1. Mode-Matching Effects

<u>def</u>: In MEMS gyros, mode-matching refers matching the drive and sense resonant frequencies:

$$\omega_d = \omega_s$$

Let's consider the effects

a. Drive Side

$$m_d \ddot{x} + c\dot{x} + kx = A_x \sin(\omega_d t)$$

solving for $x(t)$ {ELEC 5760/6760}

$$x(t) = -\frac{A_x}{c\omega_d}\cos(\omega_d t)$$

$$= -X_0 \cos(\omega_d t)$$ → assumes drive side control system (i.e. "resonator") keeps X_0 constant

$$\therefore \dot{x}(t) = X_0 \omega_d \sin(\omega_d t)$$

b. Sense Side

Coriolis Force $= F_c = 2 m_d \Omega \dot{x} = 2 m_d X_0 \omega_d \Omega \sin(\omega_d t)$

$$\therefore m_s \ddot{y} + c\dot{y} + ky + F_c = 0$$

→ This assumes $m_s = m_d$ <u>or</u> $m_s \neq m_d$

$$\therefore m_s \ddot{y} + c\dot{y} + ky = -2 m_d X_0 \omega_d \Omega \sin(\omega_d t)$$

The solution is: $y(t) = y_0 \cos(\omega_d t + \phi_s)$

where: $$y_0 = \Omega \frac{m_d \omega_d}{m_s \omega_s^2}\left(\frac{2X_0}{\sqrt{\left[1-\left(\frac{\omega_d}{\omega_s}\right)^2\right]^2 + \left[\frac{1}{Q_s}\left(\frac{\omega_d}{\omega_s}\right)\right]^2}}\right)$$

$$\phi_s = -\tan^{-1}\left(\frac{\frac{1}{Q_s}\left(\frac{\omega_d}{\omega_s}\right)}{1-\left(\frac{\omega_d}{\omega_s}\right)^2}\right)$$

where: $\omega_s = \sqrt{\frac{K}{m_s}}$ and $Q_s = \frac{m_s \omega_s}{C}$

① if $\omega_s = \omega_d$, i.e. mode-matched,

then $y_0 = \Omega \frac{2 Q_s x_0 m_d}{m_s \omega_s}$

and if $m_s = m_d = m$,

then $y_0 = \frac{\Omega \, 2 x_0}{C} = \frac{2 A_x m}{c^2 \omega_d}$

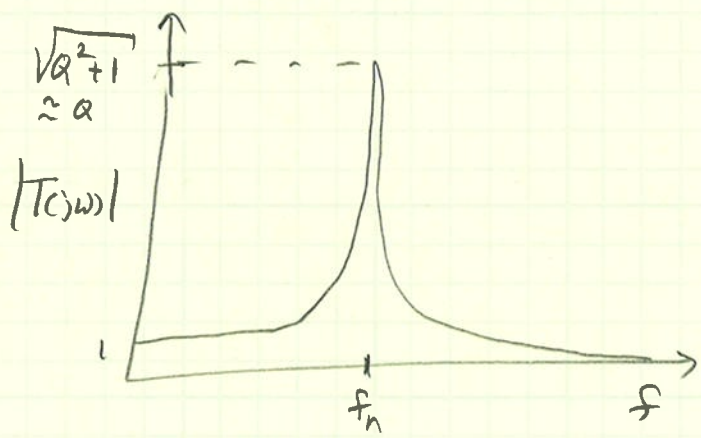
and $\phi_s = -90°$

$\therefore y(t) = \frac{2 A_x m \Omega}{c^2 \omega_d} \cos(\omega_d t - 90°)$

$= \frac{2 A_x m \Omega}{c^2 \omega_d} \sin(\omega_d t)$ → same as in ELEC5760/6760

② what if $\omega_s \neq \omega_d$, maybe due to fabrication errors

Consider a typical MEMS Resonator: high Q



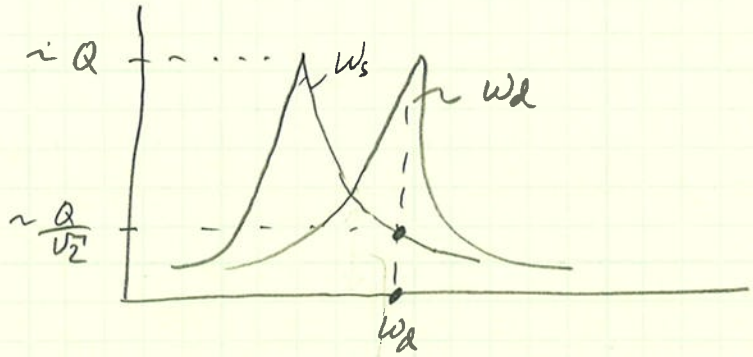
3dB BW $\simeq \frac{f_n}{Q}$

Assume $Q = 10{,}000$ and $f_n = 10\,KHz$

$\therefore BW = 1\,Hz$

So being off just ½ Hz, and the gain drops by $\frac{1}{\sqrt{2}}$ or 29.3%

So if ω_s and ω_d are off by ½ BW (0.5Hz here)



y_0 becomes: $y_0 \approx \Omega \frac{2 Q_s X_0 m_d}{\sqrt{2}\, m_s \omega_d}$ → results in gain error

and $\phi_s \neq -90°$ → problem for synchronous demodulation circuit using $V_x \sin(\omega_d t)$

③ Assume perfect mode matching: $\omega_d = \omega_s$

Let Ω not be constant

$\Omega(t) = \Omega_0 \sin(\omega_r t)$ where $\omega_r \ll \omega_d$

∴ Coriolis force: $F_c = 2 m_d \Omega(t) \dot{x}$

$$= 2 m_d \Omega_0 \sin(\omega_r t) X_0 \omega_d \sin(\omega_d t)$$

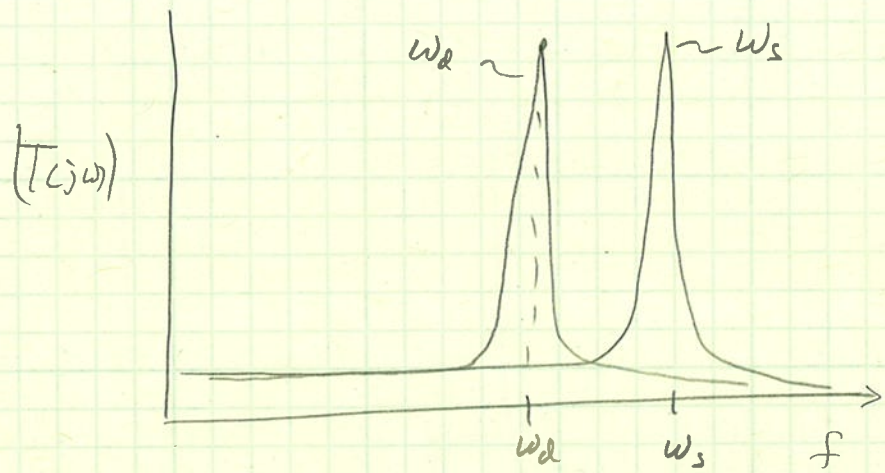
$$= \Omega_0 m_d X_0 \omega_d (\cos(\omega_d - \omega_r) - \cos(\omega_d + \omega_r))$$

Frequency of $F_c \neq \omega_d$

→ same effect as if $\omega_s \neq \omega_d$

2. <u>A Solution</u>

choose $\omega_s > \omega_d$ for high Q resonators



Now, the gain is lower (much lower) on the sense side, but the change in gain for Δf about f_d is small.

→ can better tolerate fabrication tolerances

→ can handle time varying rotations

gyro BW typically defined as $f_s - f_d$

Note, still want to closely control Q through c

→ want to maintain constant, known pressure around device in a hermetic package

→ and measure temperature to calibrate for temperature induced c (Q) changes