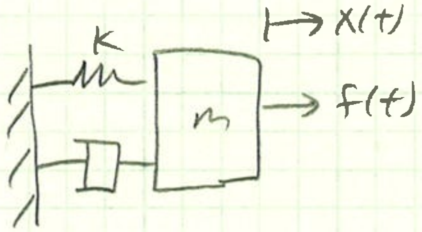
1. Applying a force to MEMS SMD device



DC Force : $F = f(t)$

steady state: $\ddot{x} = \dot{x} = 0 \therefore x = \frac{F}{k}$

$m\ddot{x} + c\dot{x} + kx = f(t)$

force $f(t)$ results in proof mass motion, $x(t)$

→ Often, a sinusoidal force is applied at $\omega_n$ to take advantage of high Q → low $f(t)$ → high $x(t)$

let $f(t) = F\cos(\omega_n t)$ where $\omega_n = \sqrt{\frac{k}{m}}$

$\therefore m\ddot{x} + c\dot{x} + kx = F\cos(\omega_n t)$

→ assume solution of the form $x(t) = A\sin(\omega_n t)$

$\therefore \dot{x}(t) = A\omega_n\cos(\omega_n t)$

$\therefore \ddot{x}(t) = -A\omega_n^2\sin(\omega_n t)$

sin tems

→ $m\ddot{x} + c\dot{x} + kx = f(t)$

cos terms

$\therefore$ equate trig terms

→ $cA\omega_n\cos(\omega_n t) = F\cos(\omega_n t)$

$\therefore A = \frac{F}{c\omega_n}$

$\therefore x(t) = \frac{F}{c\omega_n}\cos(\omega_n t) = \frac{FQ}{m\omega_n^2}\cos(\omega_n t) \rightarrow Q\uparrow : x(t)\uparrow$

$f(t) \propto \cos(\omega_n t)$ and $x(t) \propto \sin(\omega_n t)$

$\therefore$ at $\omega_n$, motion lags force by 90°

→ important in feedback systems

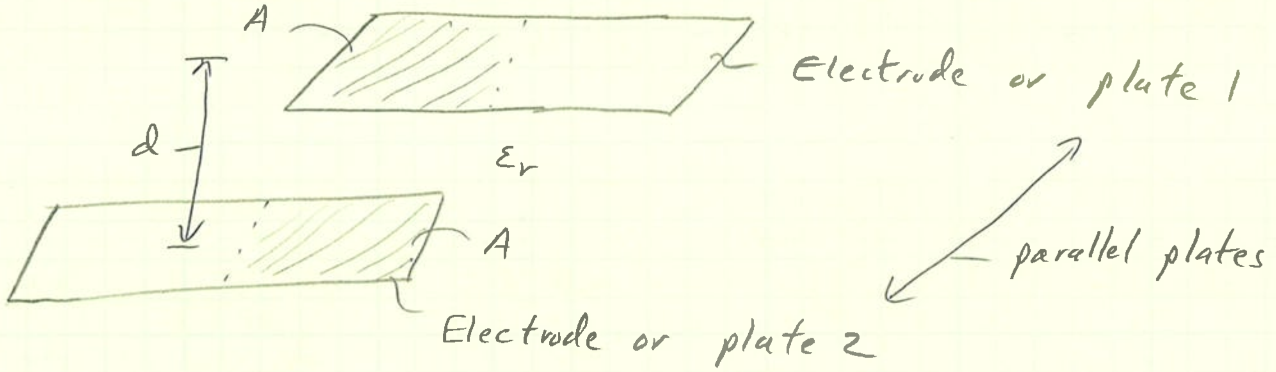
10/21/15

1. Chapter 4 (p.127+) Electrostatic Sensing and Actuation

→ Capacitors heavily used to sense + actuate in MEMS

→ Capacitance is an "area effect" → scales favorably in MEMS

a. Structure



$A \equiv$ overlapping area

$d \equiv$ plate separation distance

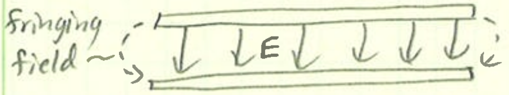
$\varepsilon_r$ = "relative permittivity" of "diectric constant" of the electrically insulating material between the plates

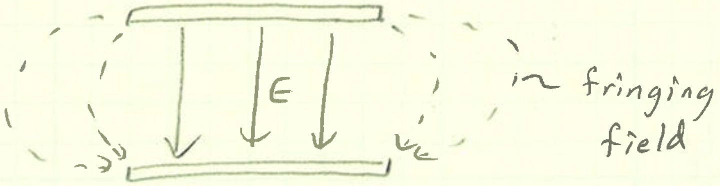
$\varepsilon_0 = 8.854\ \mathrm{pF/m}$ → permittivity of free space

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}, \quad [C] = F$$

↳ this formula ignors fringing effects

→ reasonable if $d \ll \sqrt{A}$ ⇒ often the case in MEMS





fringing fields of more importance here

$C = \frac{\varepsilon_0 \varepsilon_r A \beta}{d}$ where $\beta \equiv$ fringing effect correction factor

a. Capacitive Sensing Techniques

① plate separation : $d(t)$ or $d(x)$ → often used in MEMs

$$C(t) = \frac{\varepsilon_0 \varepsilon_r A}{d(t)} \quad \{\text{ignoring fringing}\}$$

$$C(t) \propto \frac{1}{d(t)} \text{ or } \frac{1}{\text{measurand}} \quad \{\text{measurand} \equiv \text{quantity being sensed}\}$$

② area separation : $A(t)$ or $A(x)$

$$C(t) = \frac{\varepsilon_0 \varepsilon_r A(t)}{d}$$

$C(t) \propto A(t)$ or measurand

$A(t)$ → 2-D motion

if $A(t) = X\,y(t)$ → 1-D motion where $A(t) = X\,y(t)$

$$\rightarrow C(t) = \frac{\varepsilon_0 \varepsilon_r X\, y(t)}{d}$$

$C(t) \propto y(t)$ or measurand

③ dielectric constant : $\varepsilon_r(t)$ or $\varepsilon_r(x)$

$$C(t) = \frac{\varepsilon_0 \cdot \varepsilon_r(t) A}{d}$$

$C(t) \propto \varepsilon_r(t)$ or measurand

④ fringing fields : $\beta(t)$ or $\beta(x)$

→ measurand interacts with the fringing fields

$$C(t) = \frac{\varepsilon_0 \varepsilon_r A \beta(t)}{d}$$

$C(t) \propto \beta(t)$ or measurand