1) Torsional Deflections, Continued

If the torsional beam is rectangular as shown below:

\[ J = w^3 \left[ \frac{16}{3} - 3.36 \frac{w}{L} \left(1 - \frac{T_w}{12w^4}\right) \right] \quad \text{for} \quad w \geq t \]

⇒ If \( 2t = 2w = 2a \):

\[ J = \frac{2.25a^4}{3} \]

a. Application

What is the range of motion at the end of the lever arm for a force \( F \)?

Range of motion = \( S = d \frac{\varphi}{J} \)

\[ = d \frac{T_L}{J b} \]

\[ = d \frac{(Fd)L}{J b} \]

\[ = \frac{d(Fd)L}{2.25a^4\left(\frac{E}{2(1+\nu)}\right)} \]
Advantage of Torsional System

Translational system:

\[ \text{apply } F, \text{ get displacement } d \]

Torsional system:

\[ s = d \theta \]

For some \( F \), \( s \) increases as \( d \) increases

\( \rightarrow \) can design torsional systems to get a "displacement amplification"

2. Damping

Inertial Force: \( F_i = ma = mx \)

Spring Force: \( F_s = kd = kx \)

Damping Force: \( F_d = cv = cx \)

\( \rightarrow \) damping force is proportional to velocity

\( \rightarrow \) caused by energy loss mechanisms in the system

\( C = \text{damping coefficient} : [C] = \text{Kg/s} \)
Damping Sources: Internal and External

Internal → Thermoelastic Damping → internal coupling of mechanical stress/strain and heat flow in the material

External →
1. Friction
2. Impact
3. Interaction with surrounding fluid
   → Squeeze-Film Damping → from the compression of the surrounding fluid
   Fluid is forced out by compression

   → Shear-Resistance Damping → from the resistance to shearing of a fluid as an object moves through it

where fluid shearing occurs

4. Active damping
   \[ F_0 = CX \]
   i.e., measure \( x \) and apply a force proportional to \( x \)
   → results in an ability to adjust or tune the damping

5. Eddy I damping → time varying magnetic field in a conductor
Mechanical Schematic Symbol for Damping

\[ \rightarrow \text{dashpot} \]

in our spring-mass-damper system:

\[ \rightarrow \text{x} \]

in MEMS, the device is often packaged in a low to high vacuum to control or minimize damping

\[ 10^3 \text{ Torr} \rightarrow 10^7 \text{ Torr} \]

Note: 760 Torr \approx \text{atmospheric pressure}

3. System Dynamics

\[ F_1 + F_d + F_3 = 0 \]

\[ m\ddot{x} + c\dot{x} + kx = 0 \rightarrow 2^{\text{nd}} \text{ order DIFF Eq, Linear, const. coefficients} \]

\[ \dot{x} + \frac{c}{m} \dot{x} + \frac{k}{m} x = 0 \rightarrow \dot{x} + \frac{\omega_n}{\omega_n} \dot{x} + \omega_n^2 x = 0 \]

\[ \rightarrow \text{Note: see Class Web Site for solution to this eq} \]

results: \[ \omega_n = \sqrt{\frac{k}{m}} = \text{system natural frequency} [\omega_n] = \text{rad/s} \]

\[ \zeta = \text{damping ratio} \rightarrow \text{dimensionless} \]

\[ Q = \text{quality factor (high Q = low damping)} \]

\[ Q = \frac{1}{\zeta} \]

\[ 2\zeta \omega_n = \frac{\omega_n}{Q} = \frac{c}{m} \]