Q = ratio of energy storred in the system to energy lost in the system

For MEMS, Q is usually large

→ "big devices in air" → Q ~ 20

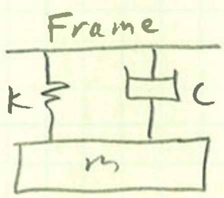
→ "small devices in air" → Q ~ 100s

→ "typical devices in vacuum" → Q ~ 1000s +

→ "specially designed MEMS in vacuum" → Q → 250,000+

a. <u>Transmissibility</u>

Apply a displacement to the frame and measure the displacement of the proof mass as a function of frequency



↑ y(t) → input displacement

↑ x(t) → output displacement

System Diff EQ: $m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0$

$$T(s) = \frac{X(s)}{Y(s)} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\frac{\omega_n}{Q}s + \omega_n^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$ ← real zero in numerator

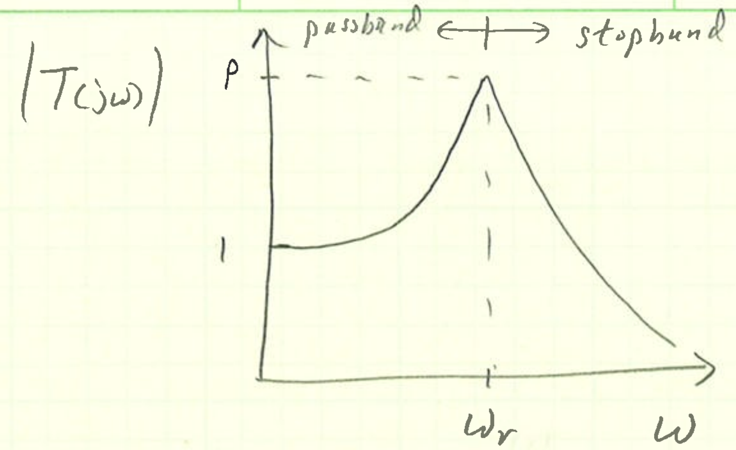
$|T(j\omega)| \equiv$ Transmissibility

$$= \sqrt{\frac{1 + \left(\frac{\omega}{Q\omega_n}\right)^2}{\left[1 + \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{\omega}{Q\omega_n}\right)^2}}$$

$$\left.|T(j\omega)|\right|_{\omega=\omega_n} = \sqrt{Q^2+1} \approx Q \text{ for } Q \gg 1$$

$$\theta_{T(j\omega)} = \tan^{-1}\left[\frac{\frac{1}{Q}\left(\frac{\omega}{\omega_n}\right)^3}{1-\left(\frac{\omega}{\omega_n}\right)^2 + \left(\frac{\omega}{Q\omega_n}\right)^2}\right] \rightarrow \left.\theta\right|_{\omega=\omega_n} = \tan^{-1}[Q]$$

if $Q=1 \rightarrow \left.\theta\right|_{\omega=\omega_n} = 45^\circ$, $\left.\theta\right|_{\omega=0} = 0^\circ$, $\left.\theta\right|_{\omega\to\infty} = 90^\circ$



$\frac{c}{m} = \frac{\omega_n}{Q}$

$\omega_n^2 = \sqrt{\frac{k}{m}}$

For MEMS, $Q \gtrsim 5$:

$\omega_r \approx \omega_n$

$P \approx Q$

∴ read ω_n + Q of plot of $|T(j\omega)|$

→ a highly underdamped lowpass response

stopband attenuation ↑ when Q ↑

<u>but</u> resonant peak ↑ when Q ↑

measure motion with laser interferometers

→ easy to measure $|T(j\omega)|$ by shaking the MEMS device

For high Q system: similar to a 2^{nd} order LPF: ~−40dB/Decade

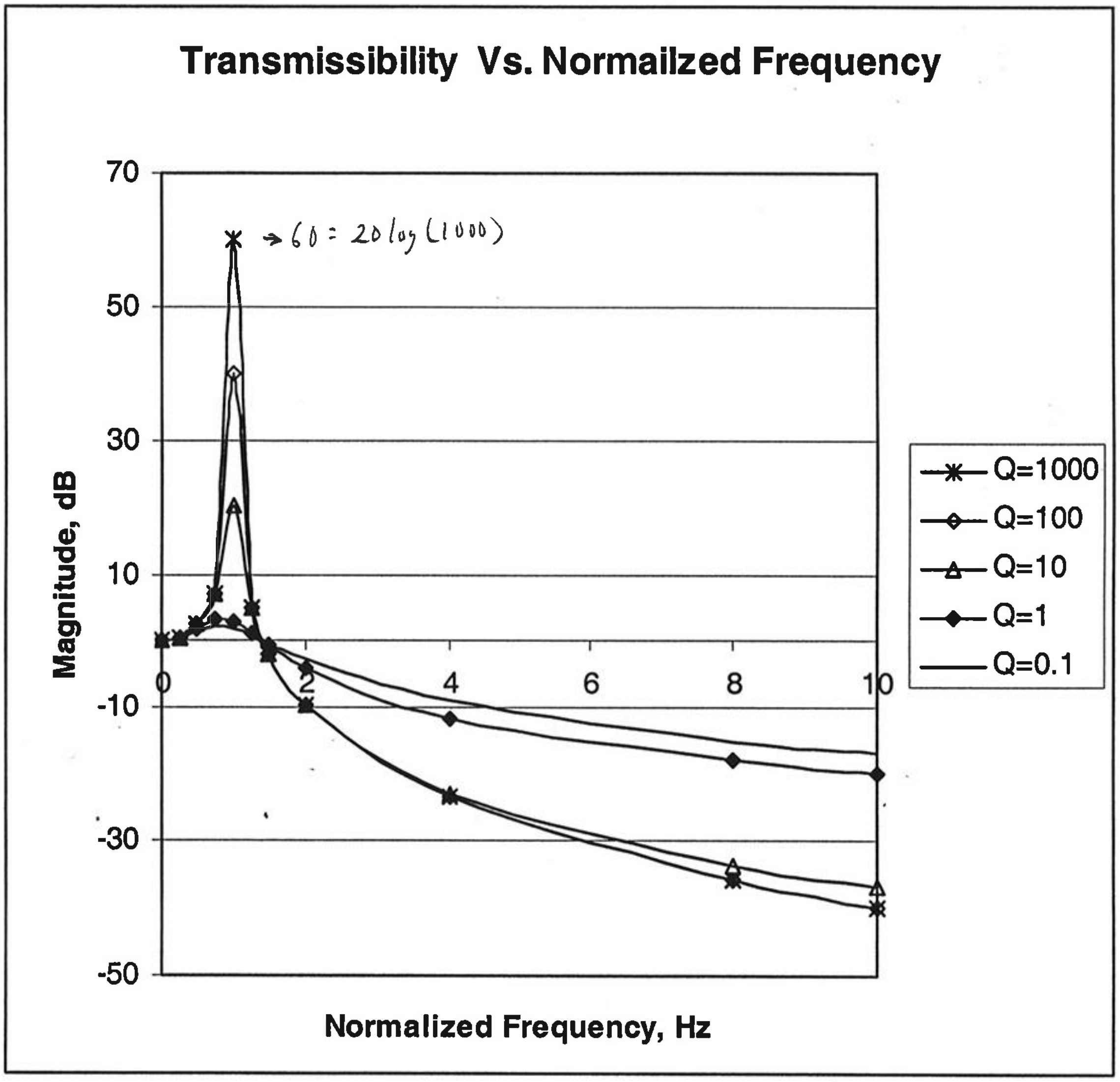
For low Q system: similar to a 1^{st} order LPF: ~−20dB/Decade

ex: Q=1 → atten. from $2\omega_n$ to $20\omega_n$ → 21.85dB ≈ 1^{st} order LPF

Q=10 → atten from $2\omega_n$ to $20\omega_n$ → 35.64dB

Q=1000 → atten from $2\omega_n$ to $20\omega_n$ → 42.48dB ≈ 2^{nd} order LPF

Handout 4



Transmissibility Vs. Normailzed Frequency
→ 60 = 20 log (1000)
70
50
30
10
-10
-30
-50
Magnitude, dB
0
2
4
6
8
10
Normalized Frequency, Hz
Q=1000
Q=100
Q=10
Q=1
Q=0.1