1. Mode-Matching Effects

**def**: In MEMs gyros, mode-matching refers to matching the drive and sense resonant frequencies:

\[ \omega_d = \omega_s \]

Let's consider the effects.

a. **Drive Side**

\[ m_d \ddot{x} + c \dot{x} + kx = A_x \sin(\omega_d t) \]

Solving for \( x(t) \) \( \in \) ELEC 5760/67603

\[ x(t) = \frac{-A_x \cos(\omega_d t)}{c\omega_d} \]

\[ = -X_0 \cos(\omega_d t) \rightarrow \text{assumes drive side control system (i.e. 'resonator') keeps } X_0 \text{ constant} \]

\[ \therefore \dot{x}(t) = X_0 \omega_d \sin(\omega_d t) \]

b. **Sense Side**

Coriolis Force \( F_c = 2m_d \omega \omega_d \ddot{x} = 2m_d X_0 \omega_d \omega \sin(\omega_d t) \)

\[ m_s \ddot{y} + c \dot{y} + k_y + F_c = 0 \]

\[ \rightarrow \text{This assumes } m_s = m_d \text{ or } m_s \neq m_d \]

\[ \therefore m_s \ddot{y} + c \dot{y} + k_y = -2m_d X_0 \omega_d \omega \sin(\omega_d t) \]

The solution is: \[ y(t) = Y_0 \cos(\omega_d t + \phi_s) \]

where: \[ Y_0 = \frac{\omega}{\omega_d} \frac{m_d \omega_d}{m_s \omega_s^2} \left( \frac{2 X_0}{\omega_d} \right)^2 \left( \sqrt{1 - \left( \frac{\omega_d}{\omega_s} \right)^2} \right)^2 \]

\[ \phi_s = -\tan^{-1} \left( \frac{\frac{1}{\omega_s} \left( \frac{\omega_d}{\omega_s} \right)}{1 - \left( \frac{\omega_d}{\omega_s} \right)^2} \right) \]
where: \( w_s = \sqrt{\frac{K}{m_s}} \) and \( Q_s = \frac{m_s w_s}{c} \)

0) if \( w_s = w_d \), i.e. mode-matched,

then \( y_0 = \frac{\Omega}{\sqrt{2}} \frac{Q_s x_0 m_a}{m_s w_s} \)

and if \( m_s = m_d = m \),

then \( y_0 = \frac{\Omega}{\sqrt{2}} \frac{x_0}{c} = \frac{2 A_x m}{c^2 w_d} \)

and \( \phi_s = -90^\circ \)

i.e. \( y(t) = \frac{2 A_x m}{c^2 w_d} \frac{\Omega}{\sqrt{2}} \cos (w_d t - 90^\circ) \)

\[ = \frac{2 A_x m}{c^2 w_d} \frac{\Omega}{\sqrt{2}} \sin (w_d t) \rightarrow \text{same as in ELEC 5784/67860} \]

2) what if \( w_s \neq w_d \), maybe due to fabrication errors

Consider a typical MEMS resonator:

\[ |T_c(w)| \]

high \( Q \)

3dB \( BW = \frac{f_n}{Q} \)

Assume \( Q = 10,000 \) and \( f_n = 10kHz \)

i.e. \( BW = 1Hz \)

so being off just \( \frac{1}{2}Hz \), and the gain drops by \( \frac{1}{2} \)

or 29.3%
So if \( w_s \) and \( w_d \) are off by \( \frac{1}{2} w_d \) (0.5Hz here)

\[
\begin{align*}
\text{i} & \quad \text{-- -- -- -- -- \( w_s \)} \\
\frac{\text{i} Q}{V_z^2} & \quad \text{-- -- -- -- -- \( w_d \)}
\end{align*}
\]

\[ y_0 \text{ becomes: } y_0 \propto \frac{\sqrt{2} Q_s x_0 m_s}{V_z m_s w_d} \rightarrow \text{results in gain error} \]

and \( \phi_s \neq -90^\circ \) \rightarrow \text{problem for synchronous demodulation circuit using } V_x \sin(w_d t) \]

(3) Assume perfect mode matching: \( w_d = w_d \)

Let \( S_d \) not be constant

\[ S_d(t) = S_{0_d} \sin(w_r t) \text{ where } w_r \ll w_d \]

\( \therefore \) Coriolis force: \( F_c = 2m_d S_d(t)x \)

\[
\begin{align*}
&= 2m_d S_{0_d} \sin(w_r t) x_0 w_d \sin(w_d t + \phi) \\
&= S_{0_d} m_d x_0 w_d (\cos(w_d - w_r) - \cos(w_d + w_r))
\end{align*}
\]

Frequency of \( F_c \neq w_d \)

\( \rightarrow \text{same effect as if } w_s \neq w_d \)
2. A Solution

choose $w_s > w_d$ for high $Q$ resonators

Now, the gain is lower (much lower) on the sense side, but the change in gain for $\Delta f$ about $f_d$ is small

→ can better tolerate fabrication tolerances
→ can handle time varying rotations

Gyro BW typically defined as $f_s - f_d$

Note, still want to closely control $Q$ through $C$
→ want to maintain constant, known pressure around device in a hermetic package
→ and measure temperature to calibrate for temperature induced $C$ ($Q$) changes