Oscillation for zero input occurs when the system becomes unstable $\rightarrow 1 + A\beta = 0$

or $A\beta = -1$

The condition where $A\beta = -1 \rightarrow$ called the "Barkhausen criterion"

for $A\beta = -1 \rightarrow |A\beta| = 1$ and $\angle A\beta = -180^\circ$

a. Electronic Oscillators

(1) amplifier provides needed gain so that $|A\beta| = 1$

(2) passive network provides 180° phase shift

$\rightarrow$ the circuit oscillates at whatever frequency produces 180° phase shift

RL & RC circuits $\rightarrow$ 1 pole per pair $\rightarrow$ up to 90° phase shift

LC circuits $\rightarrow$ 2 poles per pair $\rightarrow$ up to 180° phase shift

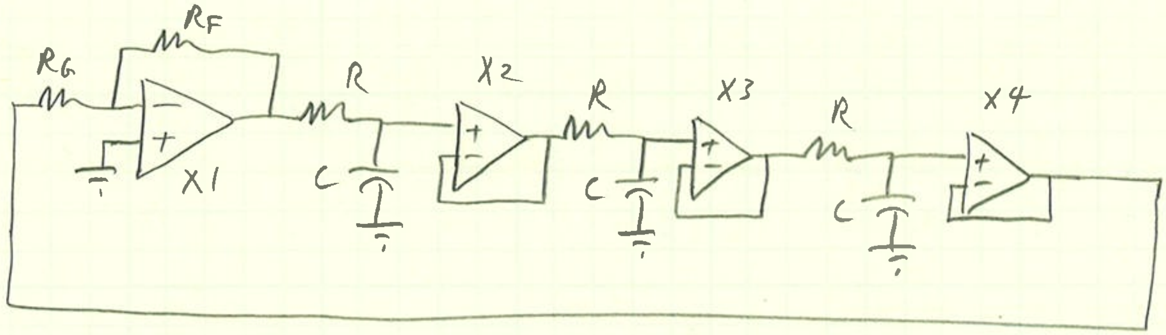
$\rightarrow$ |Loop gain| needs to be one at 180° phase shift

or: (1) oscillation may cease: <1

(2) output waveform will be distorted: >1

$\therefore$ AGC circuit can be used to adjust the gain to unity at the frequency resulting in a 180° phase shift, at a desired amplitude.

Ex: Buffered Op amp Phase Shift Oscillator



$R_G, R_F, X_1 \rightarrow$ gain stage $\rightarrow$ gain "A"

$\rightarrow$ each RC stage can produce up to -90° phase shift.

$\rightarrow$ for -180° phase shift $\rightarrow$ each RC stage produces -60° phase shift to generate oscillation

$$A\beta = A\left(\frac{1}{RCs+1}\right)^3 \rightarrow -\tan^{-1}(\omega RC) = -60^\circ$$

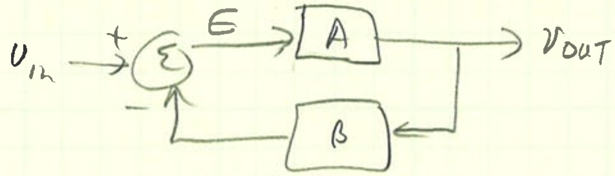
$\rightarrow$ select RC for desired oscillation frequency $\rightarrow f_0$

$\rightarrow$ determine gain A needed for oscillation

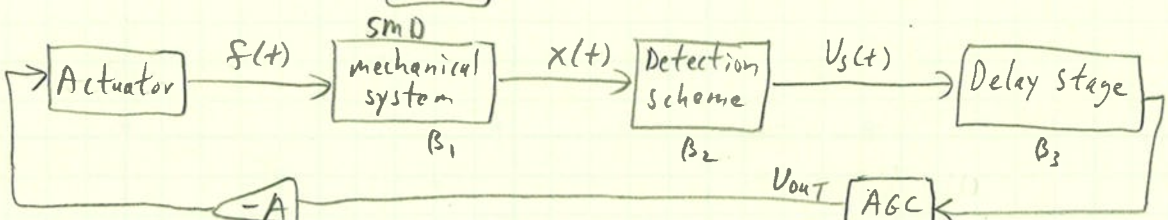
$\rightarrow$ size R_G and R_F accordingly

$\rightarrow$ select op amp so that $A f_0 \ll$ GBW product

$\rightarrow$ Now apply this to realize a MEMS electromechanical oscillator $\rightarrow$ a "MEMS Resonator"



Oscillation: $A\beta = -1$ at f_{osc}

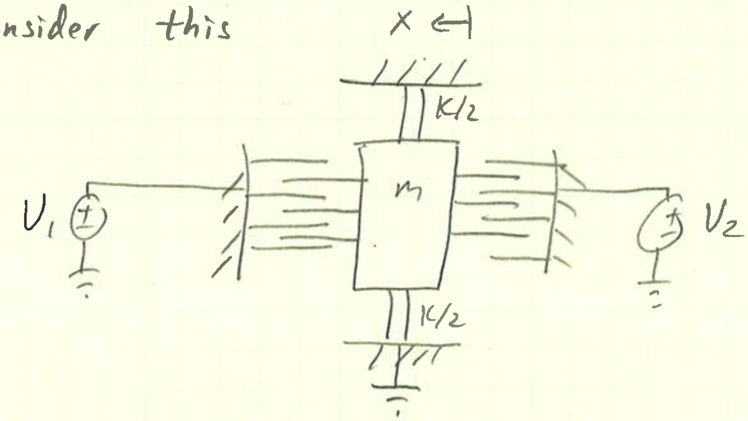


and f_{osc} : $|A\beta_1\beta_2\beta_3| = 1$ and $\theta_{\beta 1} + \theta_{\beta 2} + \theta_{\beta 3} = -180^\circ$

For a CDA: $F = \frac{n\varepsilon_0\varepsilon_r b\beta V^2}{d_0} = \alpha V^2$ → nonlinear function of $V(t)$

If $V(t) = V_1 \cos(\omega t)$ → $V^2 = \frac{1}{2}V_1^2[1 + \cos(2\omega t)]$ → not good for making an oscillator

Consider this



let $V_1 = V_B + \Delta V$ and $V_2 = V_B - \Delta V$

$$\therefore F_{NET} = F_1 - F_2 = \alpha V_1^2 - \alpha V_2^2$$
$$= \alpha\left[(V_B + \Delta V)^2 - (V_B - \Delta V)^2\right]$$
$$= \alpha\left[V_B^2 + 2V_B\Delta V + \Delta V^2 - V_B^2 + 2V_B\Delta V - \Delta V^2\right]$$
$$= \alpha\left[4V_B\Delta V\right]$$

now Force is linearly proportional to ΔV

Dynamics: $m\ddot{x} + c\dot{x} + kx = f(t)$

or $X(s)(ms^2 + cs + k) = F(s)$

$$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

$$\angle\frac{X(j\omega)}{F(j\omega)} = -\tan^{-1}\left[\frac{\frac{\omega_n\omega}{Q}}{\omega_n^2 - \omega}\right] = \theta_{mech}$$

$$\theta_{mech}\Big|_{\omega=\omega_n} = -\tan^{-1}[\infty] = -90^0$$

motion lags force by 90^0 at ω_n