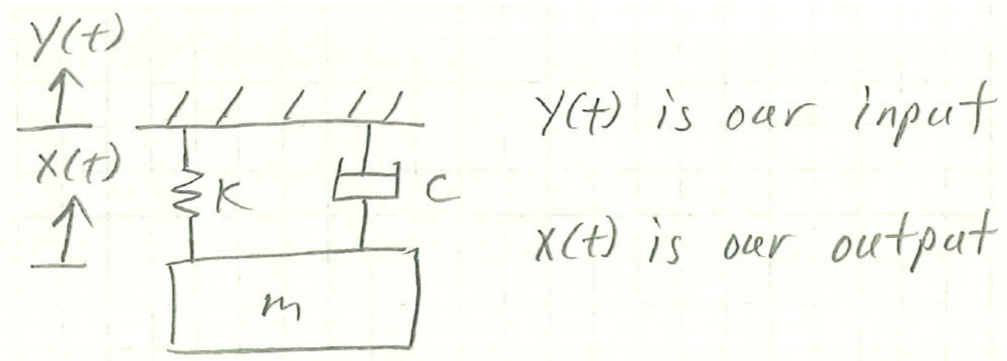
Lecture 10/26/22

Inertial Sensors (Accelerometers)

1) Accelerometer Sensitivity

Consider this model for a MEMS accelerometer:



For a constant acceleration, a, when the accelerometer has reached steady state ($\ddot{x} = \ddot{y}$ and $\dot{x} = \dot{y}$):

$F_{inertial} = F_{spring}$

$m\ddot{x} = kd$, where d is the proof mass displacement

$\therefore d = a\frac{m}{k} = \frac{a}{\omega_n^2}$, since $\omega_n = \sqrt{\frac{k}{m}}$

Let's define Sensitivity ≡ S, where $S = \frac{m}{k} = \frac{1}{\omega_n^2}$, [S] = s^2

Now, d = aS

So, given an accelerometer's sensitivity, S, an acceleration, a, will result in a proof mass displacement of d = aS, once steady state is reached.

S illustrates a tradeoff in accelerometers:

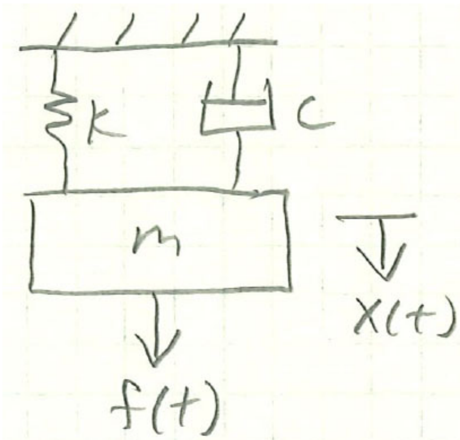
Wide bandwidth sensor: large ω_n → low sensitivity

High sensitivity sensor: large S → low bandwidth

Therefore, it is difficult to make a simple SMD accelerometer for measuring small amplitude, high frequency mechanical vibrations.

2) Damping ratio for an accelerometer

Consider this model for an accelerometer:



f(t) = inertial force = ma(t)

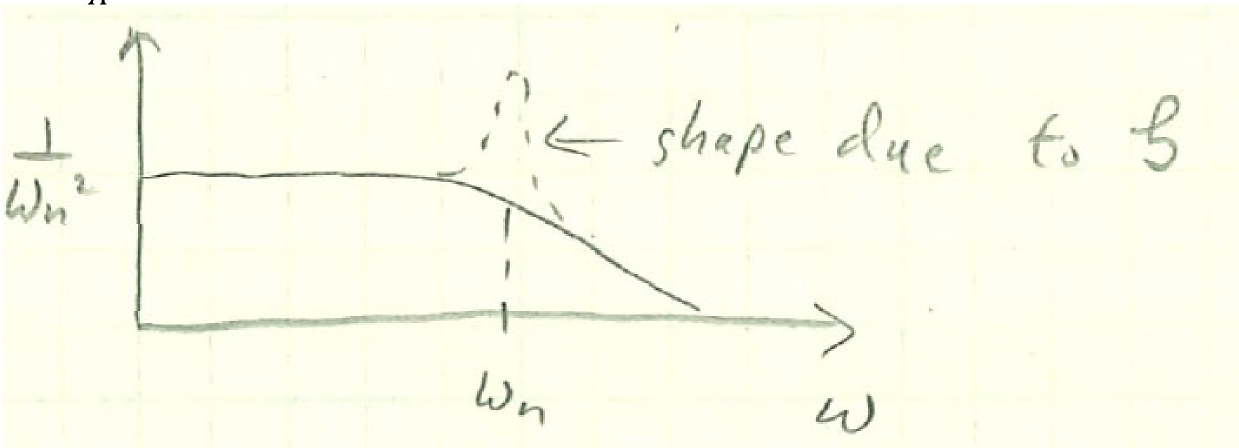
From our system's dynamics: $m\ddot{x} + c\dot{x} + kx = f(t) = ma(t)$

Taking the Laplace transform: $X(s)s^2 + X(s)s\frac{c}{m} + X(s)\frac{k}{m} = A(s)$

Rearranging: $\frac{X}{A}(s) = \frac{1}{s^2+\frac{c}{m}s+\frac{k}{m}} = \frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$

Consider a plot of $|\frac{X}{A}(j\omega)|$ versus ω:

$|\frac{X}{A}(j\omega)|$



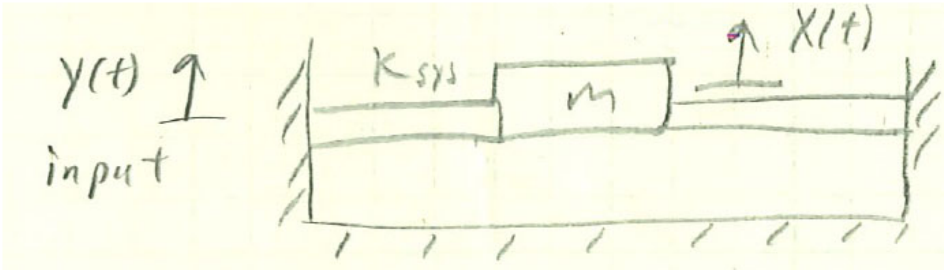
The overshoot in the frequency and time domains is due to the value for ζ. A value of ζ = 1 (critically damped) results in a fast response time with no overshoot (or oscillation) in the time response.

Remember that most MEMS devices are high Q (low ζ). So how do we increase damping to achieve ζ = 1 (Q = 0.5)? One way is to package the MEMS SMD in a hermetically sealed package with an inert gas to add sufficient fluidic damping. However, damping will change with temperature, and all hermetic packages leak over time.

3) Transient terms in the accelerometer output

So far, we have just considered the steady state operation of our MEMS SMD accelerometer. However, since it is a second order dynamic system, it has a steady state response and a transient response, and S (Sensitivity) is only defined for the steady state response. Also, we need to wait until the output signal has reached steady state to accurately determine the acceleration from the sensor's output signal.

Assuming that ζ = 1, consider this model for our accelerometer:



We know that $k_{sys} \approx \frac{N_{leg}}{N_{zig}} \frac{Ewt^3}{L^3}$

Then, for the system spring model: $F_s = k_{sys}d$,

where $d(t) = y(t) - x(t)$ or $D(s) = Y(s) - X(s)$

Also: d = aS, where a is a constant acceleration

We need the Laplace transform for y(t) when a is constant:

Acceleration: $a = \ddot{y}$, [a] = m/s^2

Velocity: $\dot{y} = \int_0^t a dt = at + v_o$, v_o is initial velocity

Displacement: $y = \int_o^t (at + v_o) dt = \frac{1}{2} at^2 + v_o t + y_o$,

y_o is initial displacement

Observe that a double integration is required to determine distance traveled from acceleration measurements. Any errors in acceleration measurement will accumulate over time, leading to a significant error between actual distance traveled and calculated distance traveled from acceleration measurements. Therefore, very accurate acceleration measurements are required to accurately estimate distance traveled for other than very short time periods.

For the case where $y_o = 0$ and $v_o = 0$: $y(t) = \frac{1}{2}at^2$

Let y(t) be the input to a MEMS device. It would be useful to take the Laplace transform of y(t).

From a Laplace Transform table: $\frac{t^{n-1}}{(n-1)!} \xrightarrow{L} \frac{1}{s^n}$

For $\frac{1}{2}at^2$: n – 1 = 2 $\rightarrow$ n = 3

$$\therefore \frac{1}{2}at^2 \xrightarrow{L} \frac{a}{s^3}$$

From transmissibility lectures earlier in the semester:

$$T(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Therefore X(s) = Y(s)T(s), and

$$\begin{aligned} D(s) &= Y(s) - X(s) \\ &= Y(s) - Y(s)T(s) \\ &= Y(s)(1 - T(s)) \\ &= Y(s)\left[\frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right], \text{ with } \zeta = 1 \end{aligned}$$

Y(s) is then due to the constant acceleration: $Y(s) = \frac{a}{s^3}$

From algebra then: $D(s) = \frac{a}{s(s+\omega_n)^2}$

Using partial fraction expansion and inverse Laplace Transforms:

$$d(t) = \frac{a}{\omega_n^2} - \frac{a}{\omega_n^2}e^{-\omega_n t} - \frac{a}{\omega_n}te^{-\omega_n t}$$

$\frac{a}{\omega_n^2}$ is the steady state term: d = aS, where $S = \frac{1}{\omega_n^2} = \frac{m}{k}$

$-\frac{a}{\omega_n^2}e^{-\omega_n t}$ and $-\frac{a}{\omega_n}te^{-\omega_n t}$ are transient terms with a time constant,

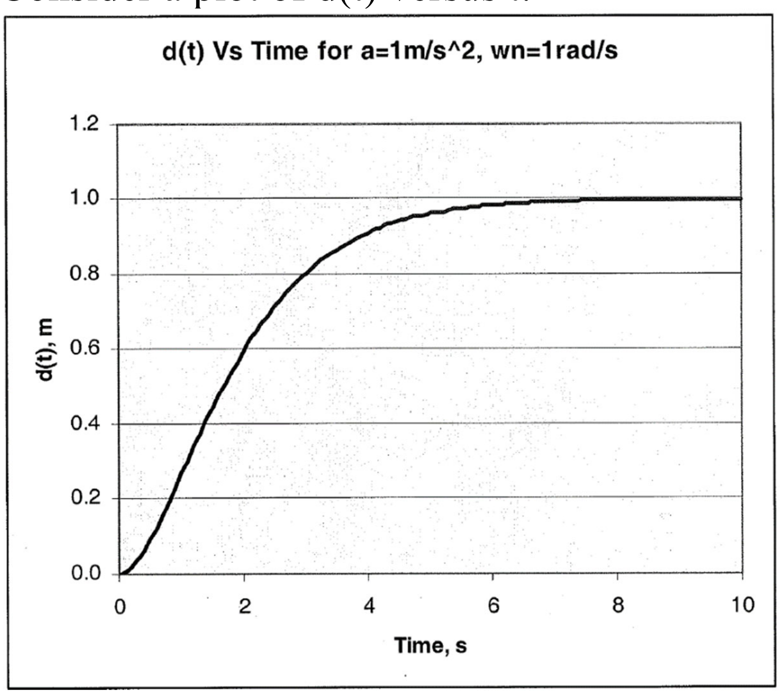
$$\tau_c = \frac{1}{\omega_n}$$

Therefore, wait at least a couple of τ_c to be approximately in steady state.

Consider a normalized example where a = 1 m/s^2 and ω_n = 1 rad/s, with ζ = 1:

Therefore, S = 1 s^2, τ_c = 1 s

Consider a plot of d(t) versus t:



Notice it took nearly 8 τ_c to approximately reach steady state.

4) Considerations for designing MEMS SMD accelerometers

Since $S = \frac{m}{k}$, and we usually want to maximize S:

make m as big as possible: large proof mass,

make k as small as possible.

Since $k \propto \frac{Ewt^3}{L^3}$, big L with small w and t would do it. However, big L means a large chip, so the springs are usually made to be short (small L), but thin (small t), in attempt to minimize k and chip size (more parts per wafer).

Packaging is a big part of the accelerometer: often it is hermetically sealed in an inert gas (N^2, Ar, etc.,) to try to make $\zeta = 1$.

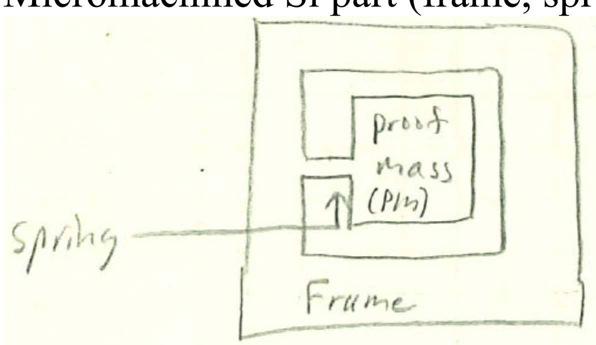
Inertial Sensors (MEMS Accelerometer Architectures)

1) Bulk micromachined accelerometer designs

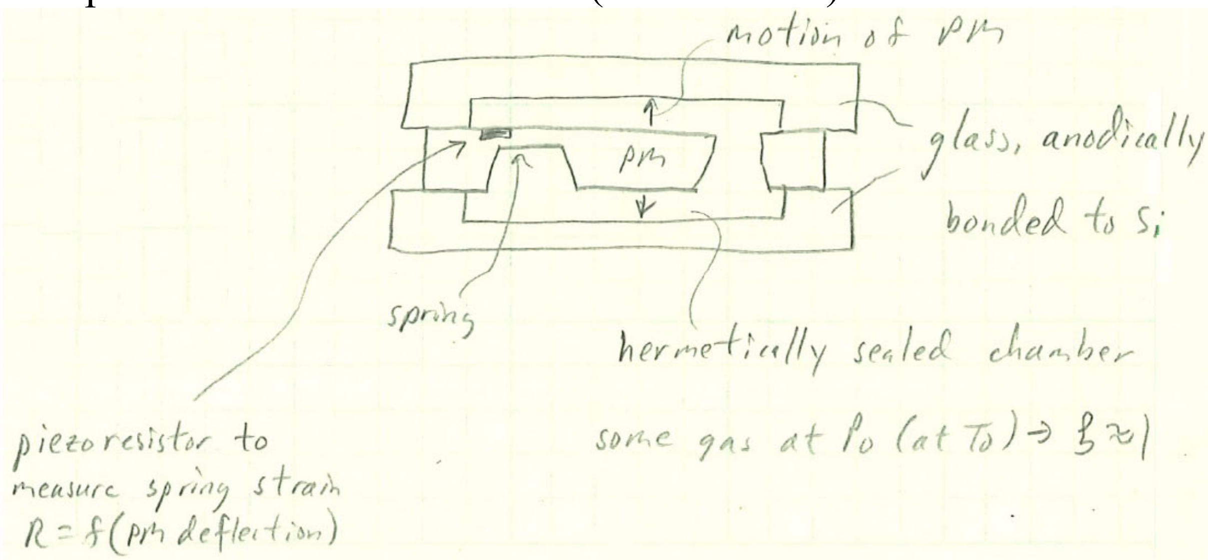
a. Single cantilevered spring design

Consider this design:

Micromachined Si part (frame, spring, and proof mass):



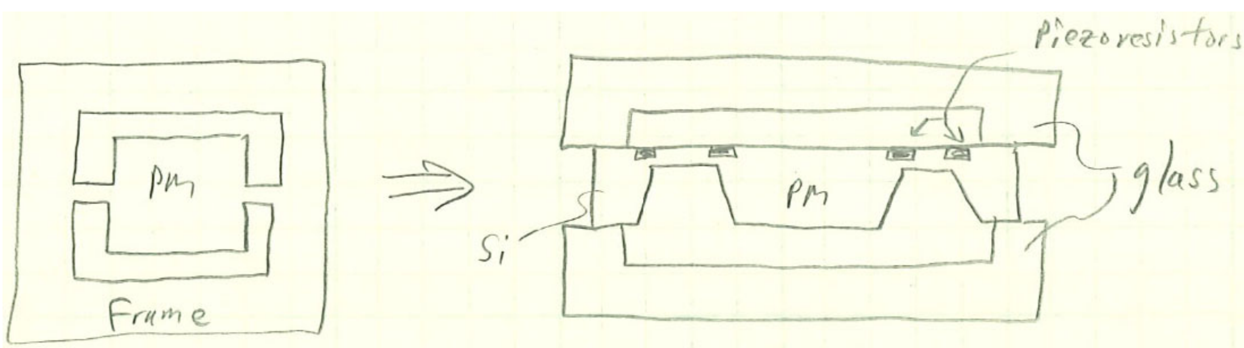
Complete accelerometer structure (cross-section):



Obviously, this simple design could have issues with lateral or torsional proof mass motion.

You should notice a similar architecture to the bossed diaphragm pressure sensor, where the diaphragm-like structure is made into a cantilevered spring and the proof mass.

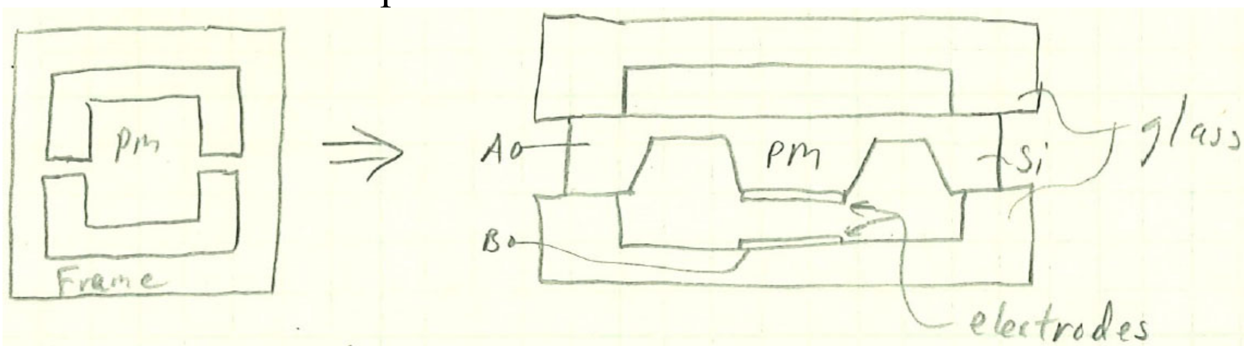
b. Double clamped beam suspension system design



Just as with the bossed pressure sensor, two PR's are in tension and two PR's are in compression (during an external acceleration event). The PR's in tension increase in R, while the PR's in compression decrease in R. Therefore, the four PR's would be connected to form a Wheatstone bridge.

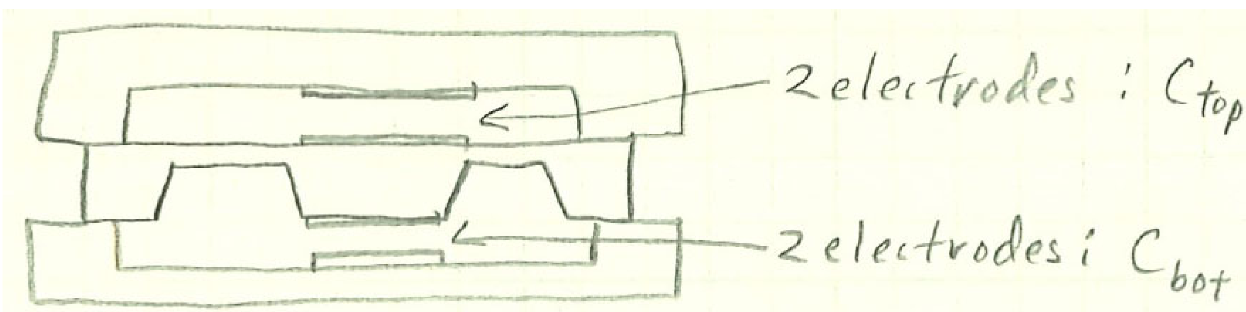
c. Single capacitance capacitive sensing accelerometer

This is also similar to pressure sensors.



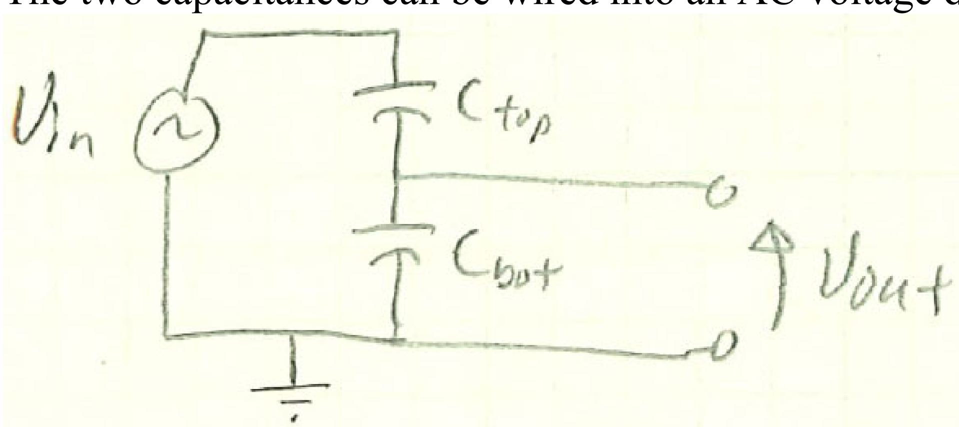
C is the capacitance between points A and B: $C = \frac{\varepsilon_o \varepsilon_r A}{d_o \pm \Delta d}$

d. Differential capacitance capacitive sensing accelerometer



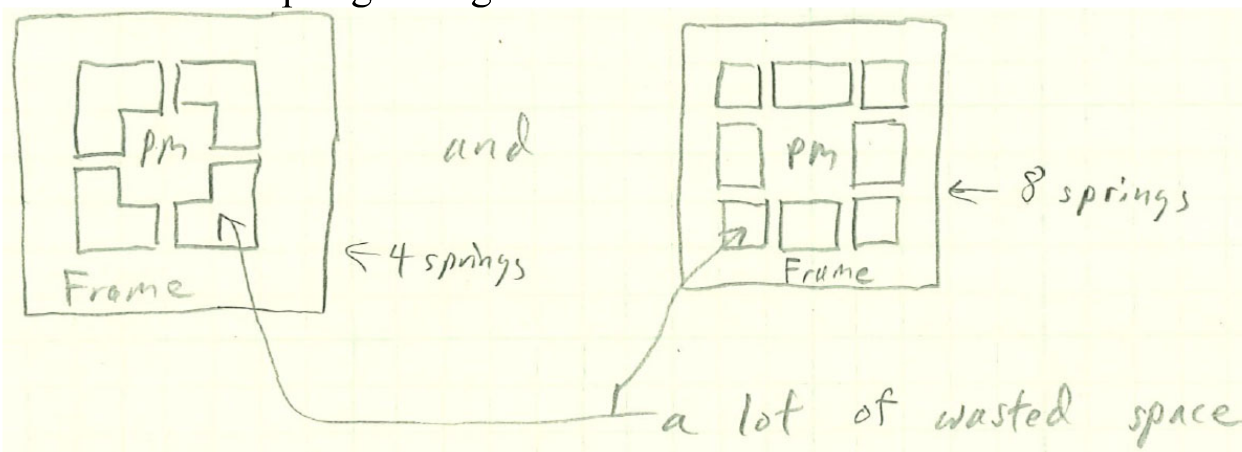
Where $C_{top} = \frac{\varepsilon_o \varepsilon_r A}{d_o \pm \Delta d}$ and $C_{bot} = \frac{\varepsilon_o \varepsilon_r A}{d_o \mp \Delta d}$.

The two capacitances can be wired into an AC voltage divider:



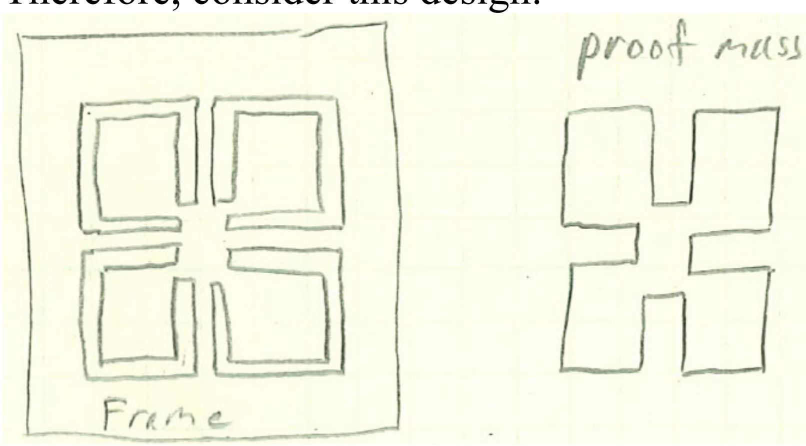
e. Other suspension system designs

Consider these spring configurations:



This wasted space causes two issues: extra cost in etching Si, and unusable real estate on the wafer. As in all real estate, space is $$.

Therefore, consider this design:



Now, much of the previously unused space is used to make a larger proof mass. However, be cautious of unexpected or undesirable bending modes with this proof mass structure.

This design yields a larger proof mass and longer springs (smaller k): which can result in a larger S.