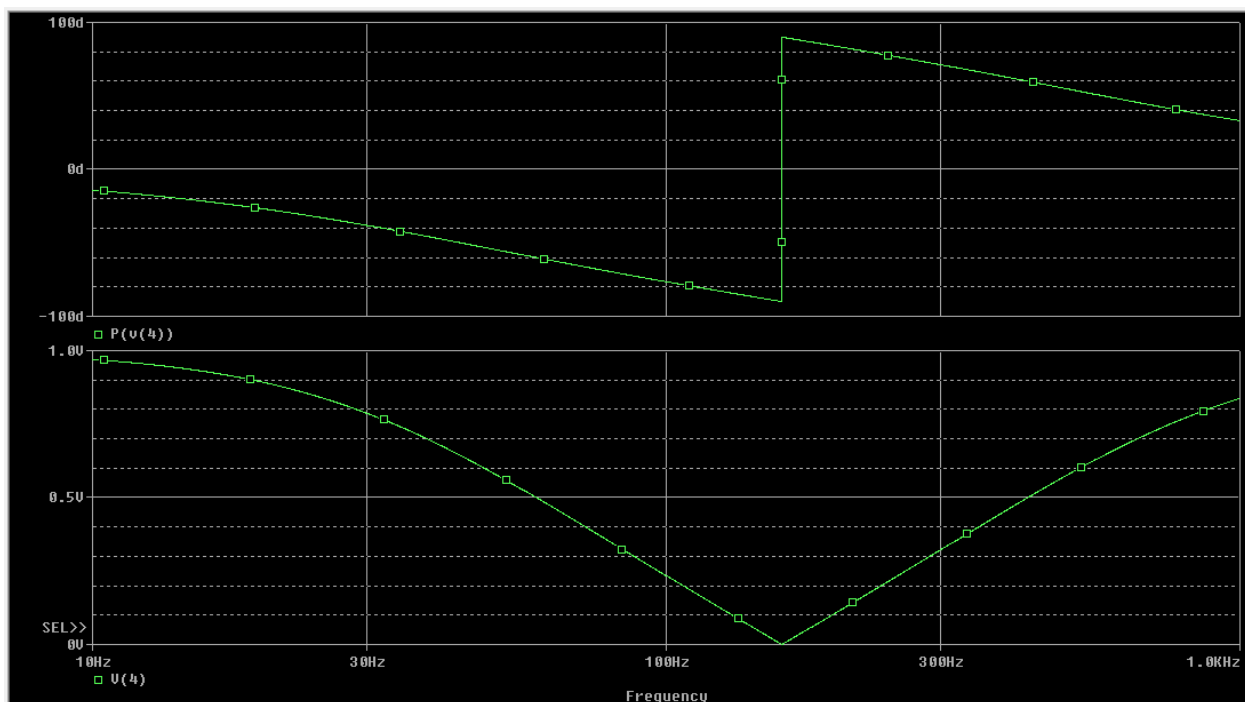
Evaluate at $\omega_o = \frac{1}{RC}$:

$$\left| \frac{V_o}{V_{in}}(j\omega) \right|_{\omega=\omega_o} = \frac{1 - 1}{\sqrt{(1 - 1)^2 + 4^2}} = 0$$

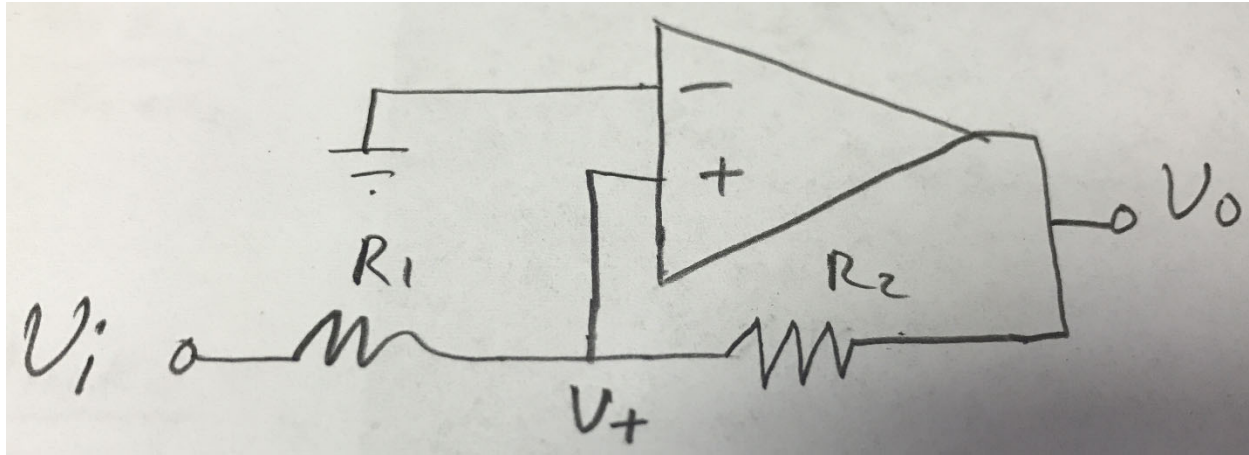
At ω_o , 100% of the transmission from V_i to V_o is blocked!

Consider this PSpice simulation of the twin T network is a notch filter:

PSpice simulation with $R = 1 \text{ k}\Omega$ and $C = 1 \text{ }\mu\text{f}$:



Consider the op amp circuit shown below:



The op amp circuit is in a positive feedback configuration.

Using superposition:

$$V_+ = V_i \frac{R_2}{R_1 + R_2} + V_o \frac{R_1}{R_1 + R_2}$$

And:

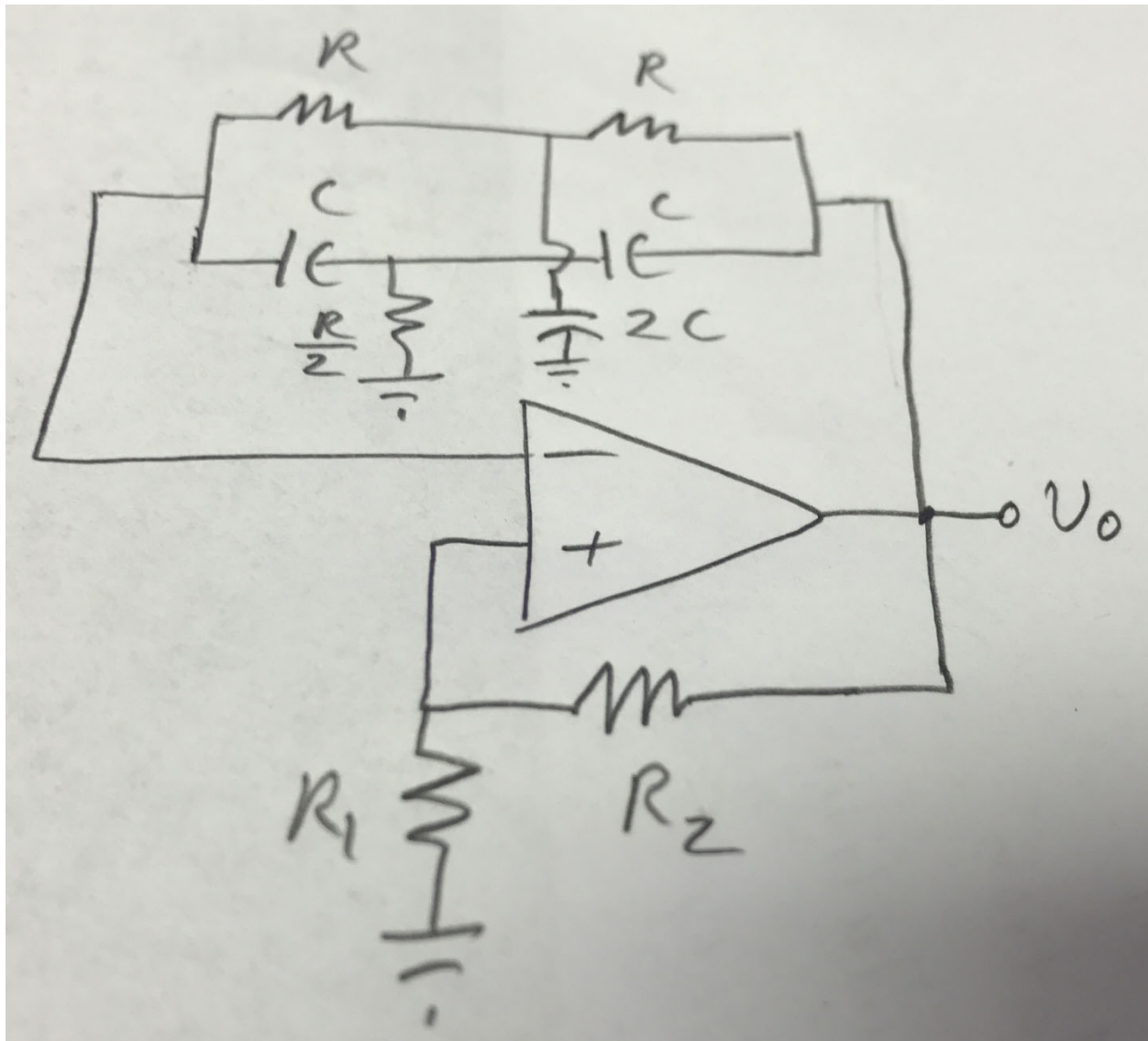
$$V_o = AV_+$$

Where A is the op amp open loop gain.

For $V_i > 0$: V_o goes to the + power supply rail.

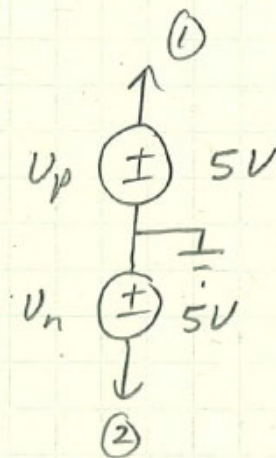
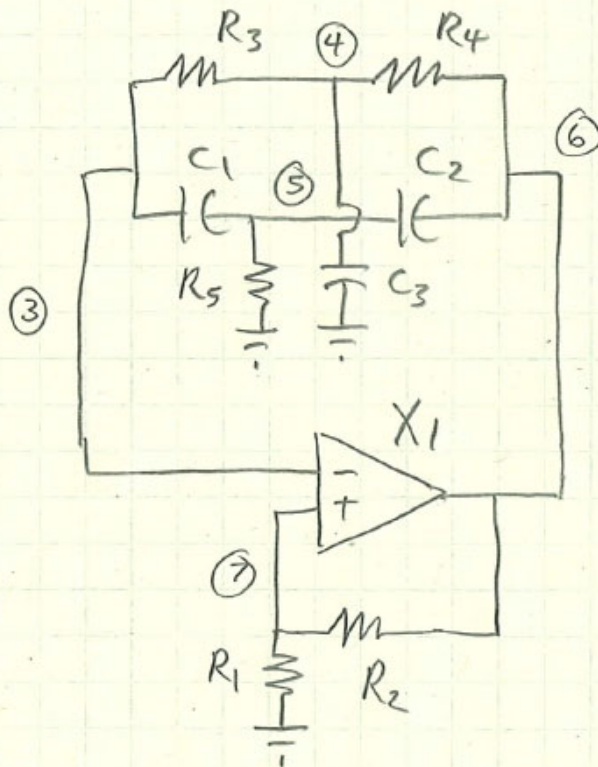
For $V_i < 0$: V_o goes to the - power supply rail.

Putting the two subcircuits together:



The op amp circuit is unstable due to positive feedback. However, the twin T network provides negative feedback except at ω_0 , which stabilizes the circuit at other frequencies. At ω_0 , the op amp circuit is unstable due to positive feedback, and oscillates at that frequency.

PSpice Simulation of Twin T Oscillator



$$f = 1\text{kHz}, R_3 = R_4 = 1\text{k}\Omega$$

$$C = \frac{1}{2\pi R f} = \frac{1}{2\pi (1000)(1000)} = 0.159\mu\text{F} = C_1 = C_2$$

$$R_5 = 500\Omega$$

$$C_3 = 0.3138\mu\text{F}$$

$$\text{let } R_1 = 1\text{k}\Omega, R_2 = 10\text{k}\Omega$$

$$X_1 = \text{AD8610 op amp}$$

* Twin T network osc 2/6/17

Vp 1 0 DC 5

Vn 0 2 DC 5

R1 7 0 1k

R2 7 6 10k

R3 3 4 1k

R4 4 6 1k

R5 5 0 500

C1 3 5 0.159u

C2 5 6 0.159u

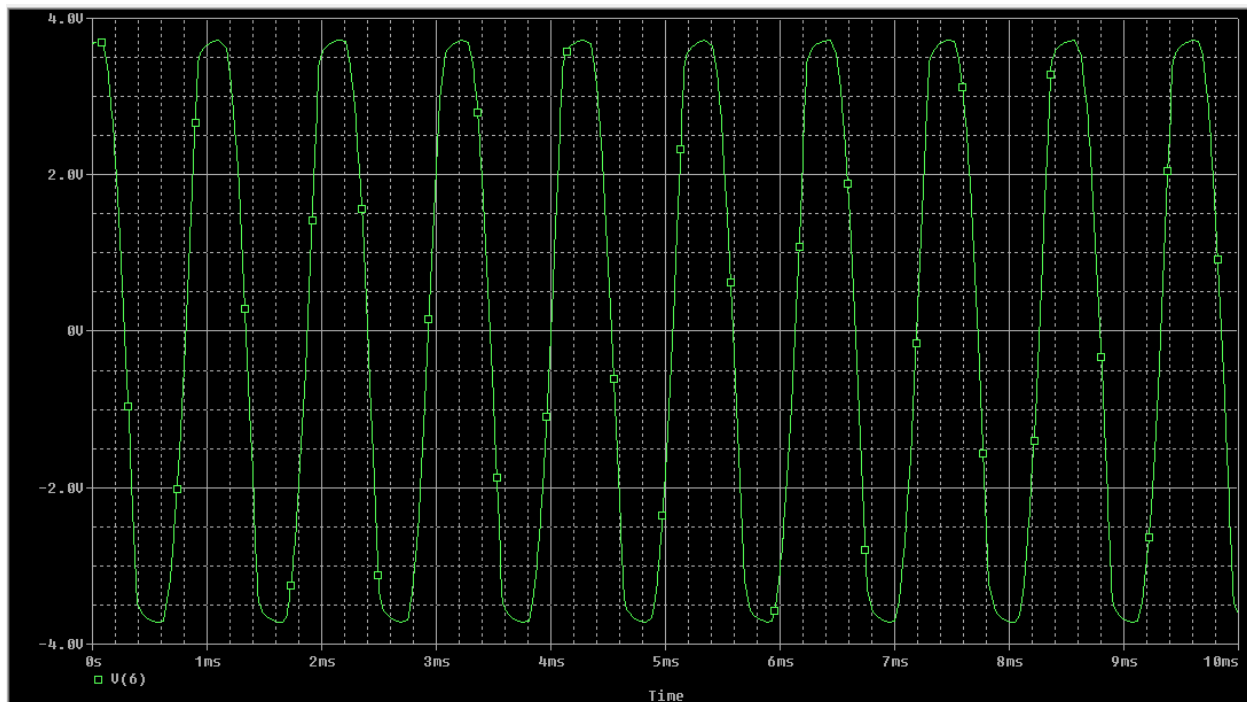
C3 4 0 .3183u ic=0

X1 7 3 1 2 6 AD8610

.tran 100u 10m

.probe

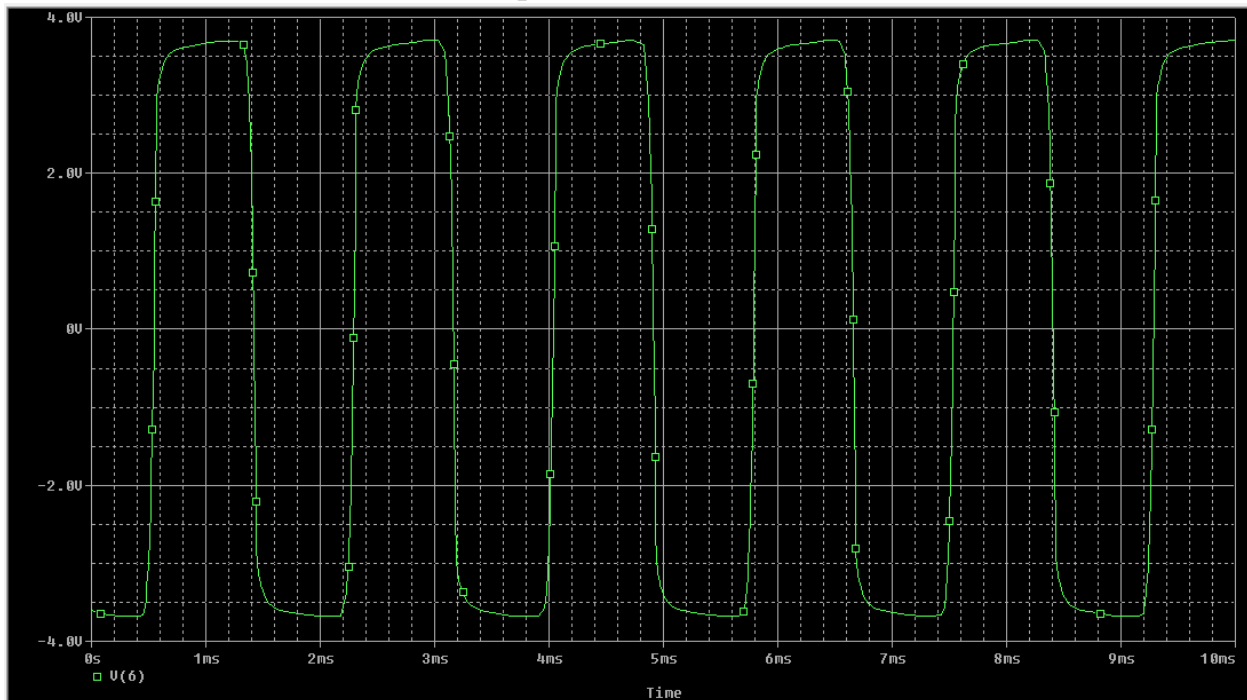
.end



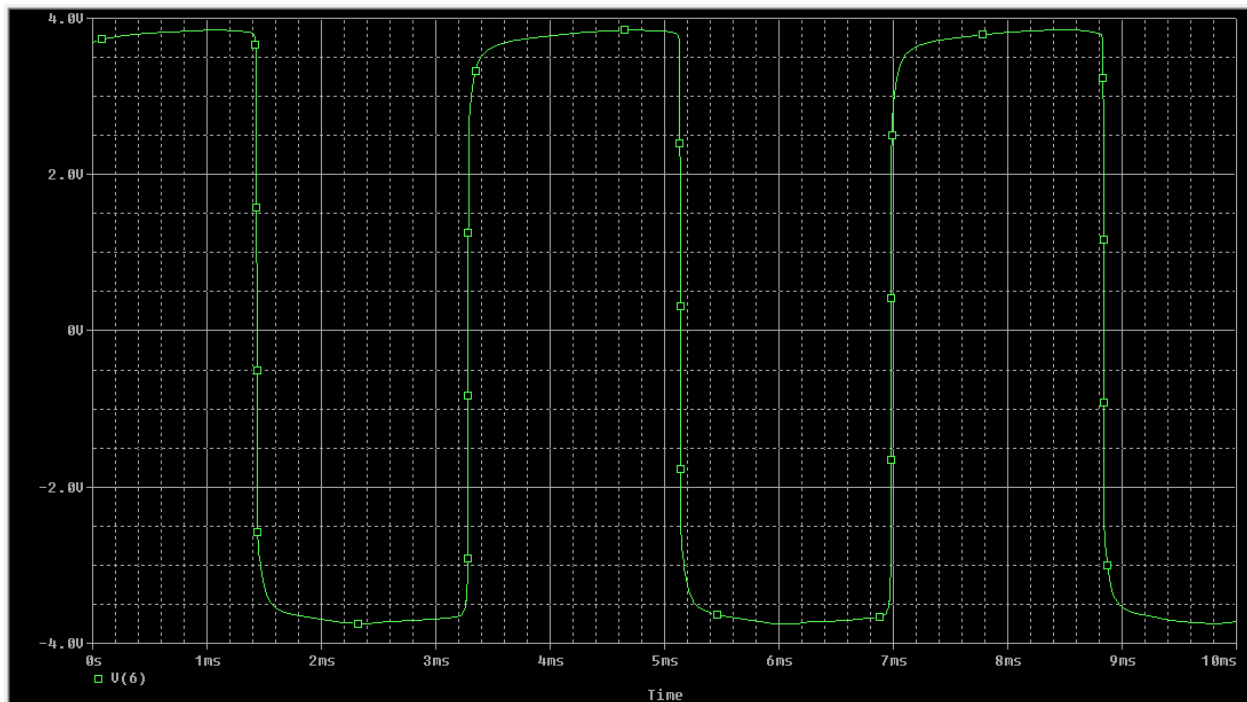
Frequency Approximately 1 KHz

Diode AGC circuit does not work here.

If $R_2 : R_1$ ratio is too small, the output is more distorted: $R_1 = 1\text{ k}\Omega$ and $R_2 = 1\text{ k}\Omega$:



$R_1 = 10\text{ k}\Omega$ and $R_2 = 1\text{ k}\Omega$:

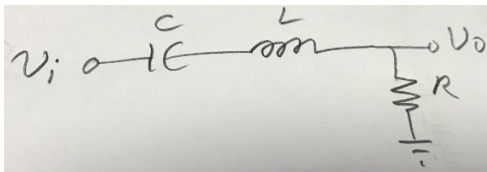


4) LC based oscillators

Comparison of RC oscillators with LC oscillators:

- (1) An RC pair yields one pole. An LC pair yields two poles.
- (2) Passive RC circuits have a maximum Q of 0.5. Passive LC circuits can have much higher Q's.
- (3) LC circuits are much better for high frequency oscillators, but inductors are physically large and lossy for low frequency applications.

Consider the RLC circuit below:



$$V_o = V_i \frac{R}{R + \frac{1}{sC} + sL}$$

$$V_o = V_i \frac{sR}{sR + \frac{1}{C} + s^2L}$$

$$\frac{V_o}{V_i}(j\omega) = \frac{j\omega R}{\frac{1}{C} - \omega^2L + j\omega R}$$

$$\left| \frac{V_o}{V_i}(j\omega) \right| = \frac{\omega R}{\sqrt{\left(\frac{1}{C} - \omega^2L\right)^2 + (\omega R)^2}}$$

$$\angle \frac{V_o}{V_i}(j\omega) = \tan^{-1}(\infty) - \tan^{-1}\left(\frac{\omega R}{\frac{1}{C} - \omega^2L}\right)$$

$$\text{At } \omega = \frac{1}{\sqrt{LC}}, \quad \left| \frac{V_o}{V_i}(j\omega) \right| = 1 \quad \text{and} \quad \angle \frac{V_o}{V_i}(j\omega) = 0^\circ$$

So, this network could be used as $\beta(j\omega)$ for a positive feedback oscillator, requiring $A(j\omega) = 1$.

Consider the circuit below.

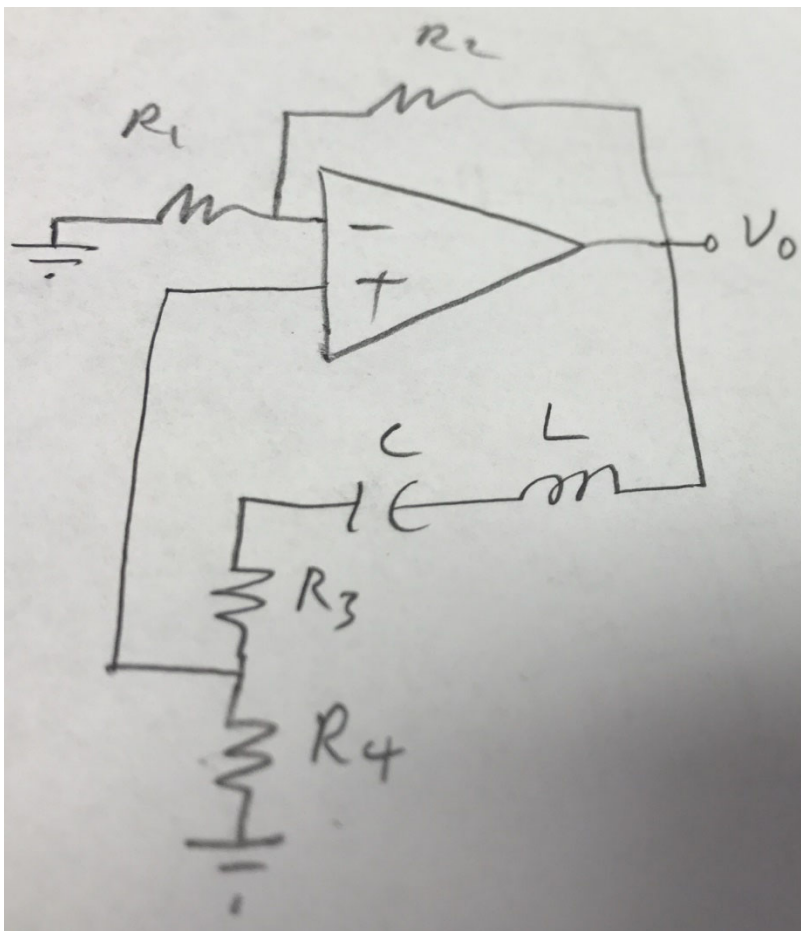
The op amp, R_1 and R_2 form a noninverting amplifier with

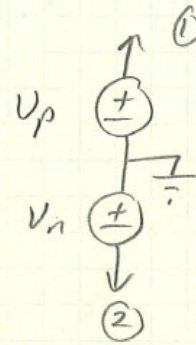
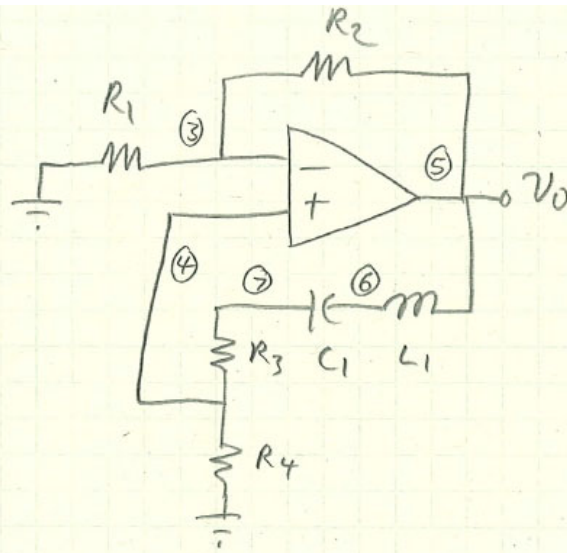
$$\text{Gain} = 1 + \frac{R_2}{R_1}$$

R_3 , R_4 , C and L form the RLC $\beta(j\omega)$ network

R_3 and R_4 form a voltage divider, allowing the loop gain to be reduced to 1 at the oscillation frequency.

Note: R_3 could represent some or all of the inductor's series resistance.





set $R_1 = R_2 = R_3 = R_4 = 10\text{ k}\Omega$

set $C_1 = 1\text{ }\mu\text{F}$ and $L_1 = 1\text{ mH}$

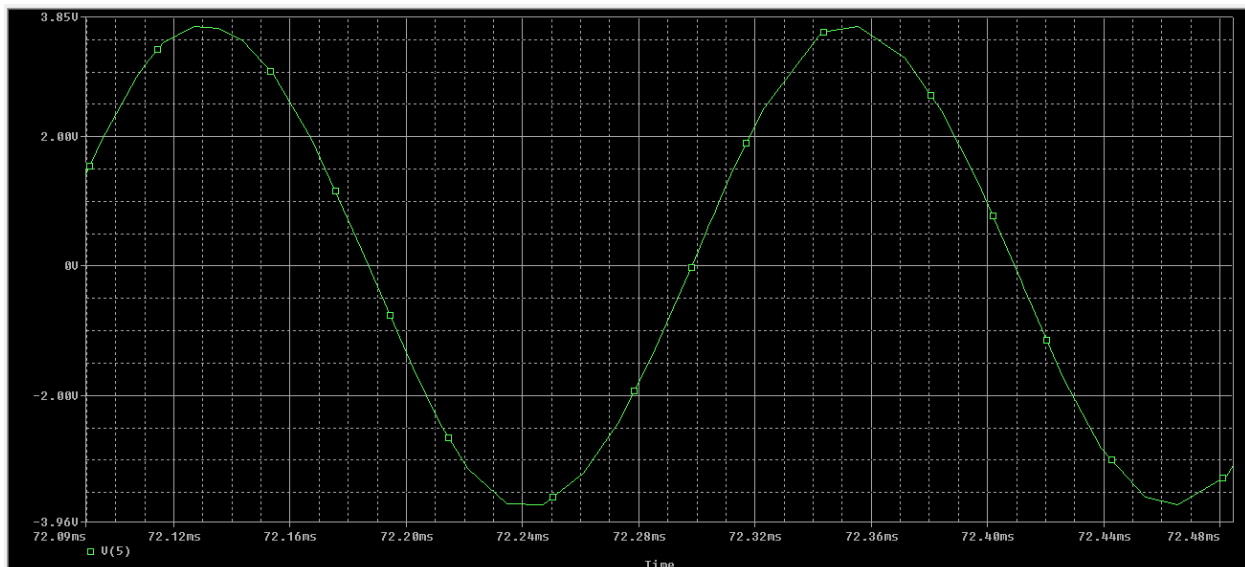
$$f = \frac{1}{2\pi\sqrt{LC}} = 5032.93\text{ Hz}$$

Result : no oscillation

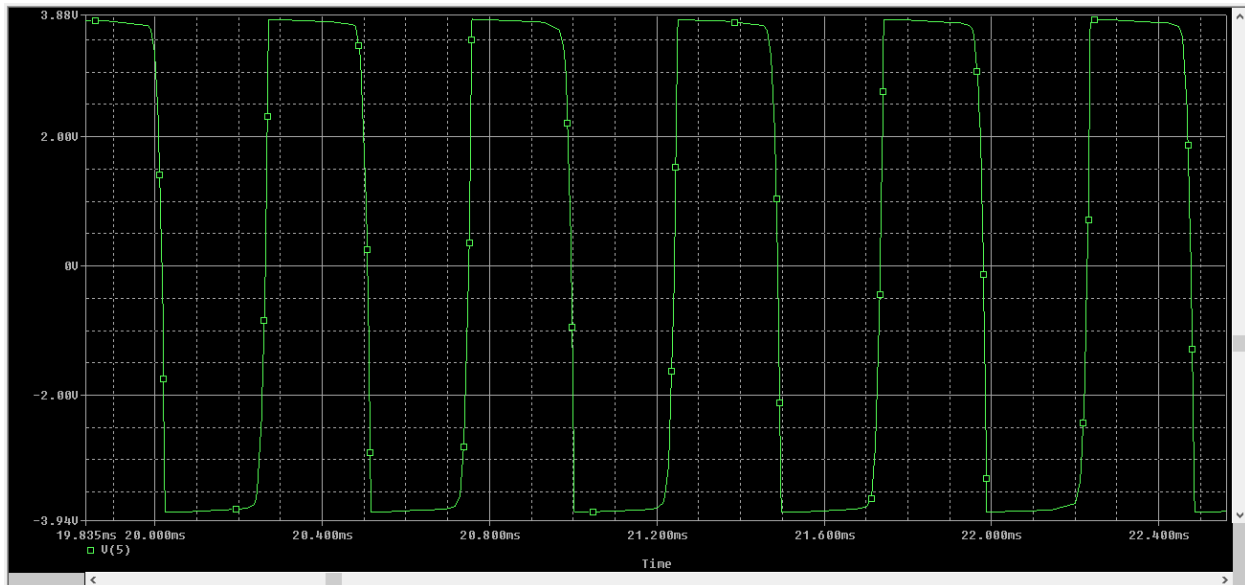
$\therefore R_2$ increased to $10.001\text{ k}\Omega$

Result : oscillation at 4464.3 Hz

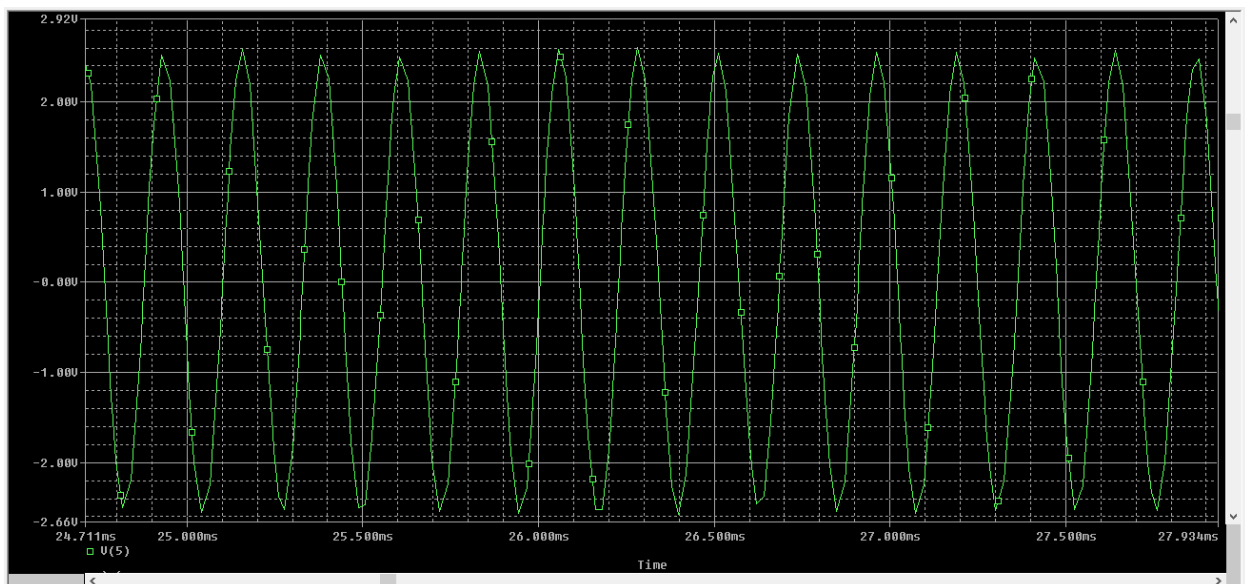
Higher gain \rightarrow increased distortion to a square wave



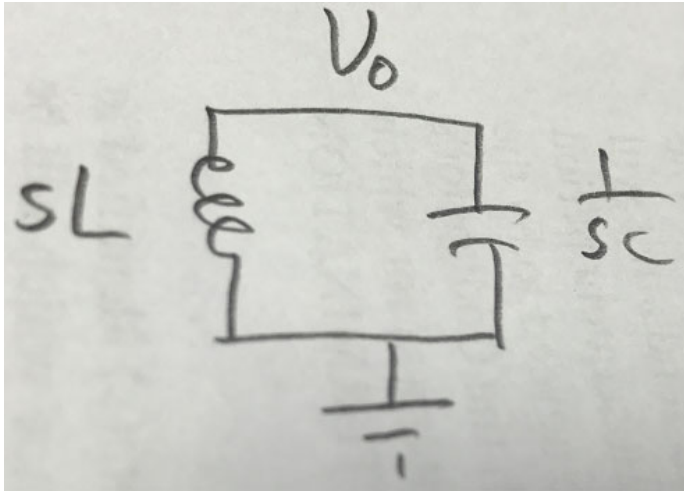
Increasing R_2 from 10.001 k Ω to 10.101 k Ω increased the loop gain, resulting in this square wave:



An AGC was added by replacing R_2 with a pot and adding two Schottky diodes between the wiper and node 5. Results shows that it basically works, with some distortion and variation in the amplitude:



The LC tank circuit:



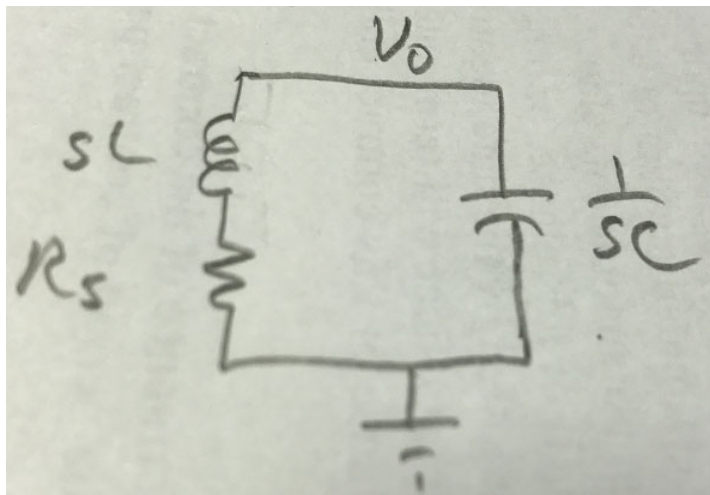
$$V_o \left(s^2 + \frac{1}{LC} \right) = 0$$

An initial condition on the inductor, or on the capacitor, or on both, will result in sustained oscillation with

$$\omega_o = \frac{1}{\sqrt{LC}}$$

This LC tank is lossless. All real LC tanks will have losses (energy dissipating mechanisms).

Consider the LC tank below:



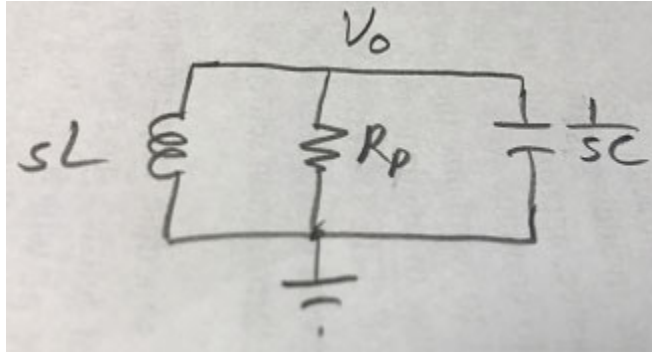
$$V_o \left(\frac{1}{sL + R_s} + sC \right) = 0$$

$$V_o (1 + s^2 LC + sR_s C) = 0$$

$$V_o \left(s^2 + s \frac{R_s}{L} + \frac{1}{LC} \right) = 0$$

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{and} \quad Q = \frac{1}{R_s} \sqrt{\frac{L}{C}}$$

R_s represents the line losses in the inductor and the traces. Sometimes, however, it may be more convenient to represent the losses like this:



$$V_o \left(\frac{1}{sL} + \frac{1}{R_p} + sC \right) = 0$$

$$V_o \left(\frac{1}{LC} + s \frac{1}{CR_p} + s^2 \right) = 0$$

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{and} \quad Q = R_p \sqrt{\frac{C}{L}}$$

$$\text{Therefore } R_p R_s = \frac{L}{C} \quad \text{and} \quad R_p = \frac{L}{R_s C}$$

R_p can represent the line losses in the inductor and the traces. It can also represent dielectric losses in the capacitor, and any load the tank circuit is driving. Observe that for a lossless system $R_s \rightarrow 0 \, \Omega$ and $R_p \rightarrow \infty \, \Omega$.