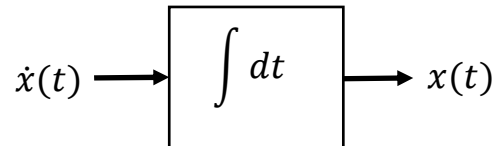


Tuesday, 1/17/23

### Simulating Dynamic Systems in MATLAB Simulink

Example: given  $A\ddot{x} + B\dot{x} + Cx = f(t)$  (1) modelling a second order linear dynamic system

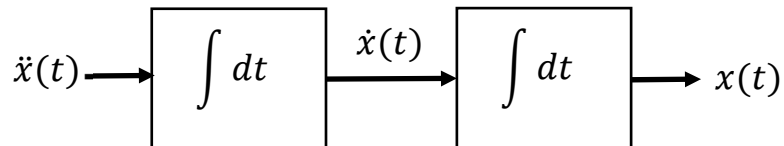
Realize that  $x = \int \dot{x} dt$ , which can be represented pictorially as:



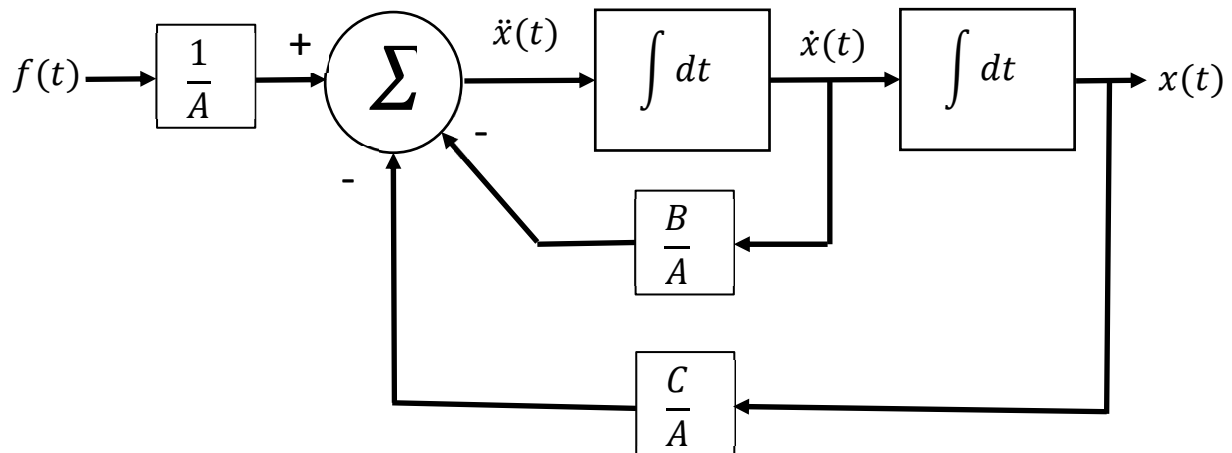
Rearrange (1) so that:  $\ddot{x} = \frac{1}{A}f(t) - \frac{B}{A}\dot{x} - \frac{C}{A}x$  (2)

This form is very easy to implement in a simulation diagram using integrators

Begin with a chain of integrators representing all states:

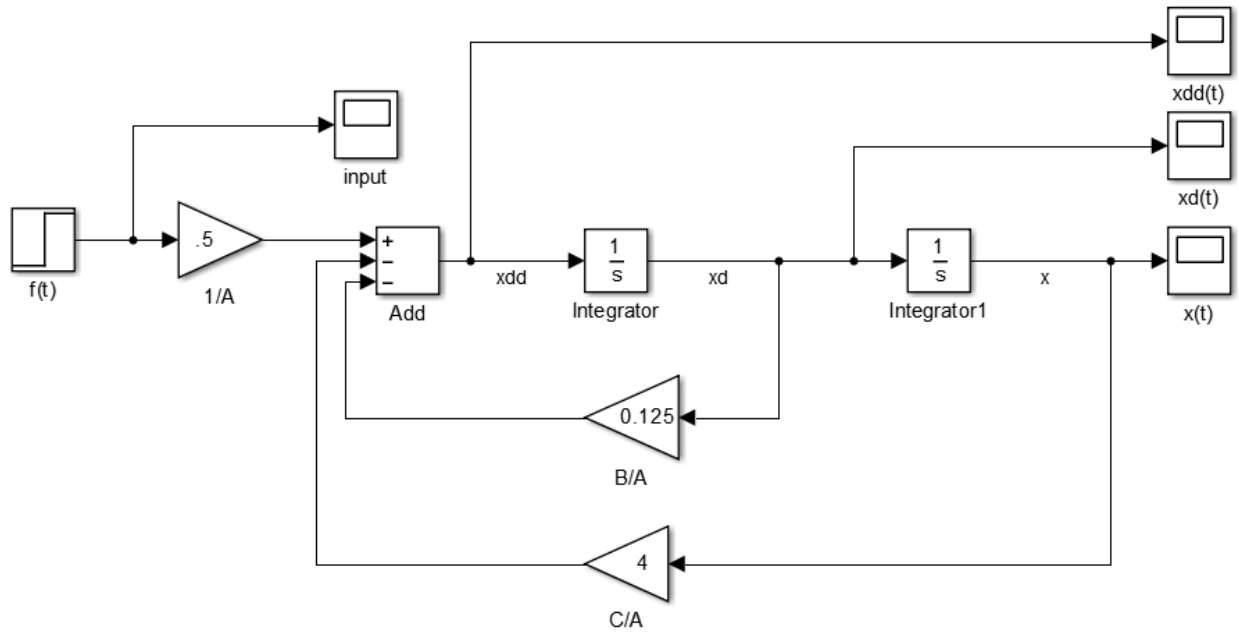


Then add a summing junction and feedback terms:

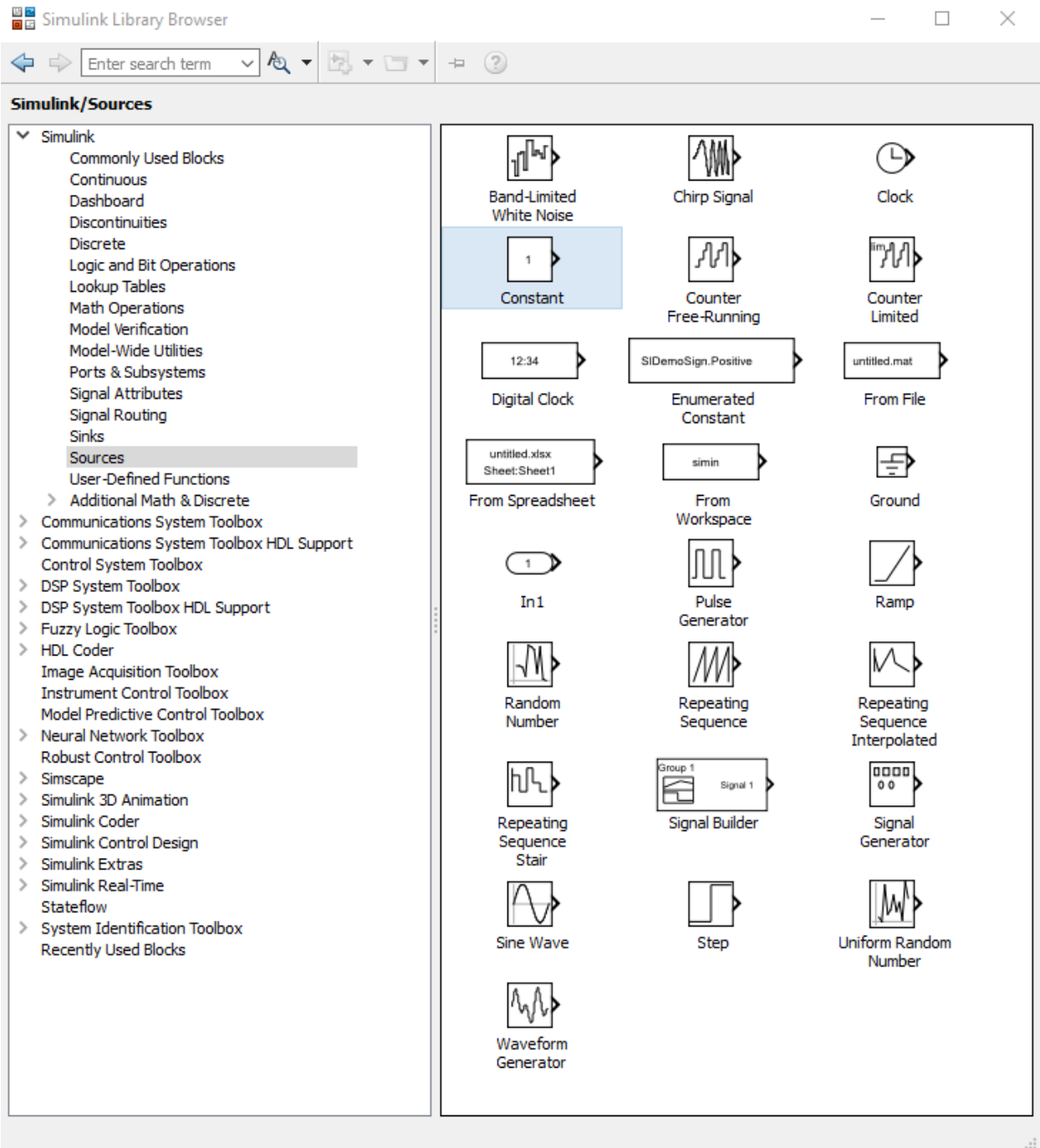


This can be built in MATLAB Simulink where the 1/s block is used for the integral block

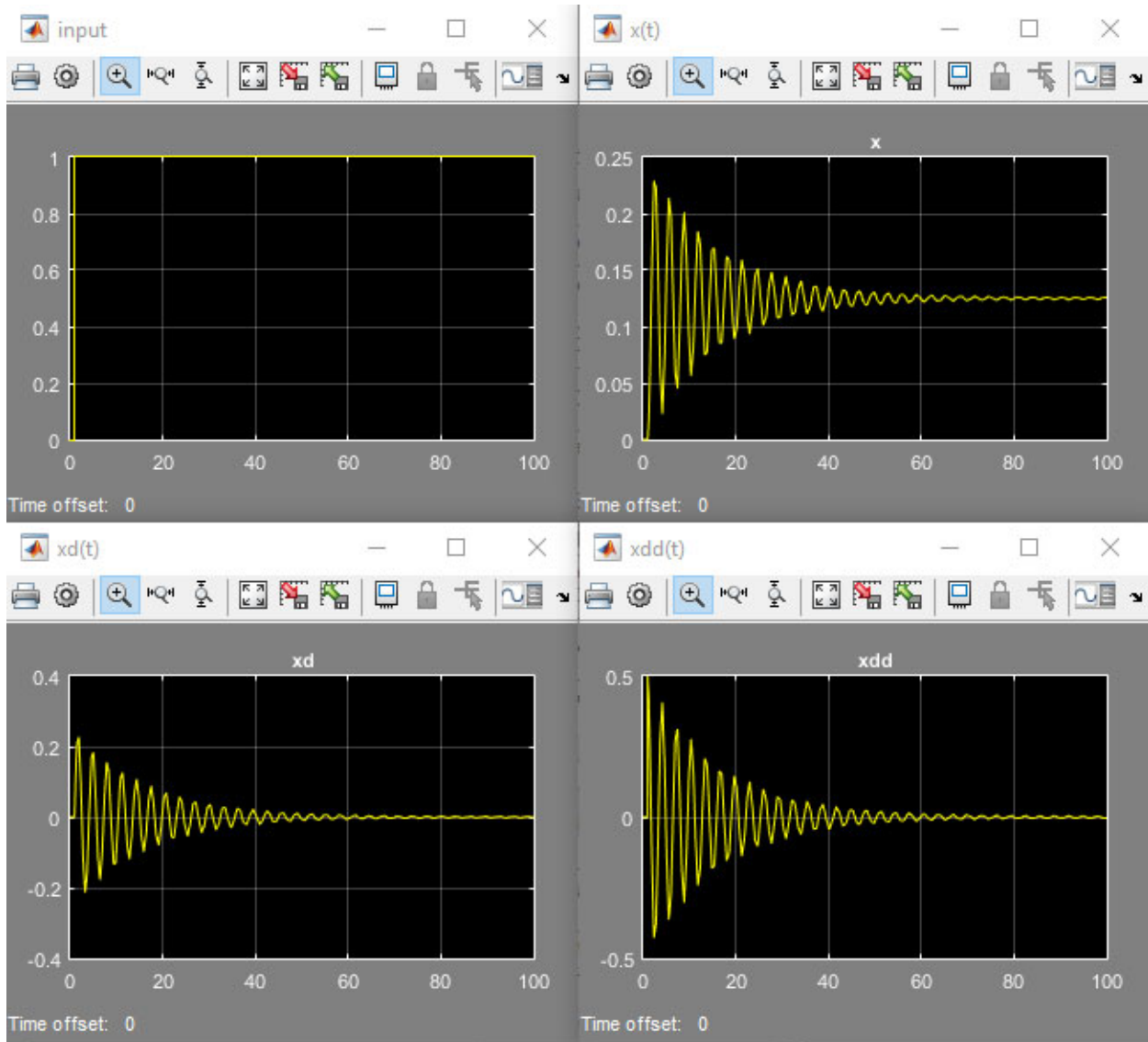
Example:  $2\ddot{x} + 0.25\dot{x} + 8x = u(t - 1)$



Simulink model of the simulation diagram

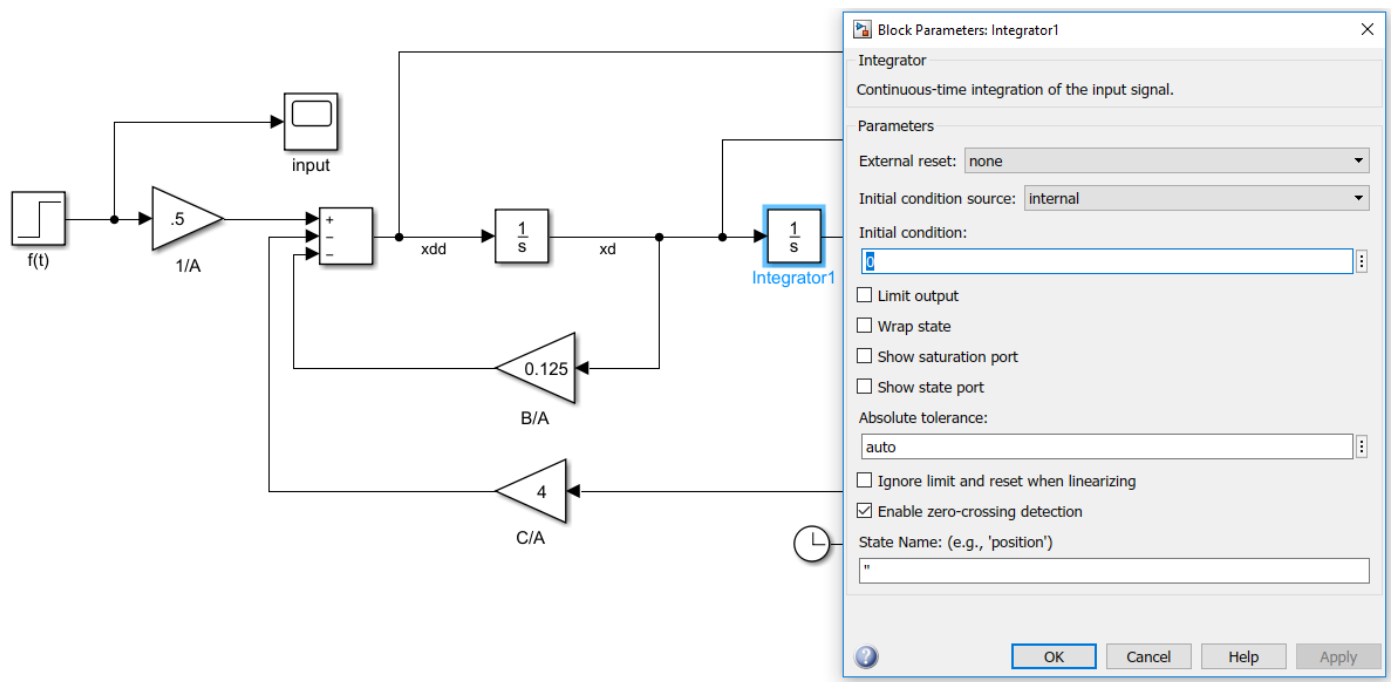


Simulink Library Browser

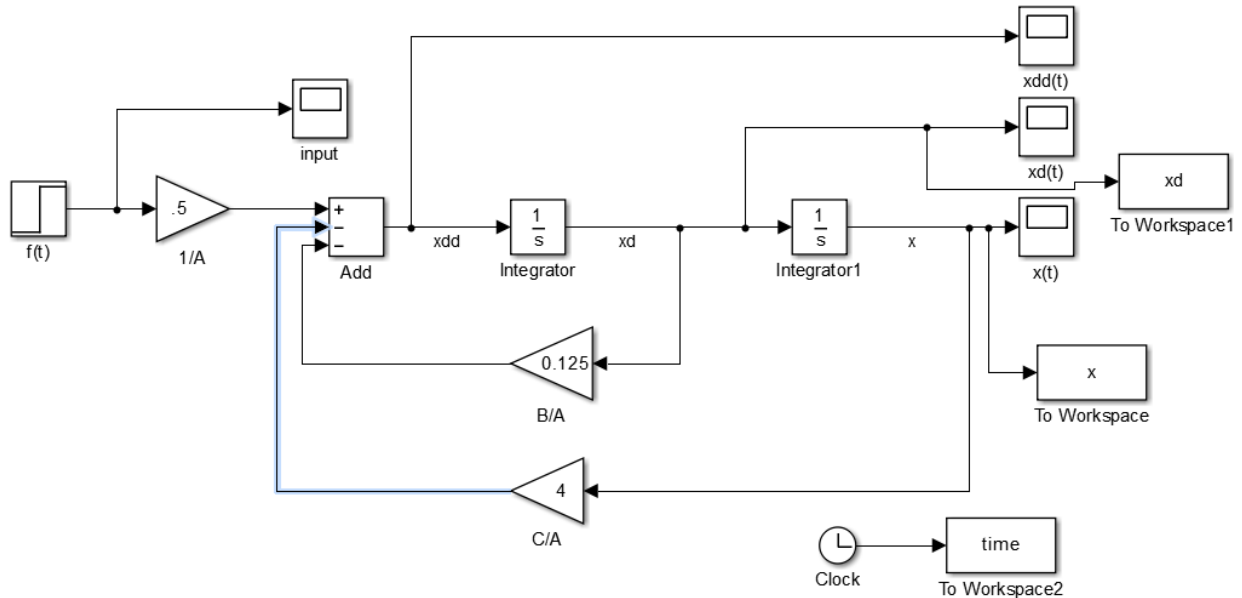


## Setting Initial Conditions:

Click on the integrator block and select the initial condition for the signal output by that integrator:

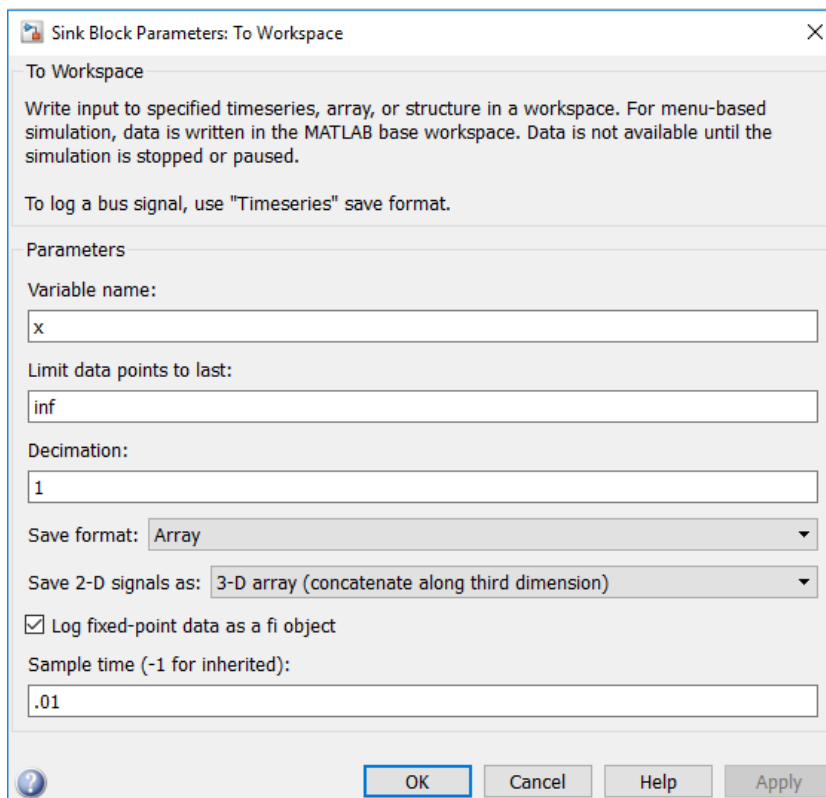


## Bringing Simulink Data into the MATLAB Workspace



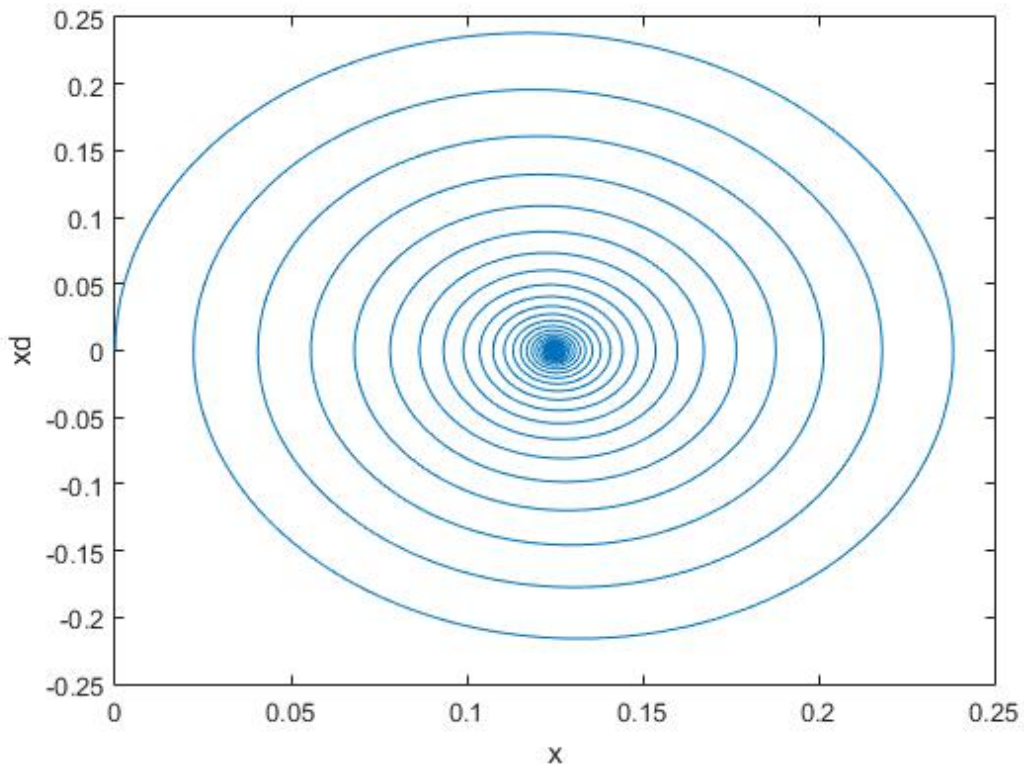
“To Workspace” blocks added to get data into the Matlab Workspace

Setting the parameters for each “To Workspace” block:



Then after running the simulation, you can process the Simulink data in the workspace or via an m-file.

Example: `plot(x,xd)`



This is a plot of  $\dot{x}$  vs.  $x$ . It is called a phase plot and is very useful in analyzing dynamical systems.

Hand out Homework 1:

## Review of Second Order Dynamic Systems

Consider systems of the form:  $A\ddot{x} + B\dot{x} + Cx = f(t)$

Example: mechanical spring-mass-damper system:  $m\ddot{x} + c\dot{x} + kx = f(t)$

Often convenient to analyze using Laplace Transforms:  $ms^2X(s) + csX(s) + kX(s) = F(s)$

Then  $X(s) = \frac{F(s)/m}{s^2 + s\frac{c}{m} + \frac{k}{m}} = \frac{F(s)/m}{s^2 + s2\zeta\omega_o + \omega_o^2} = \frac{F(s)/m}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$ , where:

$\omega_o$  is the natural frequency

$Q$  is the quality factor

$\zeta$  is the damping ratio

finally,  $x(t) = L^{-1} \left[ \frac{F(s)/m}{s^2 + s\frac{c}{m} + \frac{k}{m}} \right]$

$\zeta = 0$  or  $Q \rightarrow \infty$  : undamped system

$0 < \zeta < 1$  or  $Q \rightarrow \infty > Q > \frac{1}{2}$  : underdamped system

$\zeta = 1$  or  $Q = \frac{1}{2}$  : critically damped system

$\zeta > 1$  or  $Q < \frac{1}{2}$  : overdamped system

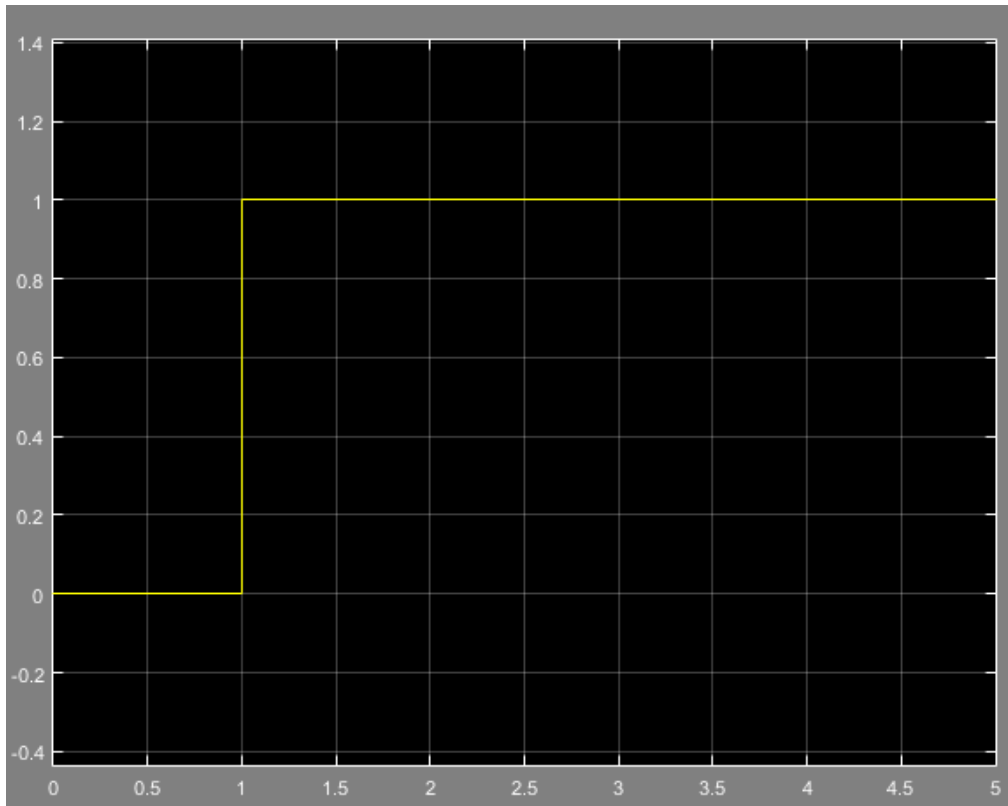
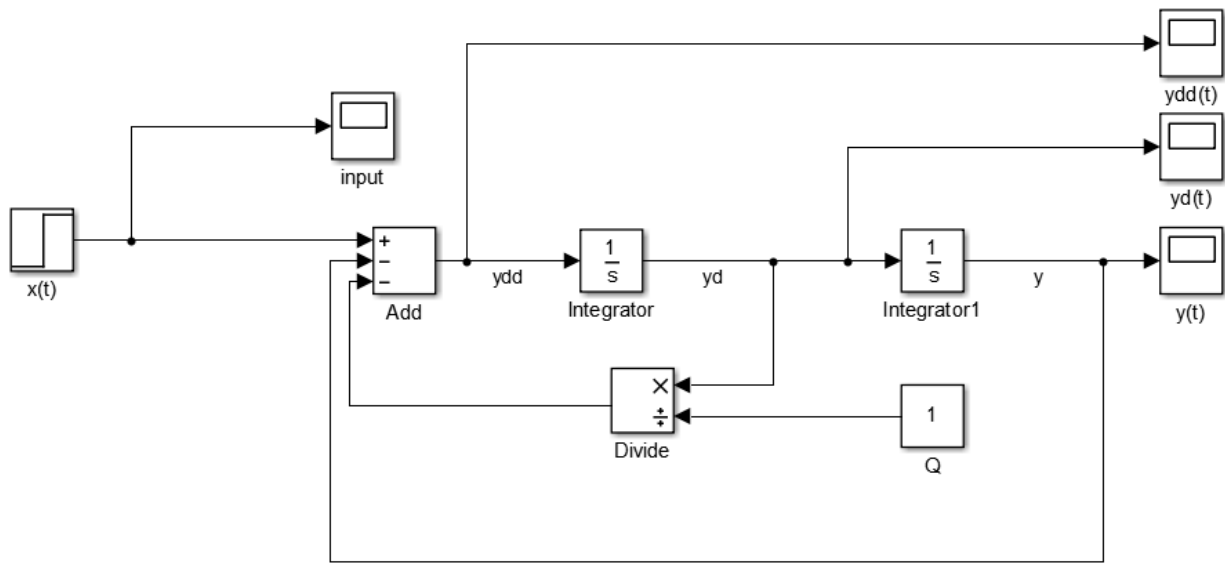
$\zeta = Q = 0.707$  : maximally flat response (no resonant peak in the frequency domain)

Example. Consider this system with  $\omega_o = 1$  rad/s:

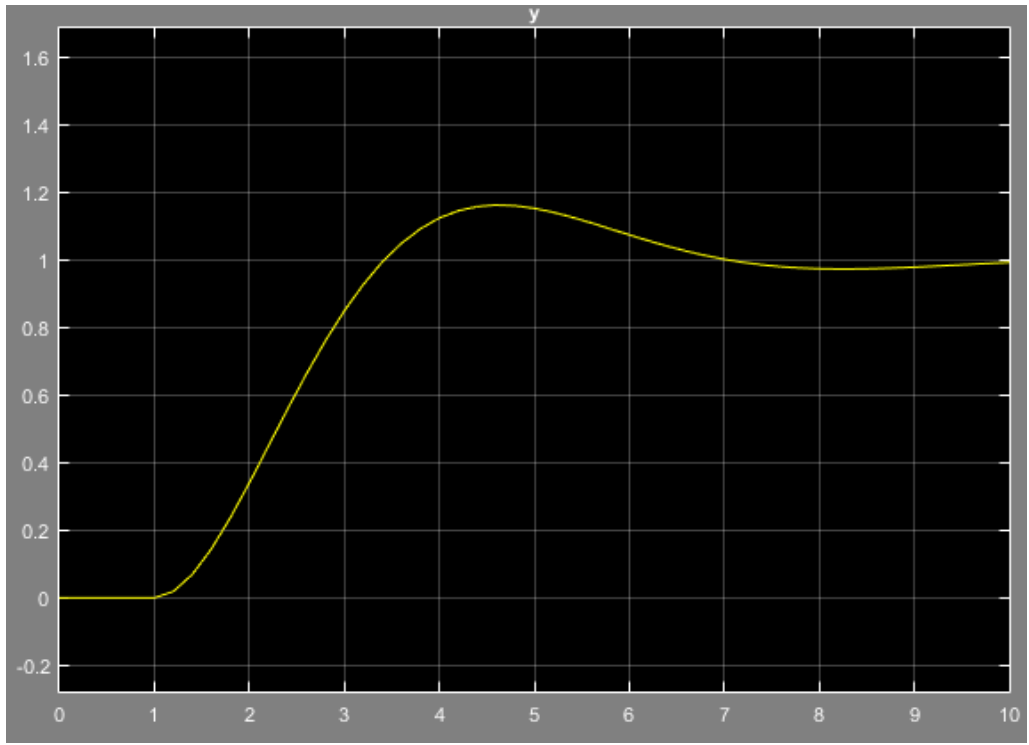
$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + s\frac{1}{Q} + 1}$$

Consider the Simulink model shown on next page with a step response:

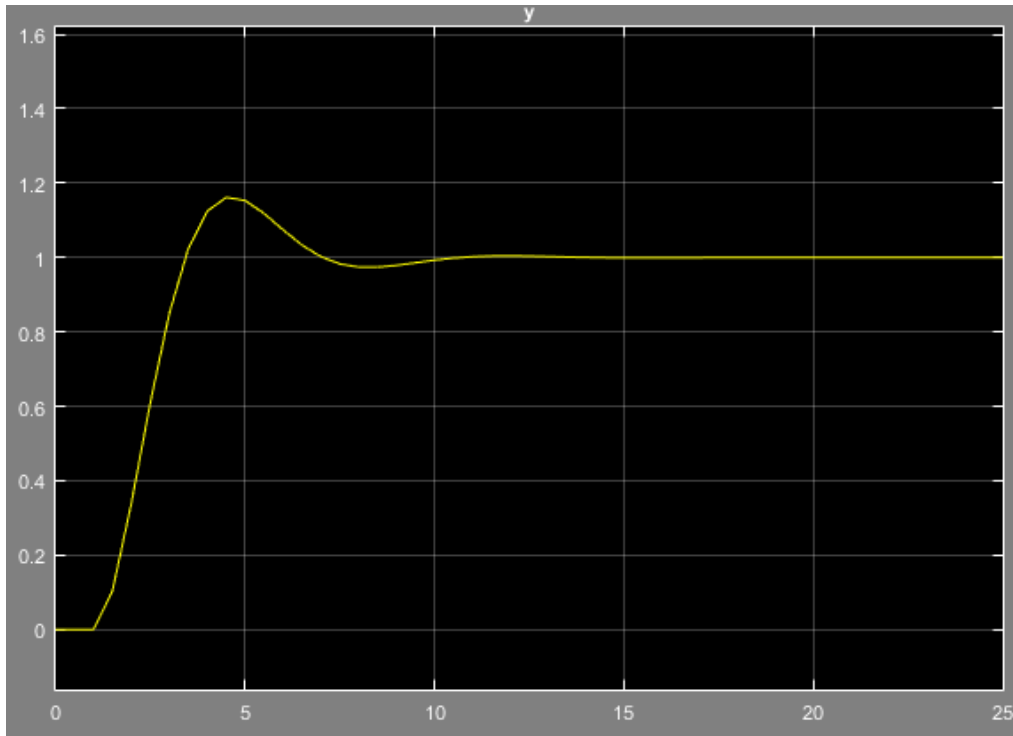




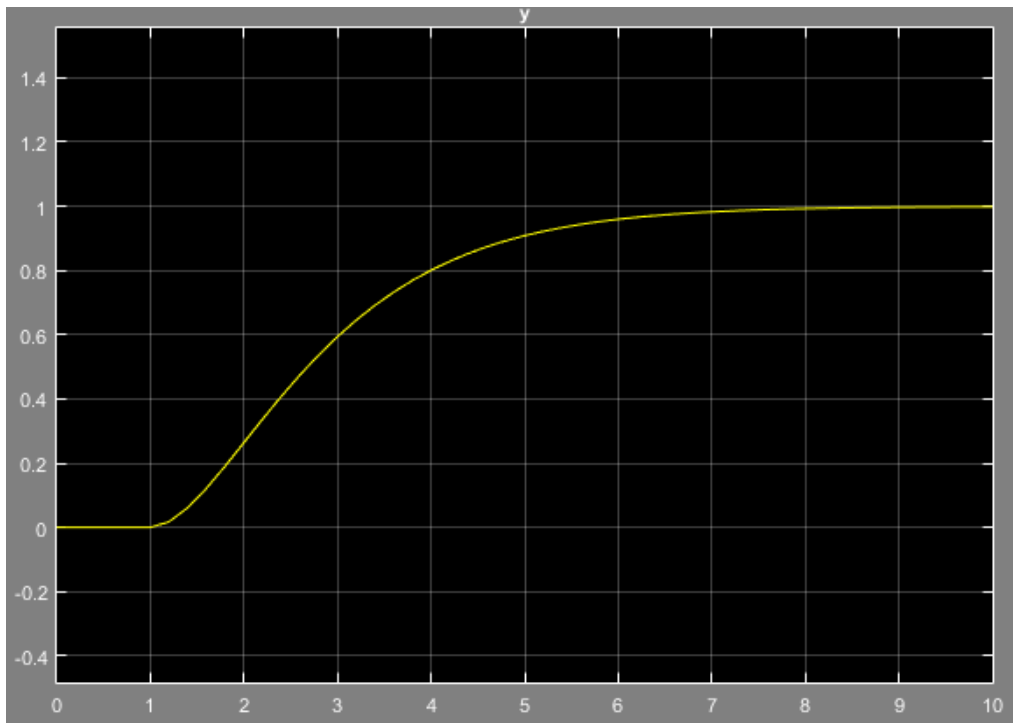
Input:  $x(t)$



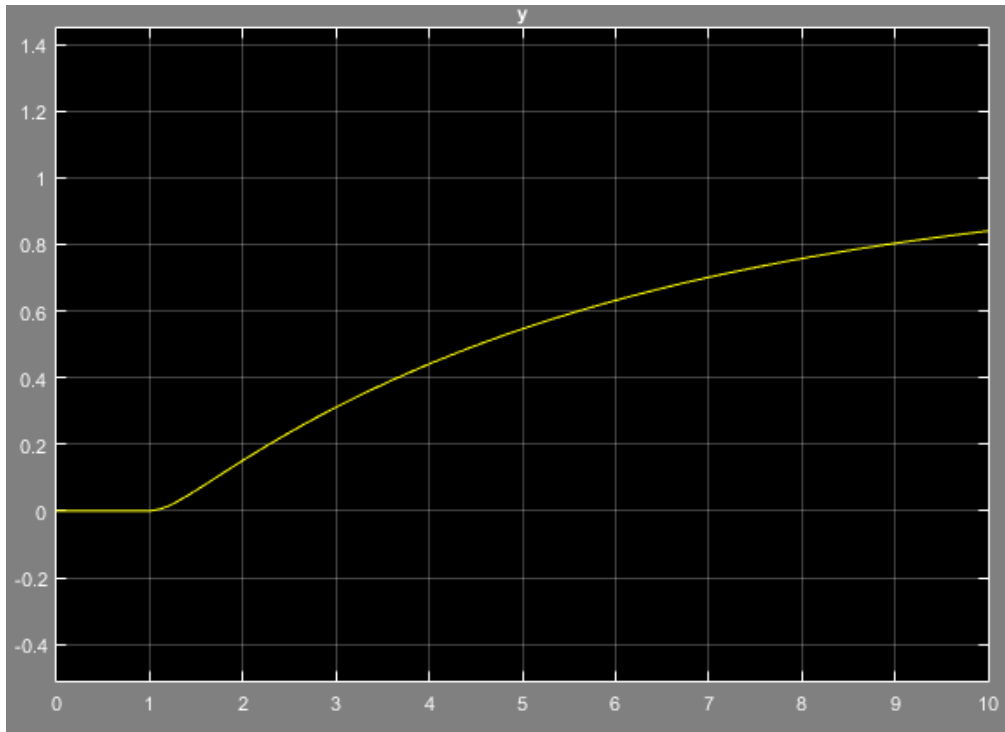
Output  $y(t)$  for  $Q = 1$ , underdamped. Observe the “ringing” at the resonant frequency.



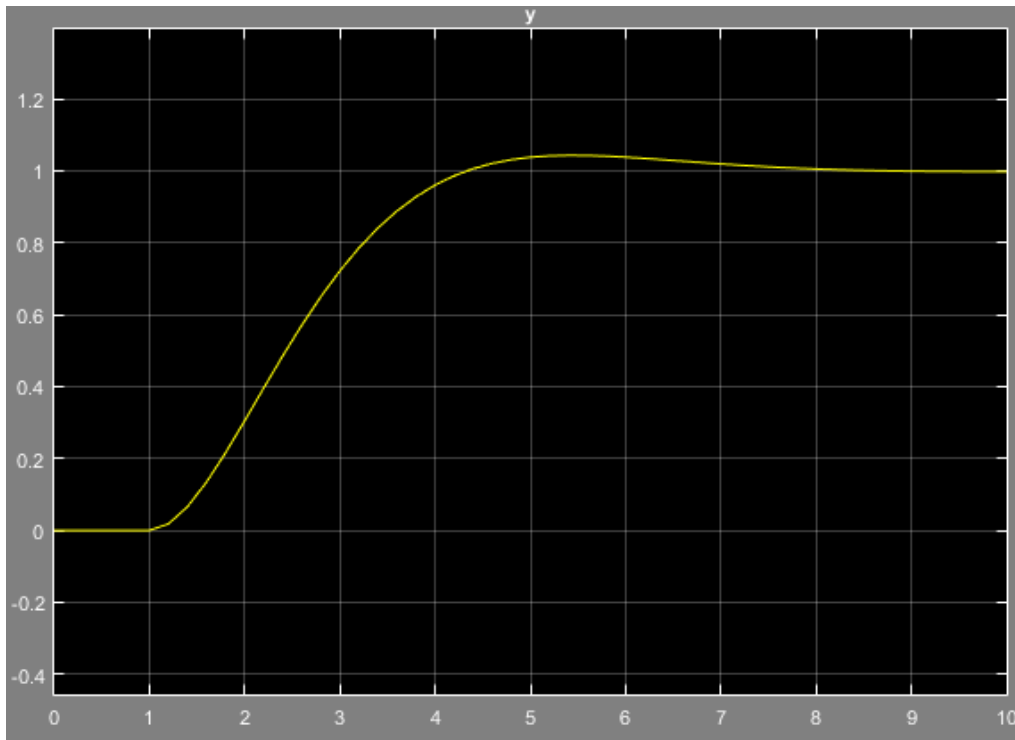
$y(t)$  for  $Q = 1$ , but response run for 25s to observe decaying ringing.



$y(t)$  for  $Q = 0.5$ , critically damped response.



$y(t)$  for  $Q = 0.2$ , overdamped response.



$y(t)$  for  $Q = 0.707$ , maximally flat response. Observe a slight overshoot with a reasonably fast response time.

## State Variable Modelling

Example:  $G(s) = \frac{Y(s)}{F(s)} = \frac{1}{ms^2+cs+k}$ , a second order system

Obviously:  $\ddot{y}(t) = \frac{1}{m}f(t) - \frac{k}{m}y(t) - \frac{c}{m}\dot{y}(t)$  (1)

Define the state variables:

Let  $x_1(t) = y(t)$  and  $x_2(t) = \dot{y}(t)$

Then:

$\dot{x}_1(t) = x_2(t)$  and  $\dot{x}_2(t) = \ddot{y}(t)$

Therefore (1) becomes:  $\dot{x}_2(t) = \frac{1}{m}f(t) - \frac{k}{m}x_1(t) - \frac{c}{m}x_2(t)$

Now the dynamical system can be represented in matrix form:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

This is a very useful way for representing dynamical systems, and it is very applicable to numerical processing techniques. The general matrix form is:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

This is also applicable to higher order systems, systems described by multiple differential equations, and even nonlinear systems.

The state-space representation is a mathematic model of a physical system consisting of the input  $\mathbf{u}(\mathbf{t})$ , output  $\mathbf{y}(\mathbf{t})$  and state variables  $\mathbf{x}(\mathbf{t})$  related by first order differential equations. The term “state space” refers to a dimensional space where the axes are the state variables. Therefore, the state of the modelled physical system can be represented as a vector within that space.