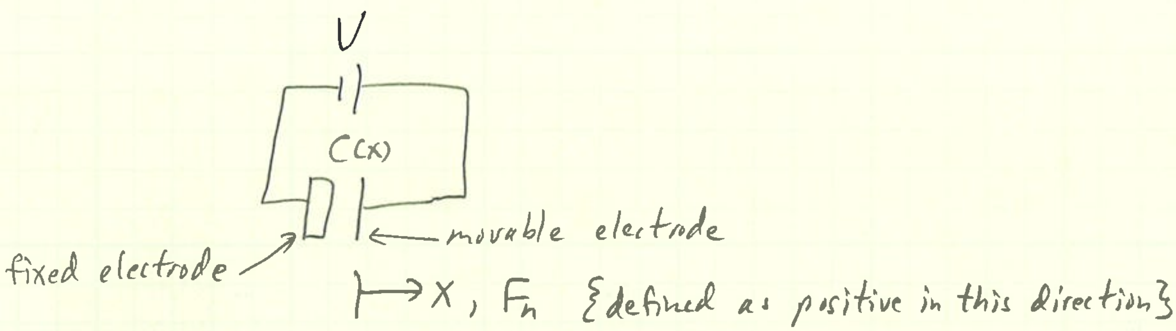
→ More difficult to measure capacitance, $C(t)$, than resistance, $R(t)$

→ covered in ELEC 5760/6760 → Solid State Sensors

b. Electrostatic Actuation

→ case where $C(x) = \frac{\varepsilon_0 \varepsilon_r A}{x}$ → variable plate gap

Consider a battery connected to this $C(x)$:



3 energy terms that sum to zero:

→ Battery: $E_B = Pt = IVt = QV$

→ Capacitor: $E_c = \frac{1}{2} CV^2$

→ Movable Electrode: $E = \text{Work} = F_n \Delta X$

Physics Perspective

$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$ → $d\downarrow \therefore C\uparrow \therefore E_c = \frac{1}{2}CV^2 \uparrow$ gains energy if $d\downarrow$

charge on C: $Q = CV$ → $C\uparrow \therefore Q\uparrow$ for constant V

this charge must come from the battery $\therefore E_c\uparrow \therefore E_B\downarrow$

Attractive force → $d\downarrow \therefore \Delta x < 0$ {as defined above}

$\therefore F_N$ will be defined as negative

Energy Balance Equation

$$F_n \Delta x + \frac{dE_c}{dx}\Delta x + \frac{dE_B}{dx}\Delta x = 0 \quad \text{for a change, } \Delta x$$

or: $F_N = -\dfrac{dE_c}{dx} - \dfrac{dE_B}{dx}$

$$\frac{dE_c}{dx} = \frac{1}{2}\frac{\partial C(x)}{\partial x}V^2 \qquad \{E_c = \tfrac{1}{2}CV^2\}$$

$$\frac{dE_B}{dx} = -\frac{\partial Q(x)}{\partial x}V \qquad \{Q = CV\}$$

$$= -\frac{\partial C(x)}{\partial x}V^2$$

$$\therefore F_N = \frac{1}{2}\frac{\partial C(x)}{\partial x}V^2 \qquad \left\{C = \frac{\varepsilon_0\varepsilon_r A}{x}\right\}$$

$$= -\frac{\varepsilon_0\varepsilon_r A V^2}{2x^2}$$

"−" because attractive force and motion is in −x direction

Textbook: p. 131: $F = \left|\dfrac{\partial U}{\partial x}\right| = \dfrac{1}{2}\left|\dfrac{\partial C}{\partial x}\right|V^2 = \dfrac{1}{2}\dfrac{\varepsilon_0\varepsilon_r A V^2}{x^2}$

This technique yields the right magnitude, but the wrong sign

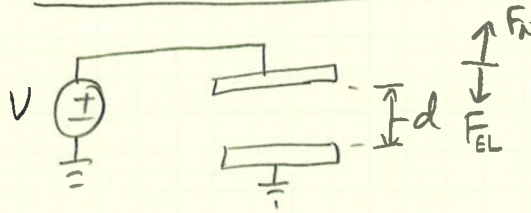
$\therefore$ For parallel plate electrostatic actuator (PPA)

$$F_n = -\frac{\varepsilon_0\varepsilon_r A V^2}{2x^2} \quad \text{or} \quad \frac{-\varepsilon_0\varepsilon_r A V^2}{2(x_0+x)^2}$$

x_0 is the rest gap distance

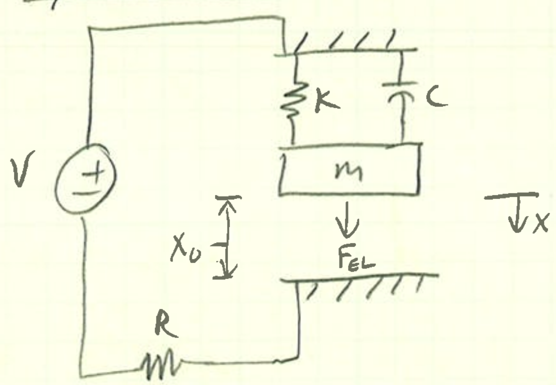
PPA → square law device: +V & −V produces the same force

1. More on electrostatic PPA(s)



$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$, $F_{EL} = \frac{\varepsilon_0 \varepsilon_r A V^2}{2d^2} = -F_N$

a. Dynamic Model



R is to limit the current if the electrodes short together

$$m\ddot{x} + c\dot{x} + k\dot{x} = F_{EL} = \frac{\varepsilon_0 \varepsilon_r A V^2}{2(x_0 - x)^2}$$

PPA → a square law device

$V_{DC} > 0 \rightarrow F_{EL} > 0$

$V_{DC} < 0 \rightarrow F_{EL} > 0$

V_{AC} : $V = V_a \cos(\omega t)$

$\rightarrow V^2 = V_a^2 \left[\frac{1}{2} + \frac{1}{2}\cos(2\omega t)\right]$

if $\omega_a \leq \frac{1}{2}\omega_n$ → PPA motion follows V_{AC} with phase delay and DC offset

if $\omega_a \gg \frac{1}{2}\omega_n$ → PPA cannot mechanically move fast enough to keep up with V_{AC}

∴ the PPA responds to the rms value of V_A

→ for $\omega_a \gg \frac{1}{2}\omega_n$ → $V \approx \underbrace{\frac{V_A}{\sqrt{2}}}_{\text{rms value of } V(t)}$ → $V^2 = \frac{V_a^2}{2}$

⇒ PPA's are generally high V, low I → ∴ could use low V, higher I at $f \gg f_n$ and use a transformer easier on the electronics side