a. If V is DC or AC and $\omega_a >> \omega_n$:

$\therefore F_{EL}$ = constant force

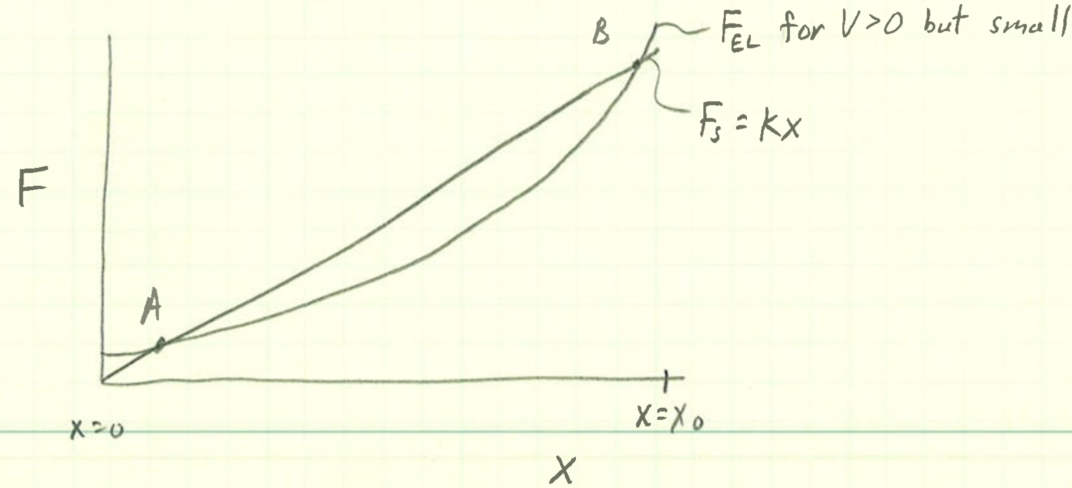
at steady state: $\ddot{x} = 0$, $\dot{x} = 0$

$$\therefore Kx = \frac{\varepsilon_0 \varepsilon_r A V^2}{2(x_0 - x)^2}$$

spring force is linear w.r.t. x

electrostatic force is nonlinear w.r.t. x

This equation can be solved graphically:



2 mathematical solutions exist: A + B

A → stable equilibrium point, w.r.t. perturbation

B → unstable " " , " "

→ Increase V slowly → F_{EL} trace moves up → A + B move closer together

→ At a voltage called the pull-in voltage (V_{pi}) A + B converge into a single unstable equilibrium point

→ For $V > V_{pi}$ → no stable solution exists + the two electrodes will snap into contact

Solving for V_p and the stable range of motion

define $F = F_{EL} - F_s = 0$

or $F = \dfrac{\varepsilon_0 \varepsilon_r A V^2}{2(X_0 - X)^2} - KX = 0$ (1)

For stability: $\dfrac{\partial F}{\partial X} < 0$

$$\frac{\partial F}{\partial X} = \frac{\varepsilon_0 \varepsilon_r A V^2 (-2)(-1)}{2(X_0 - X)^3} - K < 0$$

or: $\dfrac{\varepsilon_0 \varepsilon_r A V^2}{(X_0 - X)^3} - K < 0$ (2)

using (1) and (2) → solve for stable range of X

From (1) → $V^2 = \dfrac{2KX(X_0 - X)^2}{A\varepsilon_0 \varepsilon_r}$ ⟹ sub into (2)

$$\therefore \frac{2KX}{X_0 - X} - K < 0$$

$$\frac{2X}{X_0 - X} < 1$$

$$2X < X_0 - X$$

$$X < \frac{X_0}{3}$$

∴ the stable range of motion is $0 \le X < \dfrac{X_0}{3}$ → $\frac{1}{3}$ of the gap

To find an expression for V_{pi}, plug $X = \frac{X_0}{3}$ into (1)

$$\frac{\varepsilon_0 \varepsilon_r A V_{pi}^2}{2\left(X_0 - \frac{X_0}{3}\right)^2} - \frac{KX_0}{3} = 0$$

$$V_{pi}^2 = \frac{2(X_0 - \frac{X_0}{3})^2 K X_0}{3 \varepsilon_0 \varepsilon_r A}$$

$$= \frac{2(\frac{2}{3}X_0)^2 K X_0}{3 \varepsilon_0 \varepsilon_r A}$$

$$= \frac{2(4) X_0^3 K}{3(9) \varepsilon_0 \varepsilon_r A}$$

$$V_{pi} = \sqrt{\frac{8 X_0^3 K}{27 \varepsilon_0 \varepsilon_r A}}$$

$\therefore$ the PPA is stable for $0 \leq V < V_{pi}$

→ <u>note</u>: for high Q, a fast change in V <u>could</u> result in sufficient displacement, x, such that the proof mass enters to unstable range for $V < V_{pi}$ due to the transient response of the system

b. <u>More thoughts on PPA stability</u>

(1) F_{EL} → effect often described as being an "electrostatic spring" which softens the system spring as $V \uparrow$

At $V = V_{pi}$ → $K_m = K_E$ so that system spring constant:

$$K_{sys} = K_m - K_E = 0$$

(2) Inc V → inc F_{EL} → moves one or more poles toward the RHP of the S-plane, crossing into the RHP for $V \geq V_{pi}$

(3) Closed + Open-Loop controllers have been proposed to increase the PPA stable operating range of motion