

1. Stress and Strain continued

Si, Poly Si and Si nitride are brittle materials

→ little or no plastic deformation

→ catastrophic failure after elastic regime exceeded

a. Single crystal Si

$$130 \text{ GPa} \leq E \leq 187 \text{ GPa}$$

→ crystal plane dependant

$$0.055 \leq \nu \leq 0.36$$

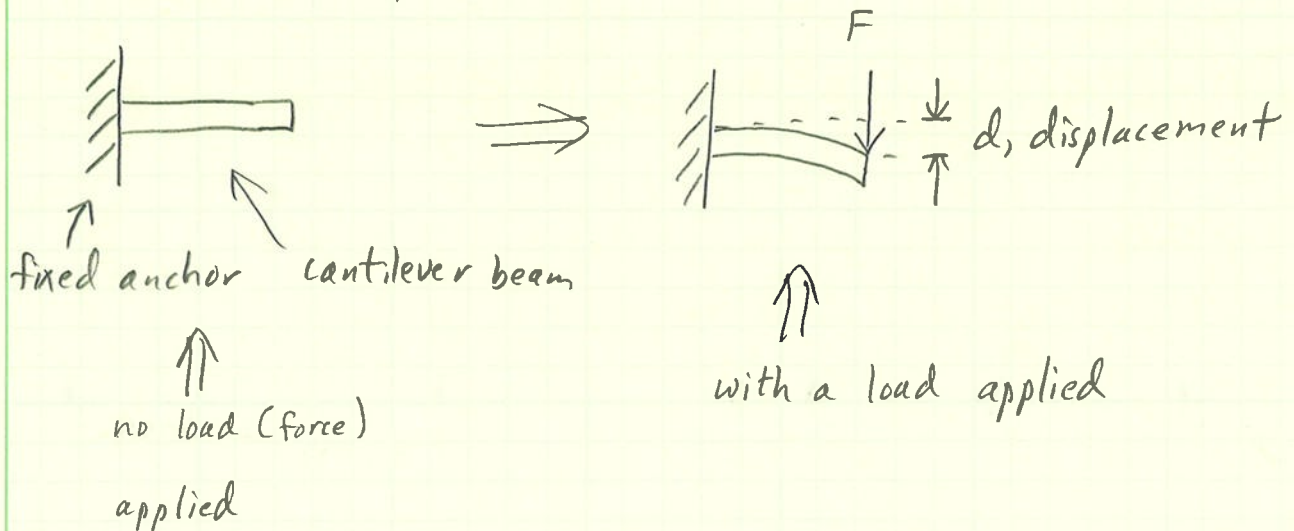
→ Compare Si + steel ($E = 200 \text{ GPa}$)

Si is 65% to 90% as strong as steel in the elastic regime

But steel deforms plastically while Si does not

2. Beams

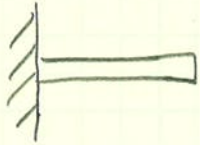
→ def: a structural member experiencing lateral loads and that responds by elastically deforming in proportion to the applied load



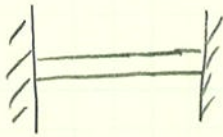
A "beam" is also called a "spring" or a "flexure"

a. Some example beams encountered in MEMS:

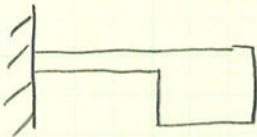
unloaded



simple cantilever
(fixed-free)

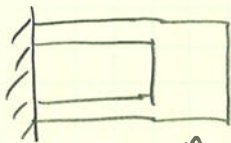


bridge
(fixed-fixed)



Cantilever with
attached proof
mass

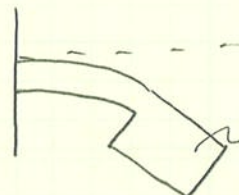
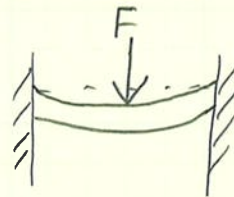
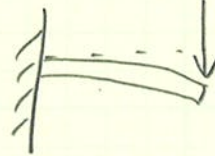
anchor and
proof mass are rigid



2 beams in
parallel

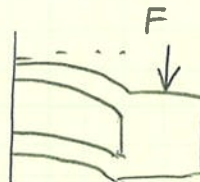
rigid section

loaded F



mass of proof
mass = m

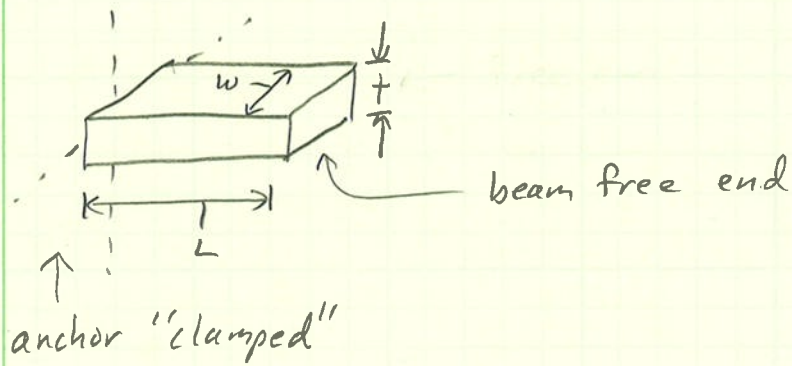
g (gravity); $F = mg$



displacement

and more complicated designs

b. Simple Cantilevered beam (rectangular)



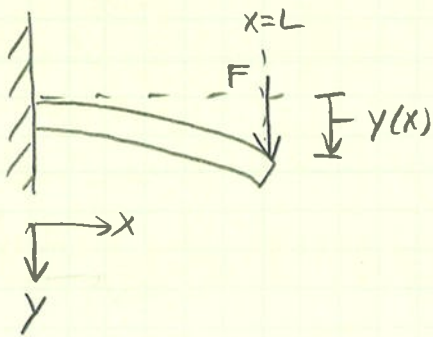
Definition: $t \rightarrow$ thickness \rightarrow direction in which beam bends or deflects

$L \rightarrow$ length \rightarrow this dimension curls as the beam deflects

$w \rightarrow$ width \rightarrow direction perpendicular to t and L

Moment of inertia $\equiv I = I_z = \frac{wt^3}{12} \rightarrow$ for a rectangular beam

Deflection due to an applied force



$$y(x) = \frac{Fx^3}{3EI}, \quad I = \frac{wt^3}{12}$$

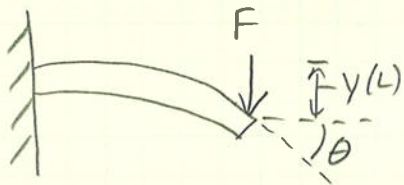
$$\therefore y(x) = \frac{4Fx^3}{Ewt^3}$$

$$\text{at } x=L \rightarrow y(L) = \frac{4FL^3}{Ewt^3}$$

Define spring constant, $k = \frac{\text{Applied Force}}{\text{Displacement}}$, $[k] = \text{N/m}$

$$k = \frac{F}{y(L)} = \frac{Ewt^3}{4L^3}$$

Deflection Angle: θ



$$\theta = \frac{FL^2}{2EI} = \frac{6FL^2}{Ewt^3}$$

note: θ is the angle at the end of the beam only

Schematic symbol for a spring: $\overset{k}{\text{---} \text{M} \text{---}}$

An anchored spring: $\text{---} \text{M} \text{---} \xrightarrow{F}$
 \xrightarrow{x} , direction of motion

Two springs in series: $\text{---} \text{M}_{k_1} \text{---} \text{M}_{k_2} \text{---} \xrightarrow{F}$
 \xrightarrow{x}
 $K_T = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} \Rightarrow$ adds like capacitors

Two springs in parallel: $\begin{array}{c} \text{---} \text{M}_{k_1} \text{---} \\ \text{---} \text{M}_{k_2} \text{---} \end{array} \xrightarrow{F}$, $K_T = k_1 + k_2$
 \xrightarrow{x}