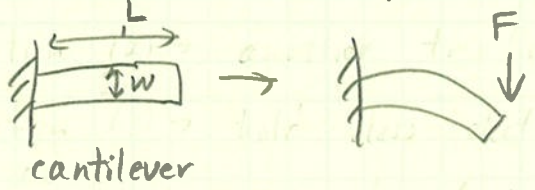
1) Review

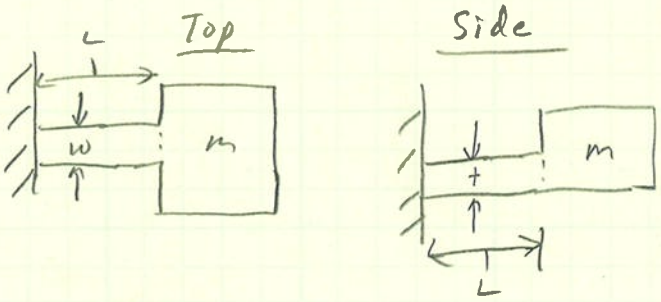
From beam theory:



$$K = \frac{Ewt^3}{4L^3} \qquad [k] = N/m$$

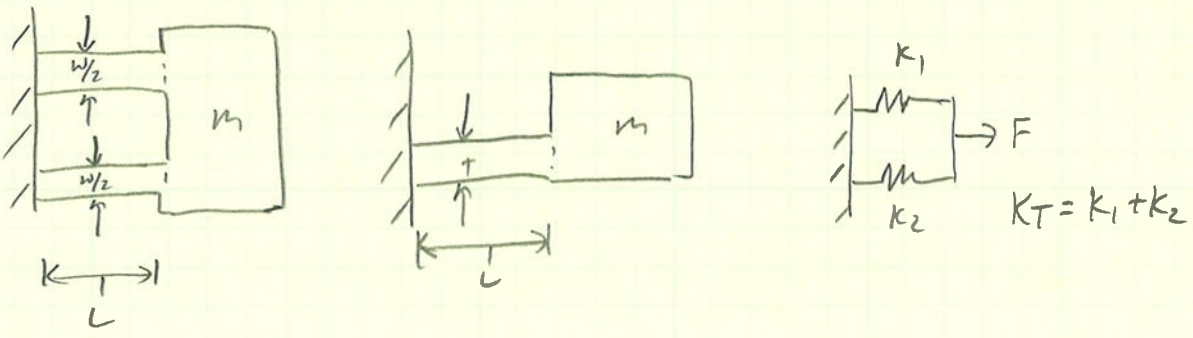
2) More on Springs

a. Fixed-Free Example



$$K = \frac{Ewt^3}{4L^3}$$

Consider this



$$K_T = K_1 + K_2$$

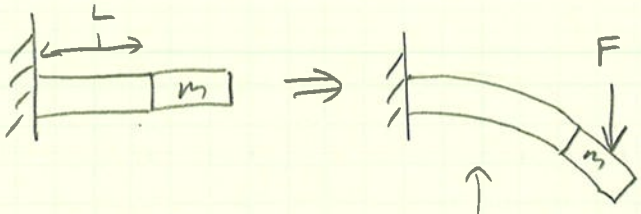
$$K_1 = K_2 = \frac{E\left(\frac{w}{2}\right)t^3}{4L^3}$$

$$\therefore K_T = K_1 + K_2 = 2K_1 = \frac{Ewt^3}{L^3} \rightarrow \text{same as before}$$

Note: other effects may be different

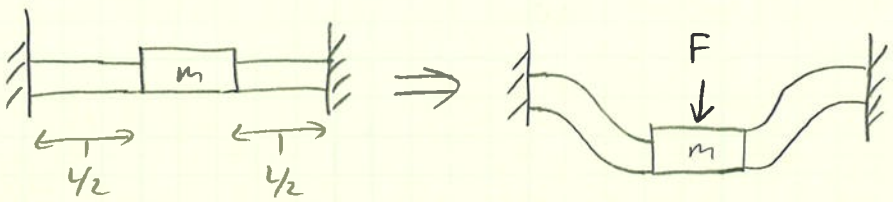
b. Fixed- Fixed Example

Fixed-Free :



spring deflects, but is not stretched

Fixed-Fixed :



spring deflects <u>and</u> is stretched (in tension)

This case is called "statically indeterminate"

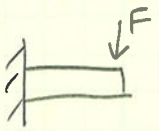
→ simple beam equations cannot be used to solve for k

→ more variables than equations

<u>Reason</u>: Deflection and tension → k higher than Fixed-Free spring

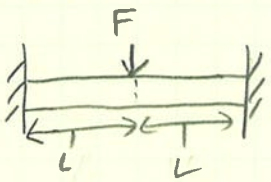
<u>Approximation</u> :

<u>Fixed - Free</u>



$K = \frac{Ewt^3}{4L^3}$

<u>Fixed-Fixed</u>



$K \approx \frac{2Ewt^3}{L^3}$ → for small deflections compared to spring length

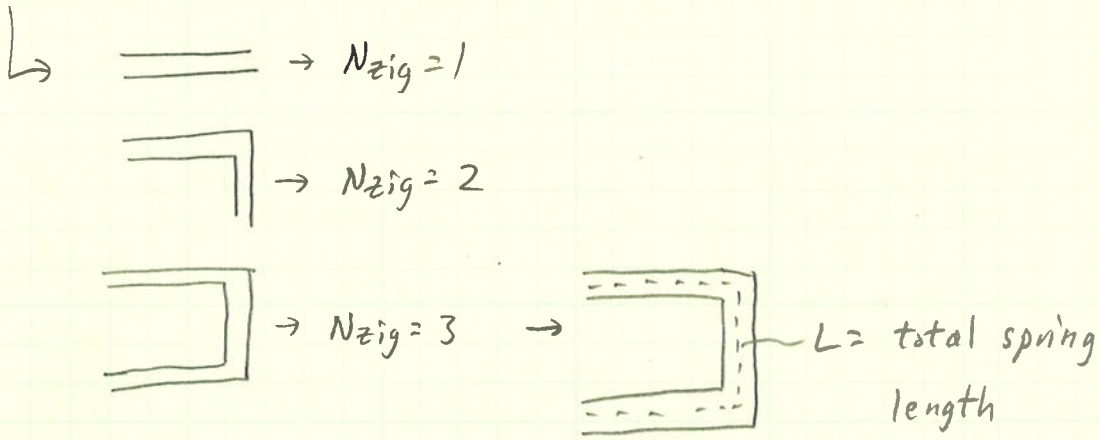
→ $k_{Fixed\text{-}Fixed} > k_{Fixed\text{-}Free}$ for same E, w, t, L

Fixed – Fixed with multiple springs (all with same E, w, t, L)

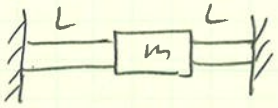
$$\rightarrow K_T = \frac{N_{Leg}}{N_{zig}} \frac{E w t^3}{L^3}$$

N_{Leg} = # of spring elements

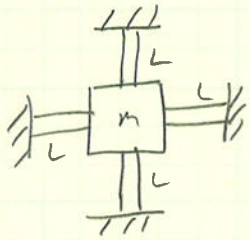
N_{zig} = # of cutbacks (straight beam = 1, folded beam = 2, ...)



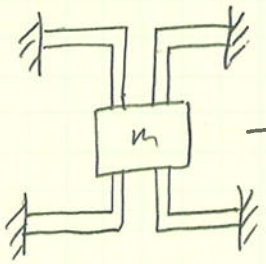
Examples

(1) 

$\rightarrow K \approx \frac{2}{1} \frac{E w t^3}{L^3}$

(2) 

$\rightarrow K \approx \frac{4}{1} \frac{E w t^3}{L^3}$

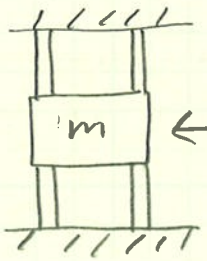
(3) 

$\rightarrow K \approx \frac{4}{2} \frac{E w t^3}{L^3}$

$[k] = N/m$

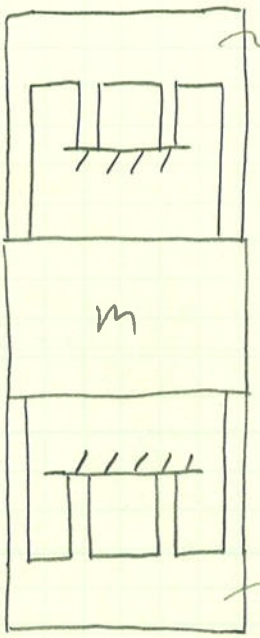
c. Suspension Systems Commonly Used

(1)



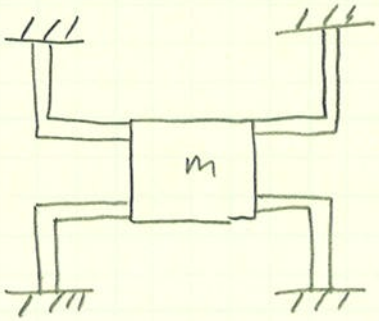
$\longleftrightarrow$ motion $\rightarrow$ "Hammock Flexture"

(2)



$\longleftrightarrow$ motion $\rightarrow$ "Folded Flexture"

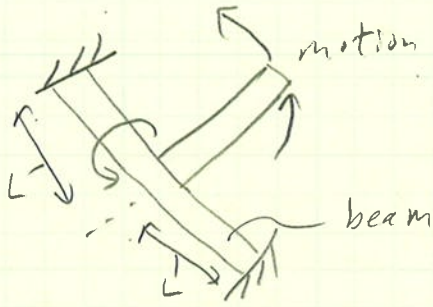
(3)



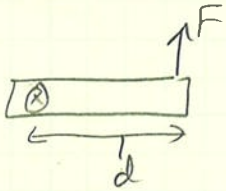
$\rightarrow$ "Crab Leg Flexture"

4 springs
2 foldbacks
$\Big\}$ $k \simeq \frac{4}{2} \frac{Ewt^3}{L^3}$

3) Torsional Springs



beam twists → torsional spring



Torque = $T = Fd$

J = Polar or Torsional Moment of Inertia

for a round member: $J = \int r^2 dA = \frac{\pi r_0^4}{2}$, radius = r_0

Angular Displacement at center = Φ

$$\Phi = \frac{TL}{JG} \qquad (\Phi) = \text{radians}$$

$$G = \frac{E}{2(1+\nu)}, \quad \nu = \text{Poisson's Ratio}$$

G = Shear modulus of elasticity