1) **Thermal Sensing and Actuation** → Chapter 5 in textbook

- **materials expand when heated** → **thermal expansion**

a. **Linear Expansion Coefficient** → material property
   - a relative dimensional change per unit temperature increase: \( \alpha = \frac{\Delta L}{L} \), \( \alpha = 10^{-6} / ^\circ C \) → Note: Book error \( \alpha \) \( \beta \)

→ also called the "Coefficient of Thermal Expansion" (CTE)

or "Thermal Expansion Coefficient" (TEC)

→ **Volumetric Expansion Coefficient** : \( \beta = \frac{\Delta V/V}{\Delta T} \) \( \approx 3 \alpha \)

**Example values for CTE** : Table 5.2, p. 183

- **Al** → 25 (2.5 is error in book)
- **Al Oxide** → 8.7
- **Si** → 2.6
- **Au** → 14.2
- **SiO\textsubscript{2}** → 0.35
- **Ni** → 13
- **Ti** → 8.6

**Differences in CTE between two attached materials results in stress, strain and sometimes mechanical failure when the temperature changes from the bonding temperature**

→ often a major problem in microfabrication

ex: \( \frac{C T E_{Si}}{C T E_{SiO\textsubscript{2}}} = \frac{2.6}{0.35} = 7.4 \)

→ Si expand or contracts 7.4 x more than SiO\textsubscript{2} for a temp increase or decrease
Ex: how much does a Si cantilever elongate for a 100\degree 
temperature increase? (beam is 1mm long)
\[
d = L \alpha \Delta T \\
= (1 \times 10^{-3})(2.6 \times 10^{-6})(100) = 0.26 \mu \text{m}
\]

b. Thermal Bimorph

Consider a beam, clamped on one end made of two 
materials with different CTEs, bonded longitudinally:

\[
\begin{array}{c}
\text{\(\square\)} \\
\text{\(\square\)}
\end{array}
\rightarrow \text{at bonding temperature}
\]

let \( \text{CTE}_1 > \text{CTE}_2 \)

if Temperature increases: 1 expands more than 2

\[
\begin{array}{c}
\text{\(\square\)} \\
\text{\(\square\)}
\end{array}
\rightarrow \text{beam bends down}
\]

if Temperature decreases: 2 contracts more than 2

\[
\begin{array}{c}
\text{\(\square\)} \\
\text{\(\square\)}
\end{array}
\rightarrow \text{beam bends up}
\]

1) Sensing Temperature: measure deflection or strain to 
determine temperature

2) Actuator \( \rightarrow \) add a resistive heater onto the bimorph and 
use Joule heating to deflect the bimorph
1. Mathematical Modeling the Bimorph

![Diagram showing bimorph bending and radius of curvature]

\[ r = \frac{d}{r} = r \cos(\theta) \]

\[ L = r \theta \Rightarrow [\theta] = \text{rad} \]

![Diagram of bimorph layers with material properties]

\[ \begin{align*}
    &\text{material 1} \rightarrow a_1, E_1 \\
    &\text{2} \rightarrow a_2, E_2
\end{align*} \]

\[ r \equiv \text{radius of curvature} \]

\[ \frac{1}{r} = \frac{6 w_1 w_2 E_1 E_2 t_1 t_2 (t_1 + t_2)(a_1 - a_2) \Delta T}{(w_1 E_1 t_1^2)^2 + (w_2 E_2 t_2^2)^2 + 2 w_1 w_2 E_1 E_2 t_1 t_2 (2t_1^2 + 3t_1 t_2 + 2t_2^2)} \]

Why use thermal Bimorph actuators?

- Tolérant of environments PPA/CDA's will not work:
  1. dirty (dust, etc.)
  2. in liquids (water, bio, etc.)
  3. easy to make an angular displacement based on input energy → current ⇒ heat ⇒ ΔT ⇒ θ
2. **Thermal Sensors** (covered in detail in Sensors course)

1. **Micro Thermocouples**
   - temperature dependent voltage between 2 dissimilar metals bonded at a point → **Seebeck effect**

2. **Thermal Resistor**
   - conductors → resistance increases with temperature
     \[ R_T = R_0 (1 + \alpha_R (T - T_0)) \]
     \[ \alpha_R = \text{temperature coefficient of resistance} \]

3. **Thermistor**
   - semiconductor temperature dependent device
     \[ R = R_0 e^{B (T_e - T_{ref})} \]
   - response is nonlinear with temperature
   - low cost → used for medical thermometers

4. **Thermistors, Thermotransistors and the PTAT**
   - temperature dependent current (also fabrication dependence)
   - **PTAT** → "Proportional To Absolute Temperature"

\[ I_0 \propto \frac{2k_B T}{q R} \ln \left( \frac{A_{eff}}{A_{ref}} \right) \]

→ removes fab. tol. dependence

→ often used in MEMS device in order to calibrate out temperature effects → most MEMS devices (i.e., sensors) are sensitive to temp. changes
3. Piezoelectric Actuation and Sensing → Chapter 7 in textbook

Consider a "special capacitor" with charge permanently trapped in between the electrodes

\[ Q_0 : V_0 \rightarrow Q_0 = CV_0 \]

\[ C = \frac{\varepsilon_0 \varepsilon A}{d_0} \]

If \( Q_0 \) is constant and \( V_0 \) is increased, \( C \) must decrease

\( Q_0 = \text{Constant}, \ V_0 \uparrow : C \downarrow \rightarrow d_0 \uparrow \)

→ Increasing \( V_0 \) results in plates moving further away

→ Decreasing \( V_0 \) """""" closer together

Likewise: Forcing the plates closer \( \varepsilon \), i.e. \( d_0 \downarrow \)
results in: \( d_0 \uparrow : C \uparrow : V_0 \downarrow \)

→ Piezoelectric crystals work in a similar way

\[ \text{"domains"} \]

→ these crystals possess multiple minute polarized crystallites that are normally randomly oriented throughout the bulk crystal

→ by exposing the crystal to a high electric field at elevated temperature, the domains most closely aligned to \( \vec{E} \) will grow at the expense of the others, and the crystal lengthens in this direction. This effect remains when temp is lowered and \( \vec{E} \) removed.

This process is called "Poling"