Thursday 10/30/25

The Inverter

An inverter is used to convert DC to AC, usually to deliver large amounts of power. It is another example of a periodically forced oscillating system, although it is not (usually) chaotic. It uses a high frequency clock in order to operate.

Typical inverter requirements:

1) High conversion efficiency
   Typically > 90%

2) Reliability
   Power electronic components tend to be the most likely ones to fail in a power electronics system. Power transistors tend to fail in the "permanent on" state.

3) Safety
   Often, inverters are high voltage and/or high current systems.

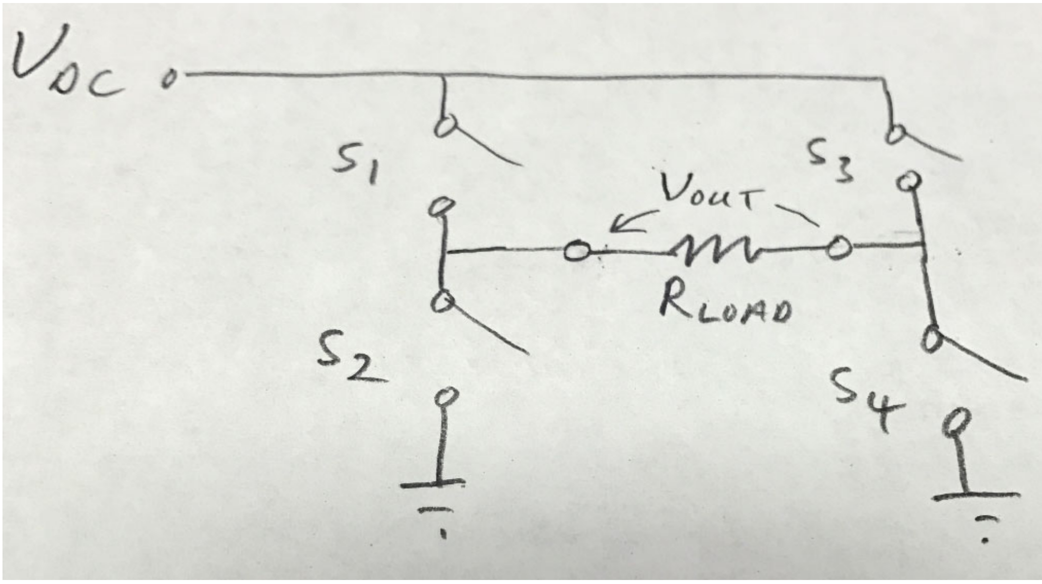
4) Output voltage regulation
   It might need to be connected and synchronized to the AC power grid.

5) Power factor correction
   This is important if connecting to the AC power grid.

6) Low EMI
   It must have low RF emissions in order to obtain (often required) FCC approval.

7) Anti-Islanding
   If it is used to transfer power onto the AC power grid, it must sense if the AC power grid goes down and then disconnect from it.

Block Diagram for a simple inverter:



The four switches are power transistors (MOSFETs, IGBTs, BJTs, etc.). They need to be sufficiently fast (turn on / turn off times), low on-resistance, able to handle sufficiently high voltages and currents, and are properly heat sinked. This configuration is called an H-bridge,

S_1 and S_4 are controlled together, and S_2 and S_3 are controlled together, typically 180° out of phase.

Call the S_1-S_4 pair the Forward Switch (FS) and the S_2-S_3 pair the Reverse Switch (RS).

Consider if the FS is on and the RS is off: Vout is positive as shown in the figure: $V_{out} = V_{DC}$.

However, if the RS is on and the FS is off: Vout is negative as shown in the figure: $V_{out} = -V_{DC}$.

Consider a square wave with an amplitude of 1 V. The Fourier series consists of a fundamental component and odd higher harmonics:

$$V(\omega t) = \frac{4}{\pi}\left[\frac{\sin(\omega t)}{1} + \frac{\sin(3\omega t)}{3} + \frac{\sin(5\omega t)}{5} + \cdots\right]$$

If the V_{DC} = 1 V, the load is 1 Ω, and the switches are lossless such that the square wave amplitude is 1 V, then:

P_{in} is 1 W.

For the square wave's fundamental component, V_{rms} is:

$$V_{rms} = \frac{4}{\pi\sqrt{2}} = 0.9\,V$$

$$P_{fun} = \frac{V_{rms}^2}{1} = 0.81\,W,$$

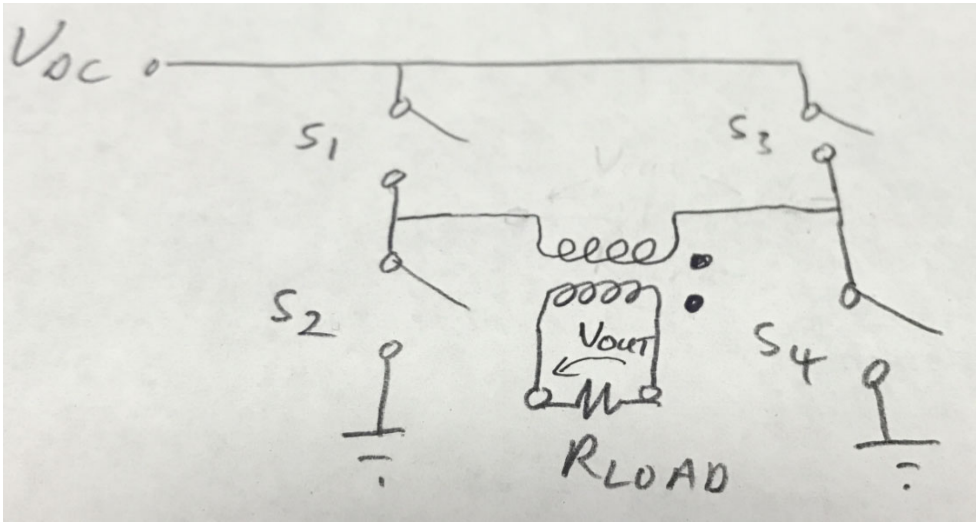
where P_{fun} is the power dissipated in the 1 Ω load due to the fundamental component.

So, almost 19% of the power is in the higher harmonics.

If a sinewave output is desired, the higher harmonics can be removed with a LPF. However, almost 19% of the power (energy) will be lost or wasted (this wasted energy is converted to heat somewhere).

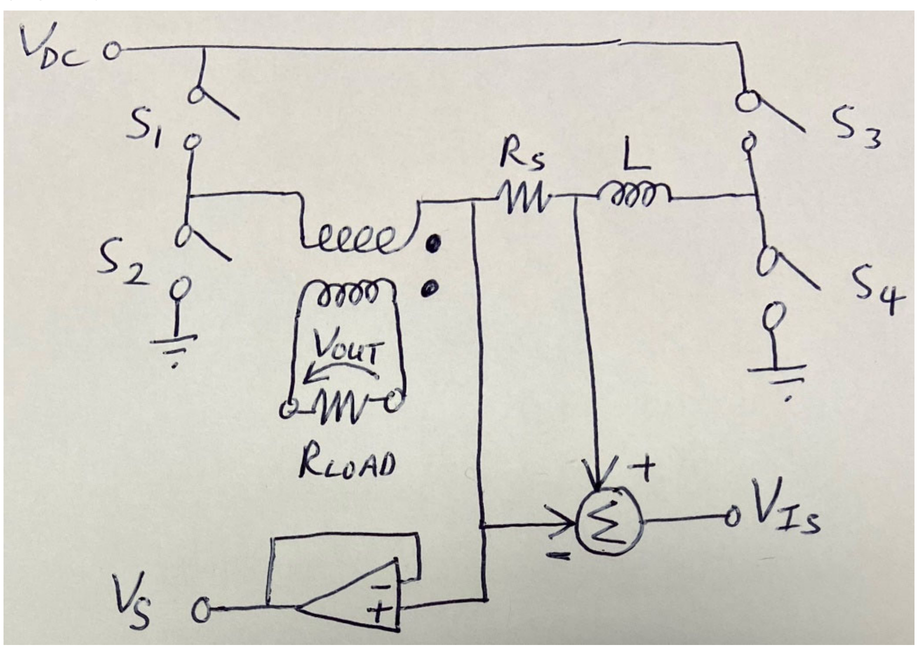
To minimize this loss, the switches can be driven with higher frequency Pulse Width Modulated (PWM) signals instead of with square waves at the fundamental frequency. This significantly reduces the amount of energy in high frequency components, increases the conversion efficiency, and makes it easier to LPF the unwanted high frequency components (since they are at higher frequencies).

A transformer can be added in front of the load to increase the output voltage:



The transformer also provides DC isolation between the input DC source and the load, which can be very important in protecting the load if a transistor switch fails, since transistors often fail in a state where they are permanently turned on.

A precision low-resistance sense resistor is often added before the load so that the output voltage and current can be monitored, and the inverter can be controlled (output voltage controlled, inverter shutoff if the load is shorted, etc.). A filter (shown as the added inductor, L) may be added to filter the PWM noise on the output waveform.



A controller (microcontroller, FPGA, etc.) uses V_s and V_{Is} in adjusting the two PWM signals: V_s is proportional to V_{OUT} and V_{Is} is proportional to the current through R_s. A temperature sensor could also be included so that the controller can shut off the inverter if it begins to overheat.

To connect to the AC grid (for delivering power to the grid), the inverter must synchronize to the grid (amplitude, frequency, and phase). It also should detect if the AC grid is down and disconnect to prevent a dangerous condition called islanding. There are two primary architectures for grid-connected inverters:

1) Self-commutated: the inverter internally produces the AC signal and locks its output to the grid (similar in operation to a PLL).

2) Line-commutated: the AC grid is sensed and used to achieve the generation of the synchronized AC output to the grid.

Also, inverters can have a 3-phase output to deliver 3-phase power.

Beyond AC power delivery, the driving PWM signals could be modified to produce other output waveforms, such as sawtooth or even non-periodic waveforms.

A linear oscillator could be used to produce AC power for a load. However, it is usually much less efficient than a PWM switching inverter.

Generally, linear oscillators are used for low power, low to high frequency, sinusoidal signal generation. An amplifier can be used to amplify the linear oscillator's output sinusoidal signal to generate a high power sinusoidal signal, if high power is needed (examples: radio transmitters and radar). AC inverters are generally used to produce high power, low frequency sinusoidal signals to deliver AC power to a load, and are usually designed for high efficiency.

Parametric Amplifiers and Oscillators

Consider this second order dynamic system:

$$\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + \omega_o^2 x = f(t)$$

Obviously, this is a 2nd order linear differential equation with constant coefficients. By now, we know how to solve it and we know how to use it to make a linear oscillator.

Now consider this second order dynamic system:

$$\frac{d^2x}{dt^2} + \beta(t)\frac{dx}{dt} + \omega_o^2(t)x = f(t)$$

Mathematically, this is also a linear differential equation, but it has variable coefficients. We normally don't work with these types of dynamic systems in most engineering problems. In this type of system, the damping and/or the natural frequency varies with time, and the system is being forced with f(t). This type of system can have two inputs: f(t) and an input signal affecting a system parameter.

Let's consider this example:

Here, we have a second order mechanical system where the system spring constant changes with time:

$k(t) = k_o + k_1 f_1(t)$,

where k(t) > 0.

So, for the mechanical system:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + k(t)x = f_2(t),$$

Where the k(t) term could be rewritten in terms of $\omega_o^2(t)$.

Let's look at some observations about this system:

1) What if $f_2(t) = 0$ and there are no initial conditions? In this case, varying the system spring constant with time has no effect: $x(t) = 0$. The mass does not move by just changing $k(t)$. The same result would happen if the damping coefficient varied with time, or both the spring constant and the damping coefficient varied with time: $x(t) = 0$.

2) What if $f_1(t) = 0$? Then the system reverts back to the 2nd order linear differential equation with constant coefficients.

3) What if both $f_1(t)$ and $f_2(t)$ are not zero? Good question!

To consider "3)", let's examine the relationship between current and voltage with a capacitor

$$i = C\frac{dv}{dt} + v\frac{dC}{dt}\ .$$

Normally, in electrical engineering applications, capacitance is a constant or it can reasonably be approximated as a constant (ex: the output of a sensor), and this equation reduces to:

$$i = C\frac{dv}{dt}.$$

Let's look at an example where:

$v(t) = Asin(\omega_1 t)$ and $C(t) = Bsin(\omega_2 t)$.

Then:

$\dot{v}(t) = A\omega_1 \cos(\omega_1 t)$ and $\dot{C}(t) = B\omega_2 \cos(\omega_2 t)$.

Therefore:

$$i = BA\omega_1 sin(\omega_2 t)\cos(\omega_1 t) + AB\omega_2 sin(\omega_1 t)\cos(\omega_2 t)\ ,$$

which can be rewritten as:

$i = \frac{1}{2}BA\omega_1\left[sin\big((\omega_2 + \omega_1)t\big) + sin\big((\omega_2 - \omega_1)t\big)\right] + \frac{1}{2}AB\omega_2\left[sin\big((\omega_1 + \omega_2)t\big) + sin\big((\omega_1 - \omega_2)t\big)\right]$.

Notice that just with a time varying capacitor being connected to a time varying voltage source, the current flowing through the capacitor has components with frequencies equal to the sum and difference of the frequencies of the two forcing functions (v(t) and C(t)). Note: here, I am loosely referring to C(t) as a forcing function for the sake of discussion.

If a resistor is added in series between the voltage source and the capacitor, realizing a complete first order system, the current components through C(t) will result in time varying voltage components across the resistor, which will in turn create more sum and difference current components through the time varying capacitor, etc. The resulting voltage across the capacitor has this form:

$$v_{cn}(t) = v_{c0}(t) + \sum_{k=1}^{n}(\dot{v}_{c(k-1)}(t)C(t) + v_{c(k-1)}(t)\dot{C}(t))R^k,$$

which is a recursive equation, and then:

$$v_c(t) = \sum_{n=0}^{\infty} v_{cn}(t) ,$$

which has many different frequency components related to:

$f_{out} = mf_1 \pm nf_2$, where m and n are integers.

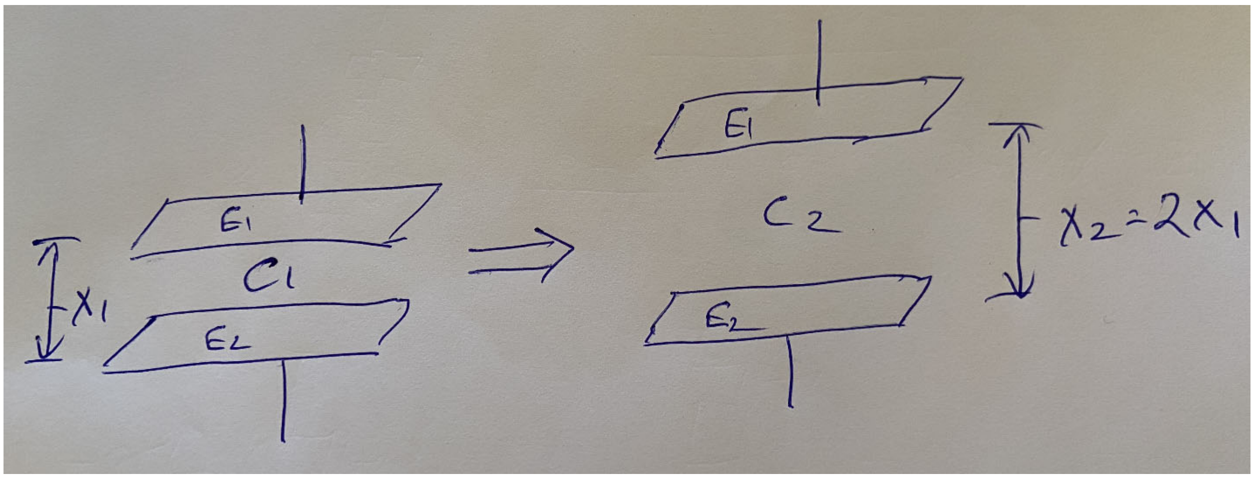
These different frequency components are called intermodulation products, and occur because of nonlinearities in the system.

Adding an inductor into the system makes it a second order system with a possibly underdamped response, further affecting the signal components in the system.

<u>The Parametric Amplifier</u>

This type of dynamic system can be used as an amplifier, and is called a parametric amplifier. Typically, the voltage or current to be amplified drives a voltage or current controlled variable impedance circuit element, such as a varactor (a diode used as a capacitor where the voltage across it varies the capacitance of the pn junction), a ferrite inductor (where the current through it causes the core to saturate, changing the inductance), a varistor (a voltage-dependent resistor), a MEMS variable capacitor (a voltage or current controlled variable capacitor moved using a MEMS actuator), or a Josephson junction (a quantum superconducting device). This drive voltage or current is called the input to the amplifier.

So, what effect does a voltage controlled MEMS capacitor have on the system? Consider a MEMS parallel plate capacitor as an example, shown below:



E_1 and E_2 are two parallel plate electrodes used to form C_1 and C_2, respectively. For C_1, the electrode separation distance is x_1, and for C_2, the electrode separation distance is x_2, where x_2 is twice the distance of x_1. It is assumed that the electrode area is much greater that the electrode separation distance squared, so that fringing effects can be neglected.

$$C_1 = \frac{\varepsilon_o \varepsilon_r A}{x_1}$$

$$C_2 = \frac{\varepsilon_o \varepsilon_r A}{x_2} = \frac{\varepsilon_o \varepsilon_r A}{2x_1} = \frac{C_1}{2}$$

Assume that charge, Q, is placed on C_1 by the rest of the circuit. Note: the MEMS actuator is NOT placing the Q on C; it only changes the electrode separation distance of C. Q results in a voltage, V_1, across the two electrodes, since:

$V_1 = \frac{Q}{C_1}$.

Let the MEMS actuator pull the electrodes further apart until the separation distance equals x_2. Assume it is a conservative event such that the amount of charge between the electrodes remains constant. Therefore,

$$V_2 = \frac{Q}{C_2} = \frac{2Q}{C_1} = 2V_1.$$

The energy stored in C_1 is

$$E_1 = \frac{1}{2}C_1V_1^2 .$$

The energy stored in C_2 is

$$E_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\frac{1}{2}C_1(2V_1)^2 = C_1V_1^2 = 2E_1 .$$

The event that pulled the electrodes twice the distance apart doubled the amount of energy stored in the capacitor. Pulling the electrodes further apart has to overcome the electrostatic force between the two electrodes, due to the charge stored on them, which is trying to pull them together. This energy is input into the electrical circuit. Note that if there is not any charge stored in the capacitor, moving the electrodes further apart does nothing to affect voltages or currents in the circuit.

For the parametric amplifier circuit, a bandpass filter is used to attenuate intermodulation products with frequencies other than that of the voltage or current modulating the time variable circuit element. The net result of generating the intermodulation products is a gain at the input signal's frequency. The advantage of this approach for signal amplification is that no resistances have to be used in the amplifier. Since the time varying circuit element can be a capacitor (such as a varactor or a MEMS capacitor), it does not have associated thermal noise (which is due to energy loss mechanisms). This approach is used to make very low noise amplifiers (LNAs) used in the frontend of RF receivers for very low power signals (example: radio telescopes).

The Parametric Oscillator

A similar second order system with time varying coefficients can be used to realize a type of oscillator called a parametric oscillator. It is a type of driven harmonic oscillator. Let's consider a mechanical example.

Consider a child's swing, where the child periodically stands and squats on the seat. If the swing is initially still, the child's standing and squatting has no effect: the swing does not move. However, if the swing is oscillating, and the child synchronizes his/her standing and squatting with the motion of the swing, actually at twice the frequency of the swing, very large sustained oscillations can be achieved in the otherwise underdamped system. The child is "pumping" the swing by changing the moment of inertia by standing and squatting. This oscillation phenomenon is different from just forcing a constant coefficient second order system because the input signal is modifying a system parameter, the moment of inertia in this case.

An equivalent electronic parametric oscillator can be realized using a varactor diode as a voltage controlled capacitor, for example, for the time varying system parameter.

Oven-Controlled Crystal Oscillator (OCXO)

As previously discussed, quartz crystals allow for precise oscillator frequency control.

However, the properties of the quartz crystals are sensitive to changes in ambient temperature, resulting in a change in oscillator frequency with temperature.

To improve performance, a crystal oscillator can be placed in a controlled temperature oven to operate at a constant temperature, often 75°C. A feedback control loop is used to control the oven's temperature. This results in a tradeoff between oscillator frequency precision and size / complexity / power usage. However, for some applications, the increased size, complexity, and power usage is worth it.

Stability Performance Comparison

Oscillator type*	Stability**	Aging / 10 year	Power	Mass (g)
Crystal oscillator (XO)[6]	10^{-5} to 10^{-4}	10...20 ppm	20 μW	20
Temperature-compensated crystal oscillator (TCXO)[6]	10^{-6}	2...5 ppm	100 μW	50
Microcomputer-compensated crystal oscillator (MCXO)[6]	10^{-8} to 10^{-7}	1...3 ppm	200 μW	100
Oven-controlled crystal oscillator (OCXO)[6] • 5...10 MHz • 15...100 MHz	 2×10^{-8} 5×10^{-7}	 2×10^{-8} to 2×10^{-7} 2×10^{-6} to 11×10^{-9}	1...3 W	200...500
Rubidium atomic frequency standard (RbXO)[6]	10^{-9}	5×10^{-10} to 5×10^{-9}	6...12 W	1500...2500
Caesium atomic frequency standard[6]	10^{-12} to 10^{-11}	10^{-12} to 10^{-11}	25...40 W	10000...20000
Global Positioning System (GPS)[7][8]	4×10^{-8} to 10^{-11}	10^{-13}	4 W	340
Radio time signal (DCF77)[9][10]		4×10^{-13}	—	87

Curtesy of Wikipedia
https://en.wikipedia.org/wiki/Crystal_oven