Q = ratio of energy stored in the system to energy lost in the system.

For MEMS, Q is usually large

- "big devices in air" \( \Rightarrow Q \approx 20 \)
- "small devices in air" \( \Rightarrow Q \approx 100 \)
- "typical devices in vacuum" \( \Rightarrow Q \approx 1000 \)
- "specially designed MEMS in vacuum" \( \Rightarrow Q \approx 250,000 \)

a. Transmissibility

Apply a displacement to the frame and measure the displacement of the proof mass as a function of frequency.

\[
\begin{align*}
\text{Frame} & \\
\frac{k}{m} & \quad \frac{c}{m} \quad \text{input displacement} \\
\uparrow \quad Y(t) \quad \text{output displacement} \\
\uparrow \quad X(t) \quad & \text{output displacement}
\end{align*}
\]

System Diff EQ: \( m\ddot{x} + c(x - \dot{y}) + k(x - y) = 0 \)

\[
T(s) = \frac{X(s)}{Y(s)} = \frac{2\beta \omega_n s + \omega_n^2}{s^2 + 2\beta \omega_n s + \omega_n^2} = \frac{\omega_n^2 s + \omega_n^2}{s^2 + \frac{\omega_n^2}{\alpha} s + \omega_n^2}
\]

\[
|T(e^{j\omega})| = \sqrt{\frac{1}{\left(1 + \left(\frac{\omega}{\alpha \omega_n}\right)^2\right) + \left(\frac{\omega}{\beta \omega_n}\right)^2}}
\]

\[
\left|T(e^{j\omega})\right| = \sqrt{Q^2 + 1} \approx Q \text{ for } Q \gg 1
\]

\[
\Theta_{T(e^{j\omega})} = \tan^{-1}\left[\frac{\omega \left(\frac{\omega}{\alpha \omega_n}\right)^3}{1 - \left(\frac{\omega}{\alpha \omega_n}\right)^2 + \left(\frac{\omega}{\beta \omega_n}\right)^2}\right] \Rightarrow \Theta = \tan^{-1}\left[\frac{Q}{\omega \omega_n}\right]
\]

If \( Q = 1 \) \( \Rightarrow \Theta = 45^\circ \), \( \Theta = 0^\circ \), \( \Theta = 90^\circ \)
For MEMS, $Q \geq 5$:

- $\omega_r \approx \omega_n$
- $p = Q$

i.e. read $\omega_n + Q$ of plot of $|T_c(j\omega)|$

→ a highly underdamped lowpass response

Stopband attenuation ↑ when $Q ↑$

but resonant peak ↑ when $Q ↑$

> easy to measure $|T_c(j\omega)|$ by shaking the MEMS device

For high $Q$ system: similar to a 2nd order LPF: ~40 dB/Decade

For low $Q$ system: similar to a 1st order LPF: ~20 dB/Decade

ex: $Q = 1$ → attenuation from $2\omega_n$ to $20\omega_n$ → 21.85 dB ≈ 1st order LPF

$Q = 10$ → attenuation from $2\omega_n$ to $20\omega_n$ → 35.64 dB

$Q = 1000$ → attenuation from $2\omega_n$ to $20\omega_n$ → 42.48 dB ≈ 2nd order LPF
Transmissibility Vs. Normalized Frequency

$60 = 20 \log (1000)$

Magnitude, dB

Normalized Frequency, Hz

$Q = 1000$, $Q = 100$, $Q = 10$, $Q = 1$, $Q = 0.1$