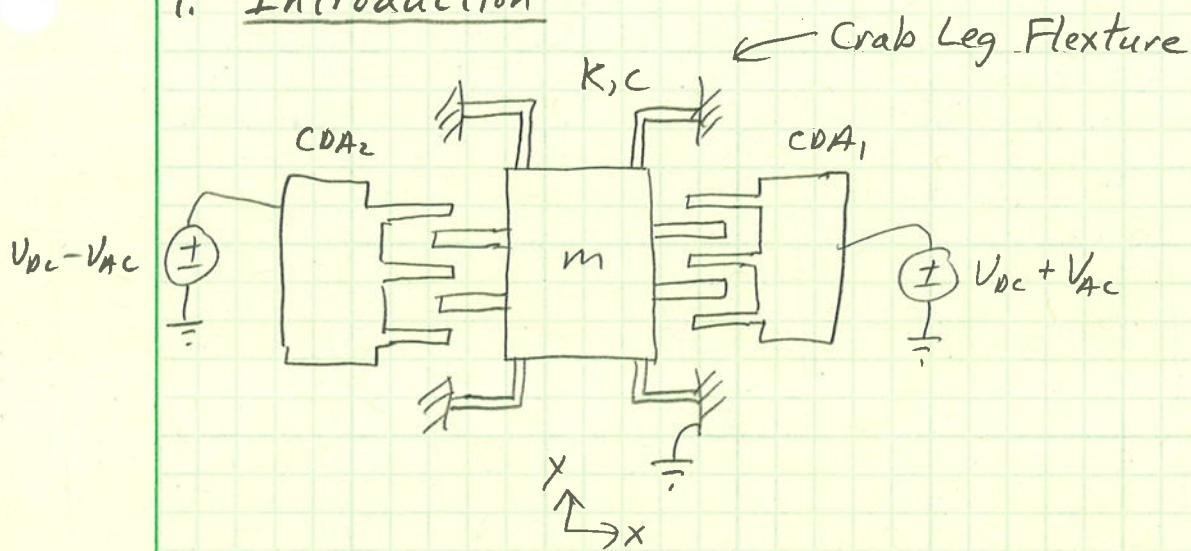


MEMS Resonators1. Introduction

mass can move along y-axis: $m\ddot{x} + c\dot{x} + kx = Ax \sin(\omega_d t)$

$$\begin{aligned}
 \text{solution is } x(t) &= -\frac{Ax}{c\omega_d} \cos(\omega_d t) \\
 &= -\frac{AxQ}{m\omega_d^2} \cos(\omega_d t) \\
 &= \frac{AxQ}{m\omega_d^2} \sin(\omega_d t - 90^\circ) \\
 &= \frac{AxQ}{K} \sin(\omega_d t - 90^\circ)
 \end{aligned}$$

At the resonant frequency:

(1) $x(t)$ lags $F(t)$ by 90°

(2) amplitude of $x(t) \propto Q$

(3) amplitude of $x(t) = \frac{Ax}{c} \sqrt{\frac{m}{K}}$

a. inc with $m \uparrow$

b. inc with $c \downarrow$

c. inc with $K \downarrow$

2. Uses of MEMS Resonators

a. MEMS gyroscope drive axis mechanism

b. Detection sensor for added mass

$$X_0 = \frac{Ax}{C} \sqrt{\frac{m}{k}}$$

example \rightarrow coat m structure with biological monolayer that only a specific species of bacteria will bond to, such as E. Coli

c. Pressure sensor for low pressure applications

- ① for pressures < few hundred Pa, $C \propto P$
- ② for "large" surface area m structures, mass-loading effect $\propto P$

d. In a sensor where the detection mechanism results in a strain in the suspension system elements affecting a change in the system spring constant

e. As a sensor to gauge the effects of aging on a mems device

f. As a sensor to evaluate microfabrication tolerances or processes

3. Open Loop Operation

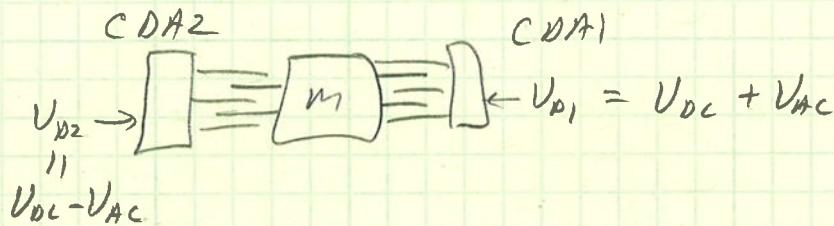
- Relatively simple

a. Combdrive Actuator : CDA

$$F_{\text{CDA}} = \frac{n \beta \epsilon_0 \epsilon_r b V_0^2}{d} \rightarrow \text{proportional to } V_0^2$$

Force is independent of displacement

① Linearization

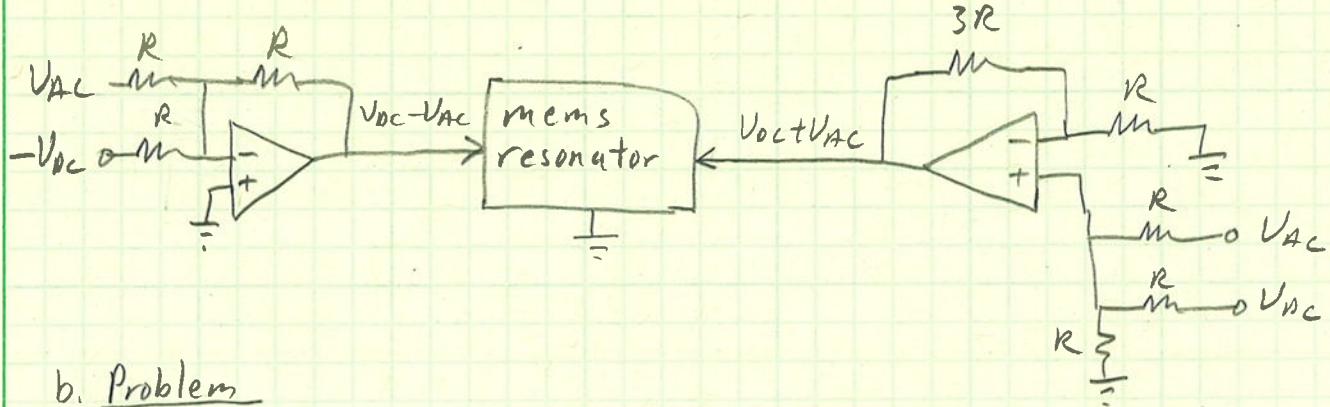


$$F_1 = \alpha (V_{0c}^2 + 2V_{0c}V_{Ac} + V_{Ac}^2)$$

$$F_2 = \alpha (V_{0c}^2 - 2V_{0c}V_{Ac} + V_{Ac}^2)$$

$$\text{net force} = F_1 - F_2 = 4\alpha V_{0c}V_{Ac} = \frac{4n\beta\epsilon_0\epsilon_r b(V_{0c}V_{Ac})}{d}$$

② Example Drive Circuit

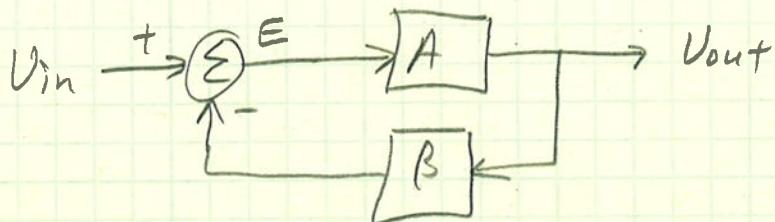


b. Problem

- ① very difficult to produce $A \sin(\omega t)$ at $\omega = \omega_d$
- ② ω_d changes with application (sensing) or with operating environment or aging

4. Oscillators

Consider a feedback system:



$$V_{out} = EA \quad (1)$$

$$E = V_{in} - V_{out}B \quad (2)$$

$$(2) \rightarrow (1) : V_{out} = A(V_{in} - V_{out}B)$$

$$V_{out}(1 + AB) = AV_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A}{1 + AB}$$

characteristic equation: $1 + AB = 0 \rightarrow AB = -1$

For oscillation: $AB = -1 = 1 \angle -180^\circ \rightarrow \text{Barkhausen Criterion}$

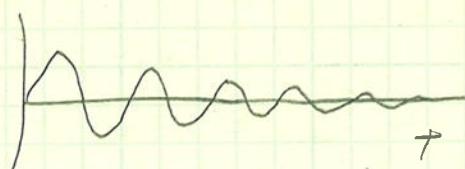
i.e. for oscillation: need $|AB| = 1$ and 180° phase shift

(a) Conditions

① $\phi = 180^\circ$ but $|AB| < 1$

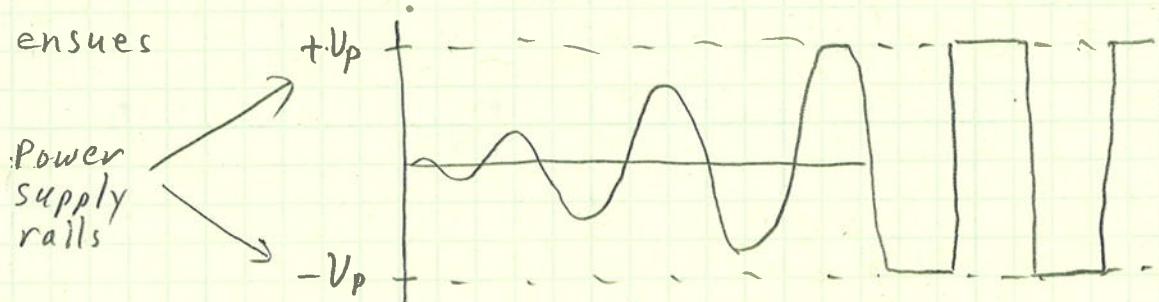
→ either no oscillation

→ or oscillation dies out



② $\phi = 180^\circ$ but $|AB| > 1$

→ amplitude grows until nonlinear, severe distortion ensues



Relaxation Oscillator