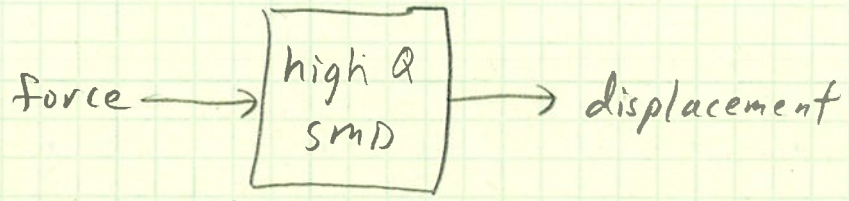
1. MEMS Resonators

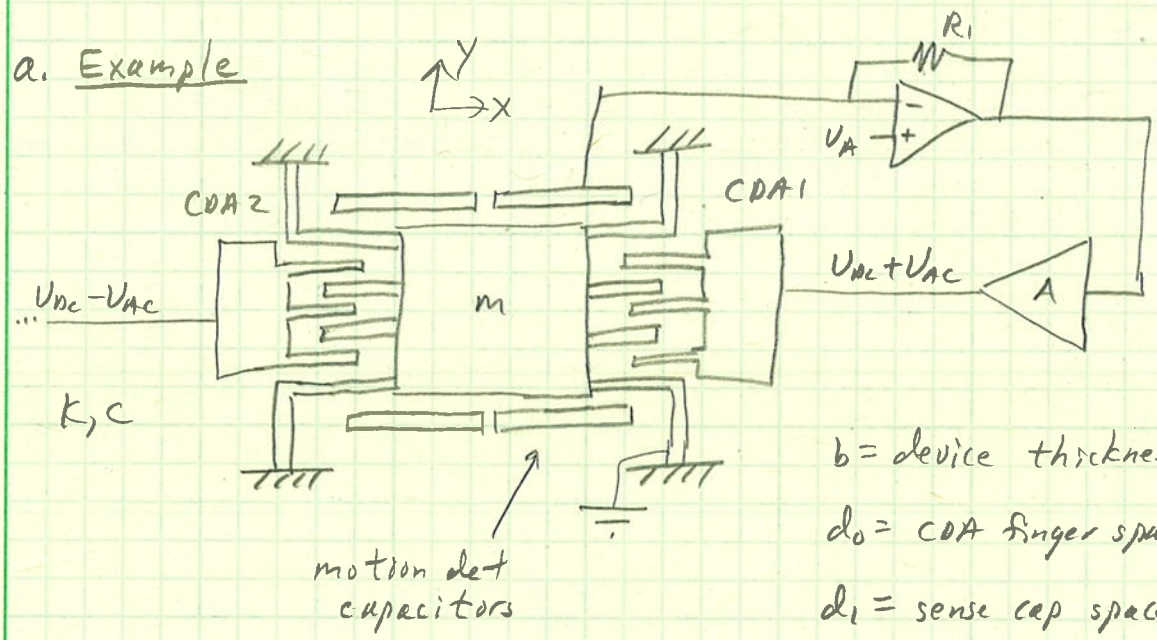


at ω_d: displacement lags force by $90°$

$\therefore$ To make it self-resonate at ω_d, add $90°$ of additional phase delay and appropriate gain such that $A\beta = 1\angle 0°$ {positive feedback model} or

$A\beta = 1\angle -180°$ {negative feedback model}

a. Example



b = device thickness

d_0 = CDA finger space width

d_1 = sense cap space width

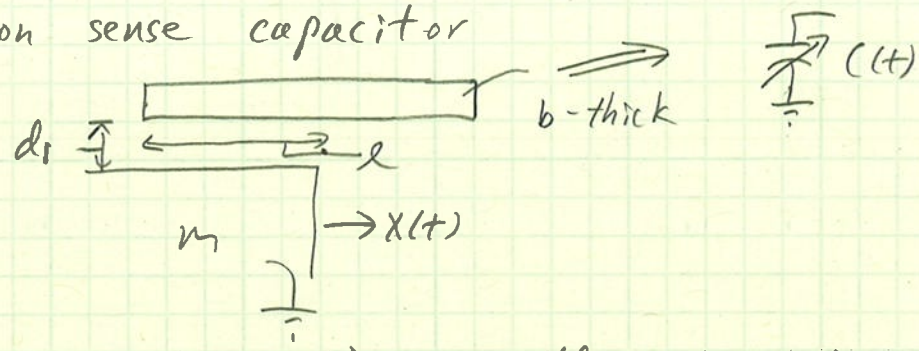
l = sense cap nominal overlap

$$\text{Net force} = \frac{4n\beta\varepsilon_0\varepsilon_r b}{d_0}(V_{DC}V_{AC})\sin(\omega_d t) = K_1 V_{AC}\sin(\omega_d t) = A_x\sin(\omega_d t)$$

$$\text{where } K = \frac{4n\beta\varepsilon_0\varepsilon_r b V_{DC}}{d_0}$$

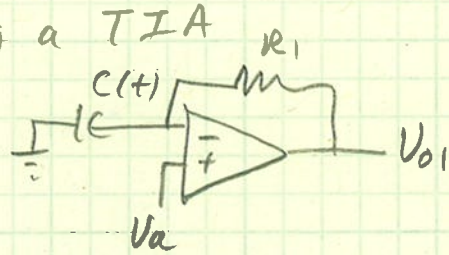
$$\therefore x(t) = -\frac{A_x}{c\omega_d}\cos(\omega_d t) = -\frac{K_1 V_{AC}}{c\omega_d}\cos(\omega_d t)$$

Motion sense capacitor



$$C(t) = \frac{\varepsilon_0 \varepsilon_r b}{d_1}\left(\ell + x(t)\right) = \frac{\varepsilon_0 \varepsilon_r b}{d_1}\left(\ell - \frac{K_1 V_{AC}}{c\,\omega_d}\cos(\omega_d t)\right)$$

Using a TIA



$V_A \equiv$ DC voltage

$$V_{01} = V_a + V_a R_1 \frac{dC}{dt}$$

$$= V_a + \frac{V_a R_1 \varepsilon_0 \varepsilon_r b}{d_1}\left(0 + \frac{K_1 V_{AC}}{c}\sin(\omega_d t)\right)$$

$$= V_a + \frac{V_a R_1 \varepsilon_0 \varepsilon_r b K_1 V_{AC}}{d_1 c}\sin(\omega_d t)$$

V_{01} is in phase with Net force $\rightarrow \angle AB = 0^\circ$ at $\omega = \omega_d$

$\therefore$ remove V_a and amplify with a gain, G

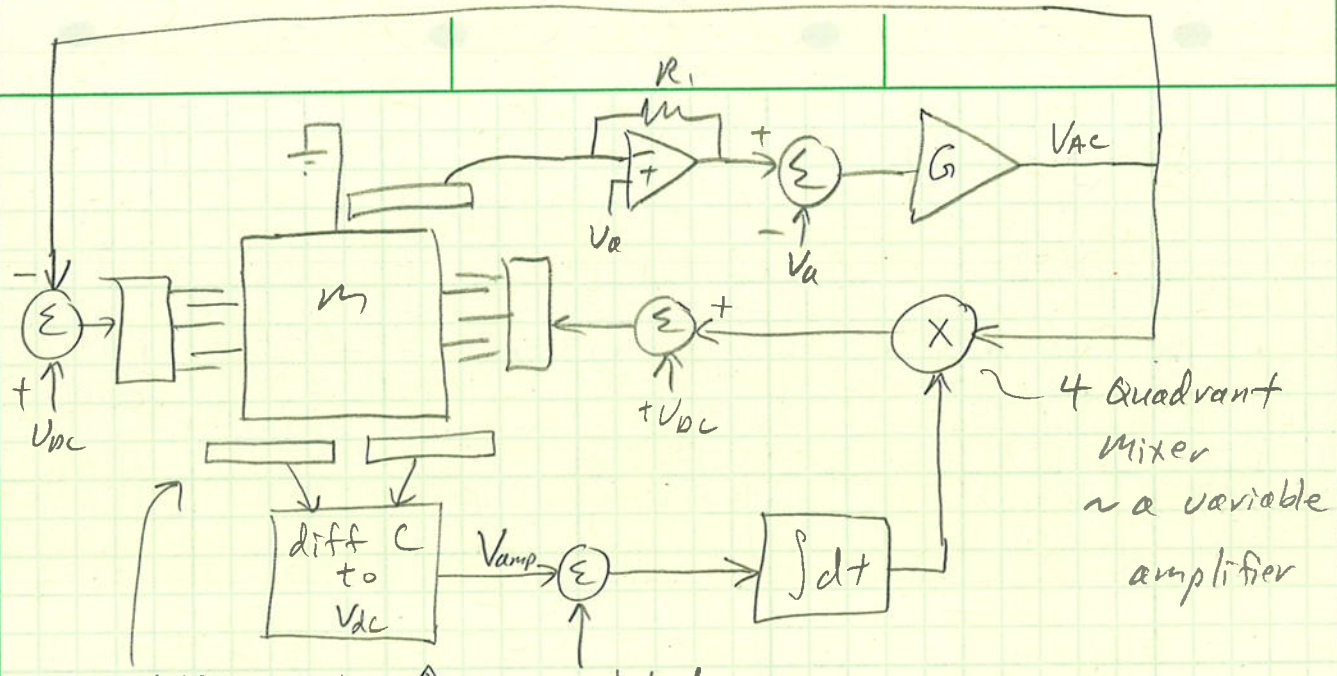
$$\therefore |AB| = 1 = \frac{G V_a R \varepsilon_0 \varepsilon_r b\, 4n\beta\, \varepsilon_0 \varepsilon_r b V_{DC}}{d_1 c d_0}$$

$$\therefore G = \frac{d_1 c d_0}{V_a \varepsilon_0^2 \varepsilon_r^2 b^2 4n\beta V_{DC} R_1} = G_{optimum}$$

if $G < G_{optimum} \rightarrow$ motion dies out

if $G > G_{optimum} \rightarrow$ mass crashes into frame and probably breaks or motion is distorted

$\therefore$ AGC is needed



amplitude ref.

Use differential cap structure to measure motion amplitude

AC voltage divider and lock-in amplifier with LPF

b. <u>Other Configurations + Notes</u>

① RC stage(s) used to produce 90° phase delay instead of TIA → differentiator {noisy}

② Piezoresistors {Wheatstone Bridge} used to detect displacement instead of capacitors

③ MEMS resonators → relatively low frequency, could possibly perform signal processing steps digitally using A/D, microcontroller, D/A and simple fixed gain amplifier

④ Non-idealities in components (MEMS device, op amps, etc.) should be considered

⑤ Check stability of the control loops