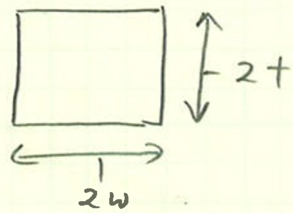
1) Torsional Deflections, Continued

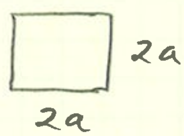
If the torsional beam is rectangular as shown below:

more typical of MEMS devices



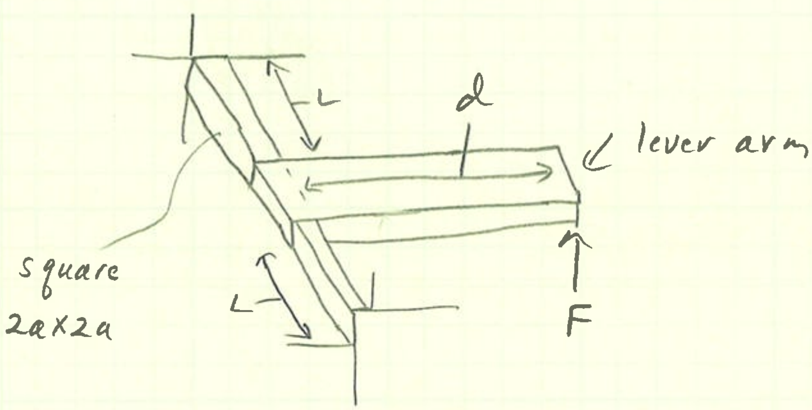
$$J = wt^3\left[\frac{16}{3} - 3.36\frac{t}{w}\left(1 - \frac{t^4}{12w^4}\right)\right] \text{ for } w \geq t$$

→ If $2t = 2w = 2a$:



then $J = 2.25a^4$

a. Application



What is the range of motion at the end of the lever arm for a force F?

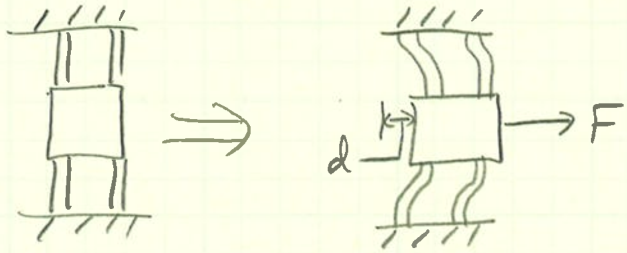
Range of motion $= S = d\Phi$

$$= \frac{dTL}{JG}$$

$$= \frac{d(Fd)L}{2.25a^4\left(\frac{E}{2(1+\nu)}\right)}$$

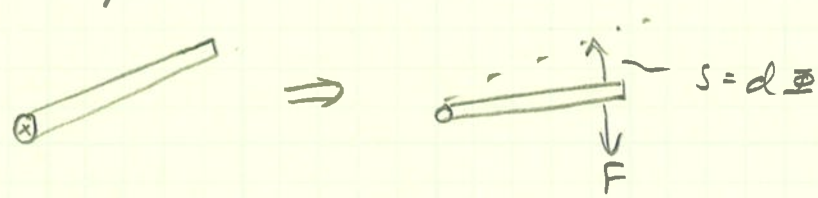
Advantage of Torsional System

Translational system:



apply F, get displacement d

Torsional system:



For same F, S increases as d increases

→ Can design torsional systems to get a "displacement amplification"

2. Damping

Inertial Force : $F_I = ma = m\ddot{x}$

Spring Force : $F_s = kd = kx$

Damping Force : $F_D = cv = c\dot{x}$

→ damping force is proportional to velocity

→ caused by energy loss mechanisms in the system

$c \equiv$ damping coefficient : $[c] = kg/s$

Damping Sources : Internal and External

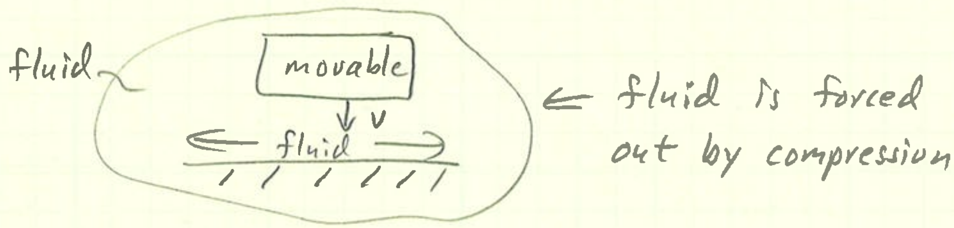
Internal → Thermoelastic Damping → internal coupling of mechanical stress/strain and heat flow in the material

External → ① Friction

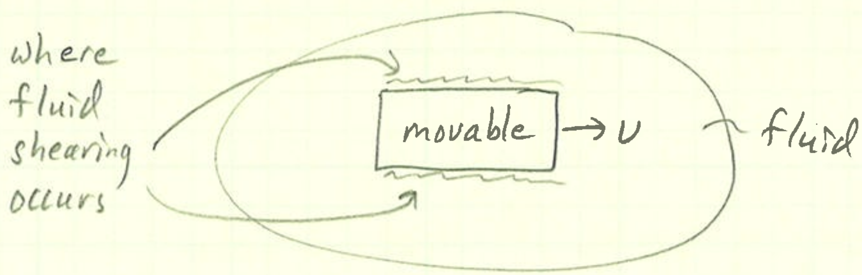
② Impact

③ Interaction with surrounding fluid

→ Squeeze-Film Damping → from the compression of the surrounding fluid



→ Shear-Resistance Damping → from the resistance to shearing of a fluid as an object moves through it



④ Active damping

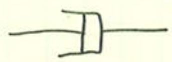
$F_0 = c\dot{x}$

∴ measure $\dot{x}$ and apply a force proportional to $\dot{x}$

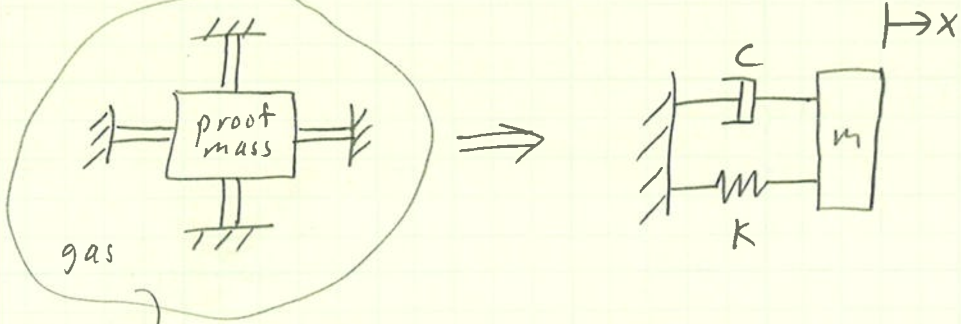
→ results in an ability to adjust or tune the damping

⑤ Eddy I damping - time varying magnetic field in a condutor

Mechanical Schematic Symbol for Damping

 → dashpot

∴ our spring-mass-damper system:



→ in MEMS, the device is often packaged in a low to high vacuum to control or minimize damping

low ↓ 10^{-3} Torr

high ↓ 10^{-7} Torr

<u>Note</u>: 760 Torr ≈ atmospheric pressure

3. <u>System Dynamics</u>

$$F_I + F_D + F_S = 0$$

$$m\ddot{x} + c\dot{x} + kx = 0 \quad \rightarrow \text{2nd order DiffEQ, Linear, const. coefficients}$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0 \quad \rightarrow \quad \ddot{x} + \frac{\omega_n}{Q}\dot{x} + \omega_n^2 x = 0$$

→ <u>Note</u>: see Class Web Site for solution to this eq

<u>results</u>: $\omega_n = \sqrt{\frac{k}{m}}$ = system natural frequency $[\omega_n]$ = rad/s

ζ = damping ratio → dimensionless

Q = quality factor (high Q = low damping)

$$Q = \frac{1}{2\zeta}$$

$$2\zeta\omega_n = \frac{\omega_n}{Q} = \frac{c}{m}$$