Application of Signal and Noise Theory to Digital VLSI Testing*

Nitin Yogi† and Vishwani D. Agrawal
Auburn University, Department of Electrical and Computer Engineering, Auburn, AL 36849, USA
nyogi@nvidia.com, *vagrawal@eng.auburn.edu*

Abstract – Circuit dependent vectors like functional verification vectors, RTL test vectors, or gate-level ATPG vectors contain circuit specific information in the form of spatial correlations (among bits of a vector) and temporal correlations (among bits of the bit stream at an input pin). Some specified bits have don't care behavior because they can be changed without affecting the relevant (testing or functional) properties of the signal. In this paper, we develop a functional analysis framework for digital signals which extracts this information content from the vectors under consideration. Our proposed method is based on spectral analysis of binary bit-streams using Hadamard transform. A bit-stream corresponding to an input pin is transformed to Hadamard spectral components. The information content is distinguished from the noise in the signal using spectral analysis of random binary bit-streams. The magnitude of the spectral components represent temporal correlations while the phases of spectral components of separate bit streams represent the spatial correlations. Applications to ATPG, test compression and BIST (combinational and sequential), as described in recent publications, will benefit from this analysis, because the previous works have used ad-hoc methods for extracting spectral components from samples of test signals. We illustrate the analysis with an application to test generation for sequential benchmark circuits.

1. Introduction

Besides structural algorithms for test generation [4], years of research has produced many vector-oriented approaches, including random vectors [20, 30], weighted random vectors [3, 14, 23, 29, 30] and property-based vectors [13]. Vector-oriented methods do work but may not be satisfactory for all circuits.

The idea of analyzing the periodicities in signals for test generation introduced the field of spectral testing. Test vectors, which provided high fault coverage, were analyzed in the frequency or the spectral domain for information content. Several published books and articles [2, 8, 16, 26] provide introduction, general properties and applications of spectral transforms for digital signals. It is believed that good quality test vectors, which give high fault coverage, exhibit certain discernible frequency or spectrum related characteristics. By preserving these characteristics good quality high defect coverage test vectors can be generated.

Spectral methods find crucial and varied applications in the field of VLSI circuit testing. Susskind [24] showed that Walsh spectrum can be used for testing a digital circuit for stuck faults by verifying the Walsh coefficients (signatures) at the outputs. Hsiao and Seth [15] further expanded that work to compact testing where the signature formed by compaction of output responses is chosen to be a coefficient from the Rademacher-Walsh (RW) spectrum of the function under test. More recently, Giani *et al.* [10, 11] reported spectral techniques for sequential ATPG and built-in self-test. In [10], starting from pseudo-random vectors the generation of new test vectors is guided by the spectral components of previously beneficial generated vectors. In [11], a spectral BIST scheme generates test vectors from stored prominent spectral components by executing a program on a processor. Virginia Tech group has published further work on spectrum-based self test and core test [5, 6, 17]. Khan *et al.* [18, 19] have designed hardware output response compactors using spectral analysis. Zhang *et al.* [39] refined the method of extracting spectra from a binary signal using a selfish gene algorithm. Recent work uses wavelet transforms for similar applications [7].

In this paper we present a functional analysis framework for analyzing the information and noise content in digital signals, applicable to applications like ATPG, test compression and BIST. In Section 2 we introduce the basic theory of Hadamard transform with its salient properties. In Section 3 we perform spectral analysis on random binary bit-streams. We develop a thresholding criteria for distinguishing information and noise contents in Section 4. We describe how to generate new bit-streams in Section 5. Section 6 illustrates few applications.

2 Hadamard Transform

The Hadamard transform was first introduced by Ohnsorg [21] as Binary Fourier Representation or BI-

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†Presently with NVIDIA Corporation, Santa Clara, CA 95050, nyogi@nvidia.com.

FORE. The idea was to introduce a transform for analyzing the frequency related characteristics for binary digital signals, analogous to the Fourier transform, which is used for analog signals. While the Fourier transform uses sine functions as their basis, the Hadamard transform [1, 22, 27] uses a set of orthogonal functions called Walsh functions.

Walsh functions consist of trains of square pulses having $+1s$ and $-1s$ as the allowed states and can only change at fixed intervals of a unit time step. For an order n, there are $N = 2^n$ Walsh functions, given by the rows of the $2^n \times 2^n$ Hadamard transform matrix $H(n)$, when the functions are arranged in the so-called "natural" order [25, 28]. The Walsh functions include patterns with varying periodicities or bit-flips which are analogous to the sine and cosine functions in the analog domain. Hence, Walsh functions can be thought of as digital counterparts of analog sine frequencies.

Any binary bit-stream can be uniquely represented as a linear combination of the orthogonal Walsh functions. This is analogous to the analog domain where any continuous signal can be uniquely represented as a linear combination of the sine and the cosine functions. Thus, by analyzing the binary signals using Walsh functions, we are actually looking into the frequency or sequency characteristics of the digital waveforms. Just as frequencies refer to periodicities for analog signals, sequencies are bit-flippings in digital waveforms [27].

Hadamard transform matrix, which incorporates the Walsh functions, can be defined in two ways [12], using a binary (base-2) representation or recursively. Using a binary (base-2) representation, the element at the j^{th} row and k^{th} column of the Hadamard matrix is,

$$H(j,k) = \frac{1}{N}(-1)^{\alpha}, \text{ where } \alpha = \sum_{i=0}^{n-1} b_j(i) b_k(i) \qquad (1)$$

Here $b_j[i]$ and $b_k[i]$ are the i^{th} binary bits of the corresponding binary numbers b_j and b_k respectively. b_j and b_k are the binary representations of the corresponding integer values j and k given by the following relations:

$$j = b_j[n-1]2^{(n-1)} + b_j[n-2]2^{(n-2)} + ... + b_j[0]2^0 \quad (2)$$

$$k = b_k[n-1]2^{(n-1)} + b_k[n-2]2^{(n-2)} + ... + b_k[0]2^0 \quad (3)$$

Hadamard matrices can also be generated using the following recurrence relation:

$$H(n) = \begin{bmatrix} H(n-1) & H(n-1) \\ H(n-1) & -H(n-1) \end{bmatrix} \qquad (4)$$

where $H(0) = 1$ and 2^n is the dimension of the nth order Hadamard matrix, $H(n)$.

2.1 Forward and Reverse Transformation

The Hadamard transform can be applied to any binary signal, provided the length of the signal is equal to the dimension of the Hadamard matrix. Given a binary signal X of length N, represented as a column vector of dimensions $N \times 1$, its transform S is,

$$S = \frac{1}{N}\left[H(n) \times X\right] \qquad (5)$$

where $N = 2^n$, H(n) is an n^{th} order Hadamard matrix of dimensions $2^n \times 2^n$ and S is a vector of Hadamard transform coefficients of dimensions $N \times 1$.

$$\frac{1}{8}\left[H(3)\right] \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 2 \\ \mathbf{6} \\ -2 \\ 2 \\ 2 \\ -2 \\ -2 \\ 2 \end{bmatrix}$$

bit stream   spectral components

Figure 1. Spectral analysis of 8-bit bit-stream.

$$\left\{\tfrac{1}{8}\left[2 \;\; \mathbf{6} \; -2 \;\; 2 \;\; 2 \; -2 \; -2 \;\; 2\right]\right\} \times H(3) \\ = \left[1 \; -1 \;\; 1 \;\; 1 \;\; 1 \; -1 \;\; 1 \; -1\right]$$

Figure 2. Obtaining original bit-stream from Hadamard coefficients.

Figure 1 shows an example of performing forward Hadamard transformation on a binary bit-stream of length 8, which is represented as $-1s$ and $+1s$, since the Hadamard matrix consists of these two values.

The original binary signal X can be faithfully recovered from the Hadamard coefficients S by the reverse transformation given by:

$$X = H(n) \times S \qquad (6)$$

Figure 2 shows the reverse transformation of obtaining the original bit-stream from Hadamard coefficients for the example shown in Figure 1.

2.2 Properties of Hadamard Transform

Here we shall discuss some of the salient properties of Hadamard transform matrix. Some of these are directly deduced from the definition of the transform, while others were proved and presented in earlier works [1, 9, 21]

2.2.1 Orthogonality and Symmetry

As observed from the definition, the Hadamard matrix is an orthogonal and symmetric matrix [1] and has the property:

$$[H(n)]\,[H(n)]^T = n \;\; I(N) \qquad (7)$$

where $I(N)$ is an identity matrix of dimension $N \times N$ and $N = 2^n$. This property allows forward and reverse Hadamard transformations defined earlier, using computationally efficient matrix multiplication.

2.2.2 Energy Conservation

Parseval's theorem can be defined for the Hadamard transform of a digital signal X, similar to a Fourier transform, as follows [1]:

$$\frac{1}{N}\sum_{k=0}^{N-1} X^2(k) = \sum_{j=0}^{N-1} S^2(j) \tag{8}$$

The above relation establishes that the total energy of a signal defined in the time domain is equal to the total energy contained in the spectral components of the Hadamard transform of that signal. This property serves crucial when interpreting the signal in the spectral domain and distinguishing the signal and noise components in the spectrum.

2.2.3 Total Energy in a Binary Signal

The energy of a binary signal of length N, represented in $[-1s, +1s]$ form, from equation 8, can be given as follows:

$$\sum_{k=0}^{N-1} X^2(k) = \sum_{k=0}^{N-1} 1 = N \tag{9}$$

From equations 8 and 9, it follows that the total energy in the spectrum is equal to unity:

$$\sum_{j=0}^{N-1} S^2(j) = \frac{1}{N}\sum_{k=0}^{N-1} X^2(k) = \frac{1}{N}\sum_{k=0}^{N-1} 1 = 1 \tag{10}$$

3 Random Binary Bit-Streams

In order to retrieve the information and noise content in binary bit-streams using spectral analysis, it is important to analyze and study the characteristics of random binary bit-streams in the spectral domain first. Here we consider binary bit-streams, which take values $+1$ and -1. Let Xr be a randomly generated binary bit-stream of length N and its Hadamard transform be Sr, as given by equation 5. The bits of Xr can be treated as independent random variables that take the values $+1$ and -1. Each component of Sr is a weighted average of those random variables and hence will have a closely approximated Gaussian distribution [38] with zero mean. The standard deviation σ of each Gaussian distributed Sr component can be found from the following relation from statistical theory:

$$\sigma = \sqrt{\frac{1}{N}\left(\sum_{j=0}^{N-1} Sr^2(j)\right) - \overline{Sr}^2} \tag{11}$$

where $\overline{Sr}$ is the mean of the values of Sr and is equal to zero. Also from equation 10, we know that $\sum_{j=0}^{N-1} Sr^2(j) = 1$. Hence, equation 11 for standard deviation of Sr reduces to $\sigma = \sqrt{\frac{1}{N}}$.

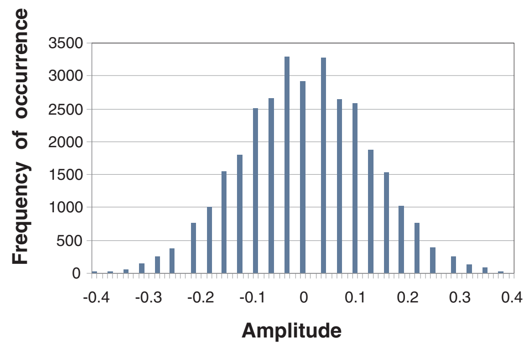

Figure 3. Frequency of occurrence of spectral coefficients for random bit-streams.

To verify this, we performed an experiment. We generated 500 samples of random binary bit-streams of length 64 and obtained the Hadamard transform of each bit-stream. Thus, a total of $64 \times 500 = 32,000$ spectral coefficient amplitudes were obtained. A histogram of those amplitudes is shown in Figure 3. For this histogram, the mean is 0.000438 (~ 0) and standard deviation equals 0.125053 ($\sim \sqrt{\frac{1}{64}}$).

4 Noise Power Level

The use of spectral analysis in testing of digital circuits, is based on the premise that circuit specific good quality test vectors (like functional verification vectors, RTL test vectors or gate-level ATPG vectors) exhibit discernible spectral characteristics. By reproducing these characteristics, effective high fault coverage test vectors can be obtained. Hence it becomes important to discriminate between information and noise content in a given signal or bit-stream. This analysis is applicable and is used in different forms, in various other fields like communication theory, signal estimation, electromagnetic theory, etc.

To perform this noise analysis, we use spectral characteristics of random binary bit-streams. Here we are interested in determining a magnitude threshold for the spectral coefficients, which can be used to discriminate between spectral coefficients which have information content and those which exhibit noise-like or random behavior. Any spectral coefficient having a magnitude above this threshold would be considered as essential, containing information, while the others would be considered as random or noise-like. A threshold of zero would classify all spectral coefficients as having information, which would reproduce the original test vectors without any modifications. From the earlier analysis, we determined that the values of the spectral coefficients of a random binary bit-stream can be approximated as a Gaussian distribution having zero mean and a standard deviation equal to $\frac{1}{\sqrt{N}}$, where N is the dimension of the Hadamard matrix. From statistical theory we know that, for a Gaussian distribution, the per-

$$\text{(a) Filtering spectra : } \begin{bmatrix} 2/8 \\ \mathbf{6/8} \\ -2/8 \\ 2/8 \\ 2/8 \\ -2/8 \\ -2/8 \\ 2/8 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ \mathbf{6/8} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(b) Probabilities obtained from filtered spectra :

$$\{[0\ \mathbf{6/8}\ 0\ 0\ 0\ 0\ 0\ 0] \times H(3)\}$$
$$= [0.75\ \ -0.75\ \ 0.75\ \ -0.75\ \ 0.75\ \ -0.75\ \ 0.75\ \ -0.75]$$
$$\rightarrow [0.875\ 0.125\ 0.875\ 0.125\ 0.875\ 0.125\ 0.875\ 0.125]$$

Figure 4. Probabilities for bit-steam generation by filtering the spectra.

centage of values of the data set lying within a range of one, two and three standard deviations from the mean is 68.27%, 95.45% and 99.73% respectively. A lower value of spectral threshold would lead to certain random spectral coefficients being classified as containing information and vice versa for a higher value of threshold. Hence an appropriate value for the threshold needs to be selected. Nominally, a value of two times the standard deviation is selected as the threshold.

Our proposed analysis method, described above, extracts information content from circuit representative test vectors in the spectral domain. This information can be used for various applications like ATPG, BIST or test compression. In the next section we describe its application to generation of new test vectors.

5 Generating Spectral Test Vectors

After the spectral coefficients containing information are distinguished from those which exhibit random or noise-like behavior, using the spectral threshold, new bit-streams can be generated using this information. We employ the method of filtering and probabilistic generation of bit-streams. The essential spectral coefficients containing information are retained, while the noise-like coefficients are set to zero. Then a reverse Hadamard transformation is performed on the modified spectral coefficients, as given by equation 6. The obtained bit-stream is then scaled from a range of $[-1, +1]$ to a range of $[0, 1]$ by using the scaling formula $(X+1)/2$ where X is the bit-stream. The obtained bit-stream values can now be viewed as probabilities for generating a logic '1' for each bit position. Using a software random number generator, new bit-streams can be generated easily using these probabilities.

Figure 4 shows the process of filtering the noise-like spectral coefficients, whose magnitude is less than the threshold, and obtaining the probabilities, using the example considered in Figure 1. For a bit-stream of length 8, the standard deviation of the values of the spectral coefficients is $\frac{1}{\sqrt{8}} = 0.3535$. We choose a threshold value twice the standard deviation equal to 0.7071. Any spectral coefficient with a magnitude less than the threshold of 0.7071 is considered noise-like and is set to zero. The probabilities obtained after the reverse transformation, in this example, are 0.875 and 0.125 for generating a logic '1'. By comparing the probabilities with Walsh function represented by the prominent coefficient, we observe that in 7 out of 8 cases, the bits generated by the probabilities will conform to the essential Walsh function. Note from Figure 1 that the original bit-stream exhibits the same property in relation to the essential Walsh function.

6 Application of Information Analysis

In this section, we shall describe the application of the information analysis framework, described in earlier sections, to test generation. The information analysis framework can be used for various other applications including BIST and test data compression as proposed in our earlier works [32, 33, 35, 37].

As mentioned earlier, certain spectral characteristics of test vectors are beneficial for obtaining high fault coverage in the Circuit-Under Test (CUT). The essential spectral characteristics are unique and representative of the CUT and can be obtained from various sources. In our experiments, we use a vector set which detects RTL faults [33] of the CUT, which are the stuck-at faults on primary inputs and outputs of the different modules/circuit and on inputs and outputs of all flip-flops. The generated RTL vectors, although have low fault coverage over all the faults, we believe they capture the essential information required for producing high fault coverage test vectors. We analyze the RTL vectors in the spectral domain by performing a Hadamard transformation. The bits being applied to each of the inputs are analyzed independently. The spectral coefficients containing information are distinguished from those which are noise-like by using an appropriate threshold, as discussed in section 4. The noise-like spectral coefficients are eliminated by setting them to zero and then probabilistic test vectors are generated as described in Section 5.

We applied our methodology to three ISCAS'89 benchmark circuits; s5378, s35932 and s38417. We compare our results with three other methods of generating vectors, which are, random vectors, weighted random vectors and randomly perturbed vectors. The weights for the weighted random vectors were obtained from the generated RTL vectors. The randomly perturbed vectors are generated by randomly perturbing a small fraction of the bits in the RTL test vector set. The different types of generated vectors were fault simulated on the CUT. Two cases were analyzed for spectral vectors; one where only spectral vectors were fault simulated and the other where original RTL vectors were fault simulated first, followed

Table 1. Amplitude and power of essential spectral coefficients and noise power for inputs of s1488 benchmark circuit.

Inputs	Spec. Coeffs.	Ampl.	Power	Noise Power
Input 1	H0	0.75	0.56	0.44
Input 2	H1	0.50	0.25	0.61
	H13	-0.38	0.14	
Input 3	H1	0.56	0.32	0.49
	H19	0.44	0.19	
Input 4	H0	0.63	0.39	0.47
	H22	-0.38	0.14	
Input 5	H1	-0.50	0.25	0.00
	H5	0.50	0.25	
	H19	0.50	0.25	
	H23	0.50	0.25	
Input 6	H4	0.50	0.25	0.50
	H22	0.50	0.25	
Input 7	H1	0.50	0.25	0.47
	H5	-0.38	0.14	
	H30	0.38	0.14	
Input 8	H4	0.38	0.14	0.58
	H12	0.38	0.14	
	H22	0.38	0.14	

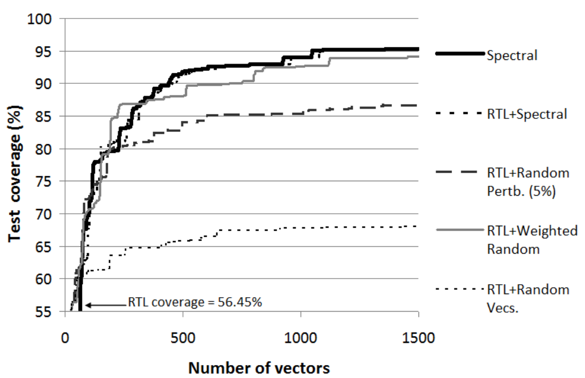

Figure 5. Test coverage for s1488 circuit.

by the spectral vectors (RTL+Spectral). RTL vectors were also appended to random, weighted random and randomly perturbed vectors. These are referred to as RTL+Random, RTL+Weighted random and RTL+Randomly perturbed vectors, respectively.

Table 1 shows the essential spectral coefficients, their amplitudes and power, along with the noise power in the spectrum, for the 8 inputs of the benchmark circuit s1488, by using a spectral threshold of twice the standard deviation. Figures 5, 6 and 7 show the growth of test coverage with the application of test vectors for Spectral, RTL+Spectral, RTL+Randomly perturbed, RTL+Weighted random and RTL+Random vectors. Clearly, coverages for Spectral and RTL+Spectral vectors are close to each other, showing that the spectral vectors capture adequately the information in RTL vectors. Also, we observe that in all three circuits,

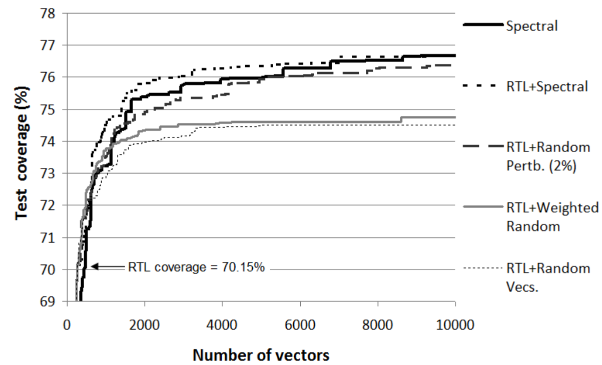

Figure 6. Test coverage for s5378 circuit.

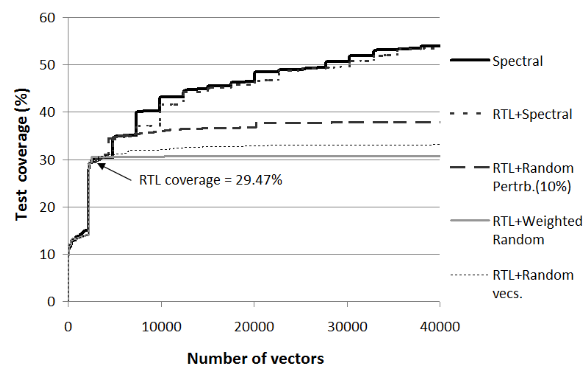

Figure 7. Test coverage for s38417 circuit.

the spectral vectors perform better, significantly or marginally sometimes, than other methods. It was also observed that simulating RTL vectors detected additional faults over those detected by the three different types of test vectors. This method of spectral analysis and vector generation can enhance previously proposed spectral ATPG algorithms [31, 32, 33, 34]. The same analysis can be extended and applied for BIST [36] and test data compression [37].

7 Conclusion

The proposed analysis identifies the information content of a bit-stream using a noise threshold. Vectors generated from spectral components and noise power level show superior quality in terms of test coverage compared with other methods of test vector generation like, random, weighted random and randomly perturbed vectors. This analysis can be easily applied to areas of test like BIST, test compression and ATPG.

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