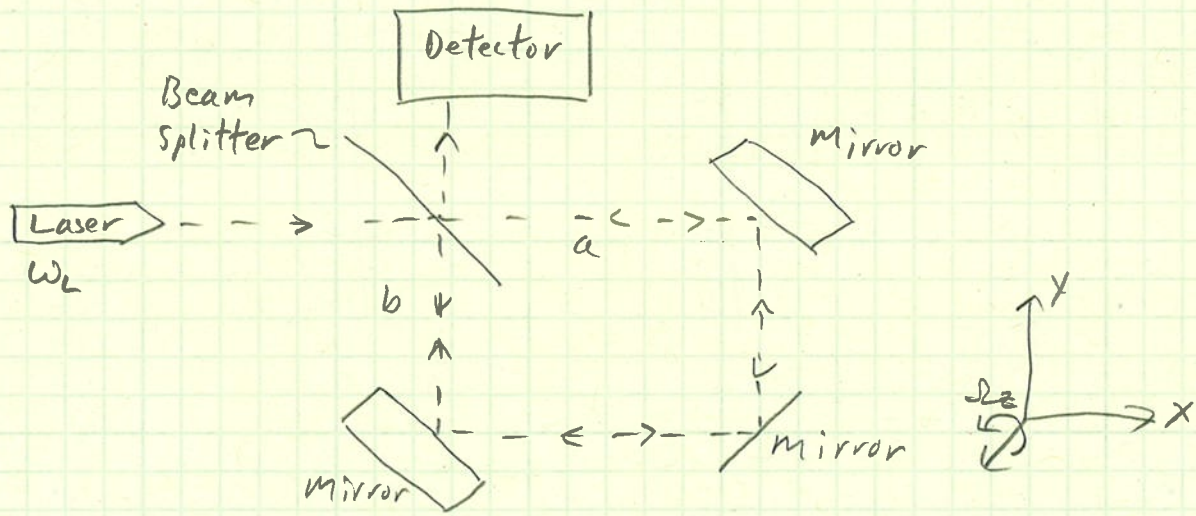
1. Optical Gyroscopes

a. Sagnac Effect

→ discovered by French scientist George Sagnac in 1913

→ Consider the Sagnac Inteferometer:



① Laser light is split so that part of it goes along path a and part along path b

② The light recombines at the beam splitter and the sum goes to the Detector

③ Without rotation ($\Omega_z = 0$), the two light beams recombine in phase → same path length

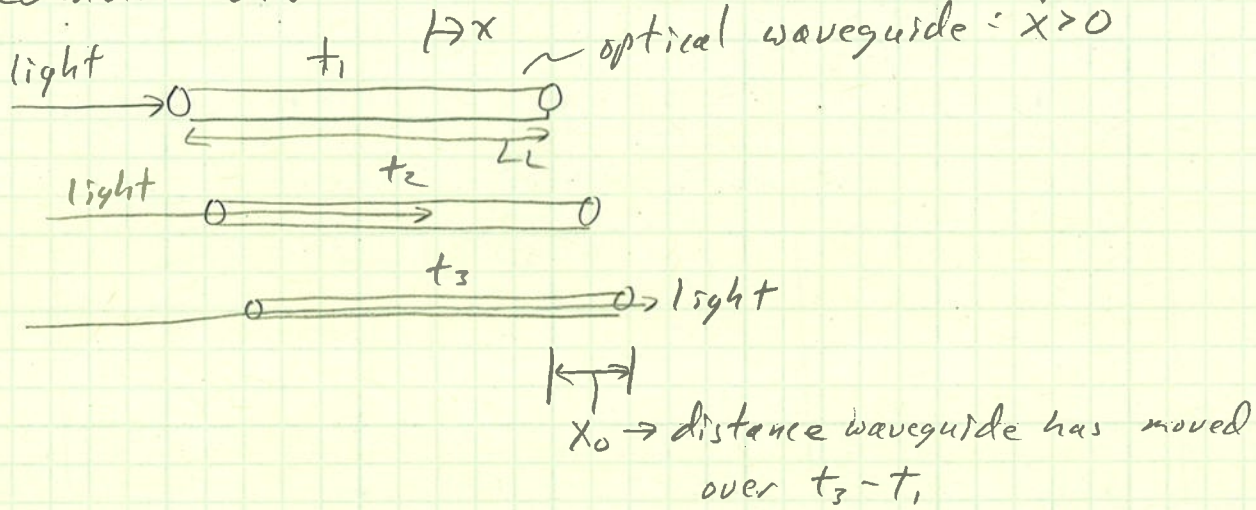
④ If $\Omega_z \neq 0$, then the two light beams do <u>not</u> have the same path lengths

∴ the recombine out of phase

∴ this is obserbed through the detector through an interference pattern

Why?

Consider this:

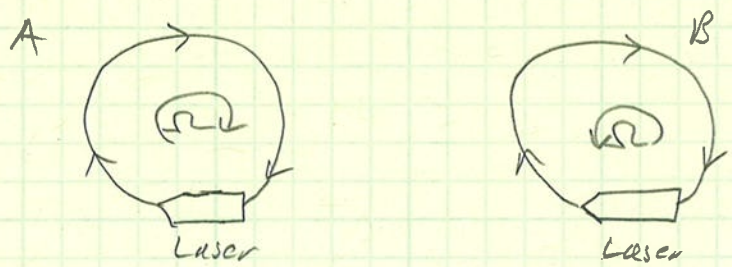


speed of light is constant: $c = 3 \times 10^8$ m/s

→ so above: waveguide is length L

→ but since it moves x_0, light must travel $L + x_0$ distance to go through it

∴ consider 2 rotating optical waveguides



$s = r\theta$: $\dot{s} = r\Omega$: $s = \dot{s}t = r\Omega t$ for Ω = constant

for L long optical waveguide,

the effective pathlength in A is longer than L

the effective path length in B is shorter than L

For a ring interferometer

$$\Delta\phi_R = \frac{4\omega}{c^2}\vec{A}\cdot\vec{\Omega} \quad \text{in rad}$$

Known as the Sagnac Equation

$\Delta\phi_R \equiv$ phase shift caused by the rotation vector $\vec{\Omega}$

$\vec{A} \equiv$ area vector for the interferometer

$\omega \equiv$ frequency of the light

$c \equiv$ speed of light

<u>Proof</u>

$$s = r\theta \rightarrow \dot{s} = r\dot{\theta} = r\Omega$$

$$s_n = \dot{s}t = r\Omega t$$

One side : $ct_1 = L - r\Omega t_1 \rightarrow t_1 = \dfrac{L}{c + r\Omega}$

Other side : $ct_2 = L + r\Omega t_2 \rightarrow t_2 = \dfrac{L}{c - r\Omega}$

$$\therefore \Delta t = t_2 - t_1 = L\left(\frac{1}{c - r\Omega} - \frac{1}{c + r\Omega}\right) =$$

$$= L\left(\frac{2r\Omega}{c^2 + r^2\Omega^2}\right)$$

$c^2 >> r^2\Omega^2 \rightarrow \therefore c^2 + r^2\Omega^2 \approx c^2$

$$\therefore \Delta t \approx \frac{2Lr\Omega}{c^2}$$

$L = 2\pi r$

$$\Delta\phi_R = \frac{2\pi\Delta t}{T} = 2\pi f\Delta t = \omega\Delta t$$

$$\Delta\phi_R = \frac{2\omega(2\pi)r^2\Omega}{c^2}$$

$\pi r^2 = A$

$$\therefore \Delta\phi_R = \frac{4\omega A\Omega}{c^2}$$

At the detector, the interference pattern is:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi_R) \qquad (W/cm^2)$$

where I_1 + I_2 are beam intensities

b. <u>Ring Laser Gyros</u> (RLG)

→ They use a ring shaped optical resonant cavity

→ optical path length = integer number of laser emission wavelengths

→ counter rotating waves interfere and form a standing wave pattern

→ as device rotates, standing wave pattern stays fixed in inertial space as the cavity rotates

∴ photodetector rotates with cavity and measures interference pattern → as it passes maximums and minimum

→ rotation info determined by counting this beat frequency, not directly detecting interference pattern

c. <u>Fiber-Optic Gyroscope</u> (FOG)

① <u>Resonant FOG</u>

→ similar to RLG but with fiber optic cable in length equal to integer number of laser wavelengths

② <u>Interferometric FOG</u>

→ multi-turn coiled fiber optic cable for counter rotating laser beams

→ Detector measures interference pattern directy by Sagnac effect