1. More on the electrostatic spring softening effect:

Given: $m\ddot{x} + c\dot{x} + kx = \dfrac{\varepsilon_0 \varepsilon_r A V^2}{2(x_0 - x)^2}$

at equilibrium: $\ddot{x} = \dot{x} = 0$

$$\therefore kx = \frac{\varepsilon_0 \varepsilon_r A V^2}{2(x_0 - x)^2} \qquad (1)$$

With (1), an equation for V as a function of x is:

$$V = \sqrt{\frac{2x(x_0 - x)^2}{\varepsilon_0 \varepsilon_r A}}, \text{ for } V < V_{PI} \text{ and } 0 < x \le \frac{x_0}{3}$$

$\therefore$ Use a Taylor's series expansion to approximate

$f(t) = \dfrac{\varepsilon_0 \varepsilon_r A V^2}{2(x_0 - x)^2}$ as $f_1 + f_2 x$

around some bias point, V_a and x_a

where $0 < V_a < V_{PI}$

and $0 < x_a < \frac{x_0}{3}$

Review of Taylor series:

Given $f(x)$ evaluated around a:

$$f(a) \approx \underbrace{\frac{f(a)}{0!}(x-a)^0 + \frac{f'(a)}{1!}(x-a)^1}_{\text{linear model approximation}} + \cdots$$

$$\therefore f(a) = \frac{\varepsilon_0 \varepsilon_r A V_a^2}{2(x_0 - x_a)^2}$$

$$f'(a) = \frac{\varepsilon_0 \varepsilon_r A V_a^2}{(x_0 - x_a)^3}$$

$$\therefore f(a) \approx \frac{\varepsilon_0 \varepsilon_r A V_a^2}{2(x_0 - x_a)^2} + \frac{\varepsilon_0 \varepsilon_r A V_a^2}{(x_0 - x_a)^3}(x - x_a)$$

$$= \underbrace{\varepsilon_0 \varepsilon_r A V_a^2 \left(\frac{1}{2(x_0 - x_a)^2} - \frac{x_a}{(x_0 - x_a)^3} \right)}_{f_1} + \underbrace{\frac{\varepsilon_0 \varepsilon_r A V_a^2}{(x_0 - x_a)^3} x}_{f_2 x}$$

$\therefore$ around V_a and X_a:

$$m\ddot{x} + c\dot{x} + \left(k - \frac{\varepsilon_0 \varepsilon_r A V_a^2}{(X_0 - X_a)^3}\right)x = \varepsilon_0 \varepsilon_r A V_a^2 \left(\frac{1}{2(X_0-X_a)^2} - \frac{X_a}{(X_0-X_a)^3}\right)$$

$\therefore$ for $x = X_a$ at $V = V_a$,

the system spring constant softens from k

to $k - \dfrac{\varepsilon_0 \varepsilon_r A V_a^2}{(X_0 - X_a)^3} = k - k_{EL}$

$\therefore$ the poles of the C.E. become:

$$\text{poles} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k-k_{EL}}{m}\right)}}{2}$$

At $k - k_{EL} \rightarrow$ the system goes unstable

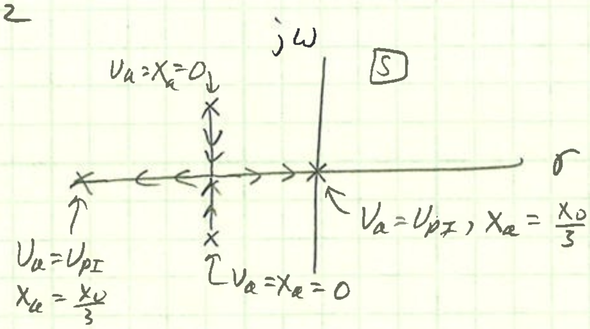
by plotting the poles in $\boxed{s}$ for $0 < V_a \le V_{PI}$ and $0 < X_a < \frac{X_0}{3}$,

a mapping of the poles in $\boxed{s}$ is obtained

ex: for $\frac{4k}{m} > \left(\frac{c}{m}\right)^2$

$\Downarrow$

high Q



Also: $\omega_n = \sqrt{\dfrac{k}{m}} = \left.\sqrt{\dfrac{k - k_{EL}}{m}}\,\right|_{\substack{x = X_a \\ V = V_a}}$

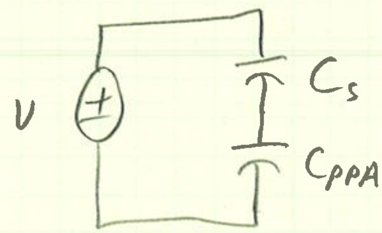
and: $Q = \dfrac{\omega_n m}{c} = \dfrac{\sqrt{km}}{c} = \left.\dfrac{\sqrt{m(k - k_{EL})}}{c}\right|_{\substack{x = X_a \\ V = V_a}}$

$\therefore$ for a bias point about $V_a, X_a \rightarrow 0 < V_a < V_{PI}$ and $0 < X_a < \frac{X_0}{3}$

ω_n and Q decrease as V_a, X_a increase

* However, keep in mind that this is a linear model for a nonlinear system about V_a, X_a

1. Series capacitance method to increase stable range of PPA



$$C_{PPA} = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad \text{at } V=0$$

let $C_s = \dfrac{\varepsilon_0 \varepsilon_r A}{D}$

$$C_T = C_s \oplus C_{PPA} = \frac{C_s C_{PPA}}{C_s + C_{PPA}} = \frac{\frac{\varepsilon_0 \varepsilon_r A}{d} \frac{\varepsilon_0 \varepsilon_r A}{D}}{\frac{\varepsilon_0 \varepsilon_r A}{d} + \frac{\varepsilon_0 \varepsilon_r A}{D}} = \varepsilon_0 \varepsilon_r A \left(\frac{\frac{1}{d} \frac{1}{D}}{\frac{1}{d} + \frac{1}{D}} \right)$$

$$= \frac{\varepsilon_0 \varepsilon_r A}{D+d}$$

For PPA to be stable over full range : $d \to 0$: $D = \frac{2}{3}(D+d)$

$\rightarrow$ so that C_T appears as a PPA with a $\frac{1}{3} D_T$ range of motion

$$\therefore 3D = 2D + 2d \rightarrow D = 2d$$

$$\therefore C_{PPA}\Big|_{rest} = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad \text{i.e. } V=0$$

$$C_s = \frac{\varepsilon_0 \varepsilon_r A}{D} = \frac{\varepsilon_0 \varepsilon_r A}{2d}$$

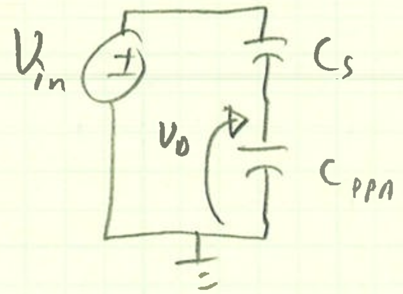
or : $C_s = \frac{1}{2} C_{PPA}\Big|_{rest}$

actually, for stability : $C_s \leq \frac{1}{2} C_{PPA}$

But, this comes with a penalty ...

This model also ignors stray capacitance in parallel with C_{PPA} and/or C_s.

Consider this:



V_D is the voltage that actually appears across the PPA and produces F_{EL}

$$V_D = V_{in} \frac{\frac{1}{sC_{PPA}}}{\frac{1}{sC_{PPA}} + \frac{1}{sC_s}} = \frac{V_{in} C_s}{C_s + C_{PPA}}$$

$C_s = \frac{1}{2} C_{PPA}$ min

$\therefore$ at $V_{in} \approx 0V$: $C_s \approx \frac{1}{2} C_{PPA}$

or $V_D = V_{in} \dfrac{\frac{1}{2}}{1 + \frac{1}{2}} = \dfrac{V_{in}}{3}$

As $x \to x_0$: $C_{PPA} \to$ very big : $V_D \to 0V$ and V_{in} gets large

→ Downside of this approach → requires very large V_{in}

→ show example PPA results on next page

Observe that V_D is maximum at $x = \frac{x_0}{3} \to V_D = V_P$

as $x > \frac{x_0}{3} \to V_D$ decreases while V_{in} increases

→ system can be modeled as a feedback controller that results in a stable system

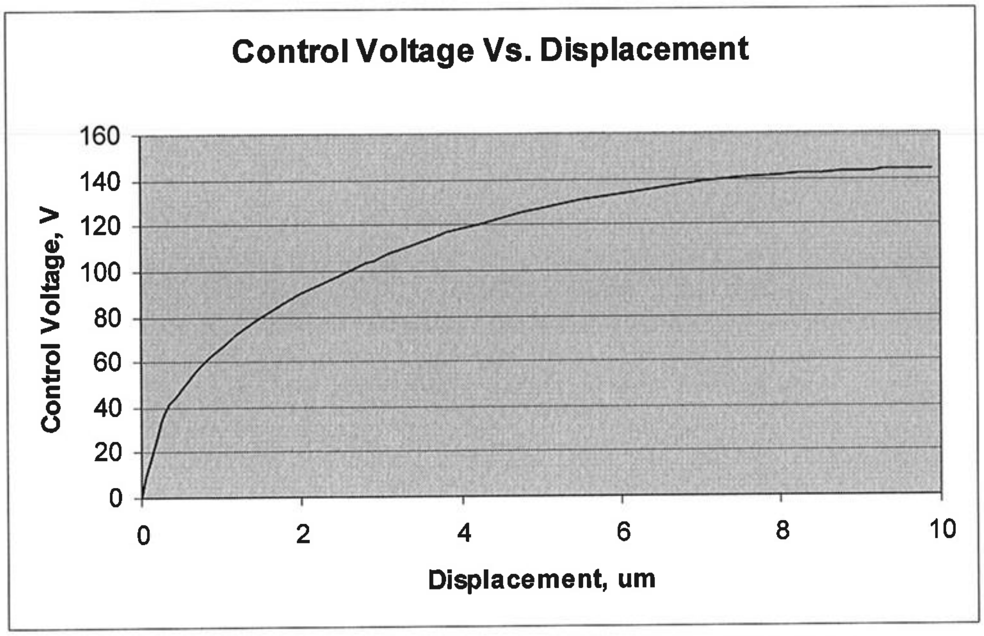


Figure 1: Required Voltage across the Series Capacitor PPA Pair.

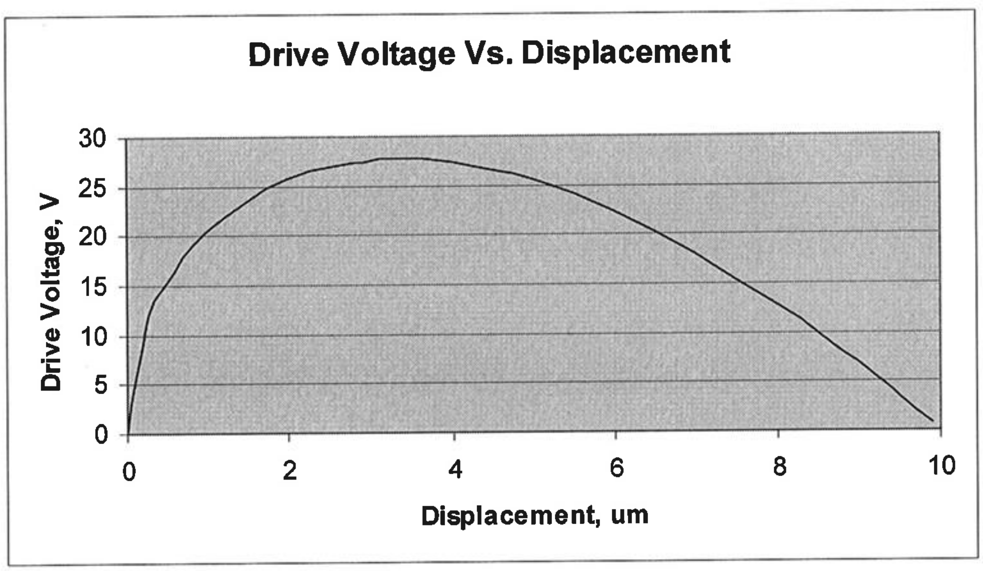


Figure 6: Drive Voltage just across the PPA.

Note: PPA has a 10μm rest gap distance