Parametric Fault Diagnosis of Nonlinear Analog Circuits using Polynomial Coefficients

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Analog Circuit Testing

To determine catastrophic (open or short) faults and fractional deviations in circuit components from their nominal values.
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In this talk
To propose a method to detect & diagnose fractional deviations of circuit components from their nominal values in a large class of circuits.
Outline

1. Motivation
2. Our Idea
3. Generalization
4. Results
5. Fault Diagnosis
6. Conclusion and Future Work
Develop an Analog Circuit Test & Diagnosis Scheme

- Suitable for large class of circuits
Motivation

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Previous Approaches

Important previous techniques

- $I_{DDQ}$ based test – Intrusive, Area overhead is high
  [Chakravarty ’97]

- Signal flow graph – Complexity order is high
  [Bushnell et al. ’97]

- Transfer function based test – Valid only for LTI systems
  [Savir and Guo ’03]

- Digital assisted analog test – Intrusive
  [Tim Cheng et al. ’06]

- Polynomial coefficient based test – DC test
  [Sindia et al. ’09]
Transfer Function Coefficient Based Test

Second order low pass filter

\[ H(s) = \frac{1}{(R_1 R_2 C_1 C_2) s^2 + (R_1 C_1 + (R_1 + R_2) C_2) s + 1} \]
Our Idea

Taylor series expansion of circuit function about $v_{in} = 0$ at Multi tones

$$v_{out} = f(v_{in})$$

$$v_{out} = f(0) + \frac{f'(0)}{1!} v_{in} + \frac{f''(0)}{2!} v_{in}^2 + \frac{f'''(0)}{3!} v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!} v_{in}^n + \cdots$$
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Ignoring the higher order terms we have

$$v_{out} \approx a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n$$

where every $a_i \in \mathbb{R}$ and is bounded between its extreme values for

$$a_{i,\text{min}} < a_i < a_{i,\text{max}} \quad \forall i \quad 0 \leq i \leq n$$
In a nutshell

- Find the $V_{out}$ v/s $V_{in}$ relationship at DC and “relevant” frequencies.
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- Repeat the same for CUT by curve fitting the I/O response
In a nutshell

- Find the $V_{\text{out}}$ v/s $V_{\text{in}}$ relationship at DC and “relevant” frequencies.
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- Compare each of the obtained coefficients with fault-free circuit range
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- Find the $V_{\text{out}}$ v/s $V_{\text{in}}$ relationship at DC and “relevant” frequencies.
- Compute the coefficients of fault-free circuit.
- Repeat the same for CUT by curve fitting the I/O response.
- Compare each of the obtained coefficients with fault-free circuit range.
- Classify CUT as **Good** or **Bad**.
Cascaded Amplifiers

Two stage amplifier with 4\textsuperscript{th} degree non-linearity in $V_{in}$

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4$$
Polynomial Coefficients

\[ a_0 = V_{DD} - R_2 K \left( \frac{W}{L} \right)_2 \left[ (V_{DD} - V_T)^2 + R_1^2 K^2 \left( \frac{W}{L} \right)_1 V_T^4 \right] - 2(V_{DD} - V_T) R_1 \left( \frac{W}{L} \right)_1 V_T^2 \]

\[ a_1 = R_2 K \left( \frac{W}{L} \right)_2 \left[ 4 R_1^2 K^2 \left( \frac{W}{L} \right)_1 V_T^3 + 2(V_{DD} - V_T) R_1 K \left( \frac{W}{L} \right)_1 V_T \right] \]

\[ a_2 = R_2 K \left( \frac{W}{L} \right)_2 \left[ 2(V_{DD} - V_T) R_1 K \left( \frac{W}{L} \right)_1 - 6 R_1^2 K^2 \left( \frac{W}{L} \right)_1 V_T^2 \right] \]

\[ a_3 = 4 V_T K^3 \left( \frac{W}{L} \right)_1 \left( \frac{W}{L} \right)_2 R_1^2 R_2 \]

\[ a_4 = -K^3 \left( \frac{W}{L} \right)_1 \left( \frac{W}{L} \right)_2 R_1^2 R_2 \]
MSDF Calculation

**Definition**

Minimum Size Detectable Fault ($\rho$) of a circuit parameter is defined as its minimum fractional deviation to force at least one of the polynomial coefficients out of its fault-free range.
### MSDF Calculation

#### Definition

**Minimum Size Detectable Fault**($\rho$) of a circuit parameter is defined as its minimum fractional deviation to force at least one of the polynomial coefficients out of its fault free range.

Overview of MSDF calculation of R1 with $V_{DD}=1.2V$, $V_T=400mV$, $\left(\frac{W}{L}\right)_1 = \frac{1}{2} \left(\frac{W}{L}\right)_2 = 20$, and $K = 100\mu A/V^2$
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Maximize $a_0$

\[
\left\{ 1.2 - R_{2,nom}(1 + y) \left( 2.56 \times 10^{-3} + R_{1,nom}^2(1 + x)^2 \right) \right. \\
\left. - 5.12 \times 10^{-4} R_{1,nom}(1 + x) \right. \\
\left. \right\}
\]

subject to $a_1, a_2, a_3, a_4$ being in their fault free ranges and

$-\alpha \leq x, y \leq \alpha$
MSDF Calculation (contd..)

Assuming single parametric faults, $\rho$ for $R_1$

$$\rho = (1 - \alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$
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$$\rho = (1 - \alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$

<table>
<thead>
<tr>
<th>Circuit parameter</th>
<th>%upside MSDF</th>
<th>%downside MSDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor $R_1$</td>
<td>10.3</td>
<td>7.4</td>
</tr>
<tr>
<td>Resistor $R_2$</td>
<td>12.3</td>
<td>8.5</td>
</tr>
</tbody>
</table>
Let us Generalize
Generalization – Fault Simulation

1. Start
2. Choose frequency for fault simulation
3. Apply sweep to input and note corresponding output voltage levels
4. Polynomial Curve fit the obtained I/O data – find the coefficient values of fault free circuit
5. Simulate for all parametric faults at the simplex of hypercube
6. Find min-max values of each coefficient \((C_i)\) from \(i = 1 \cdots N\) across all simulations
7. Repeat process at all chosen frequencies
8. Stop
Generalization – Test Procedure

1. **Start**
2. Choose a frequency
3. Sweep input and note corresponding output voltage levels
4. Polynomial Curve fit the obtained I/O data. Obtain coefficients $C_i \forall i = 1 \cdots N$
5. Start with first coefficient
6. Consider next coefficient $C_{i+1}$
7. $|C_i| > |C_{i,max}| \text{ or } |C_i| < |C_{i,min}|$?
   - If True go to step 11
8. $i < N$? If True go to step 6
9. Repeat steps 2–8 at all desired frequencies
10. Subject CUT to further tests. **Stop**
11. **CUT is faulty. Stop**
Results – Elliptic filter
Results - Curve fitting at DC

\[ v_{out} = 4.5341 - 3.498 v_{in} - 2.5487 v_{in}^2 + 2.1309 v_{in}^3 - 0.50514 v_{in}^4 + 0.039463 v_{in}^5 \]
Results - Curve fitting at 100Hz

\[ V_{out} = 3 - 7.9V_{in} - 11V_{in}^2 + 4.4V_{in}^3 - 0.78V_{in}^4 + 0.049V_{in}^5 \]
Motivation  Our Idea  Generalization  Results  Fault Diagnosis  Conclusion and Future Work

Results - Curve fitting at 900Hz

\[ v_{out} = 2.5 + 5.4v_{in} - 8.6v_{in}^2 + 4v_{in}^3 - 0.77v_{in}^4 + 0.054v_{in}^5 \]

Simulated
5th degree Polynomial

\[ a_5 = 0.054 \]
\[ a_4 = -0.77 \]
\[ a_3 = 4 \]
\[ a_2 = -8.6 \]
\[ a_1 = 5.4 \]
\[ a_0 = 2.5 \]
Results - Curve fitting at 1000Hz

\[ V_{out} = 1.2 + 2.4V_{in} - 3.9V_{in}^2 + 1.8V_{in}^3 - 0.35V_{in}^4 + 0.024V_{in}^5 \]

**Simulated**

**5th degree polynomial**

\[ a_5 = 0.0024 \]
\[ a_4 = -0.35 \]
\[ a_3 = 1.8 \]
\[ a_2 = -3.8 \]
\[ a_1 = 2.4 \]
\[ a_0 = 1.2 \]
Results - Curve fitting at 1100Hz

\[ V_{out} = 0.23 - 0.48V_{in} - 0.74V_{in}^2 + 0.34V_{in}^3 - 0.063V_{in}^4 + 0.0043V_{in}^5 \]
Results at 1000Hz – Elliptic filter

Parameter Combinations Leading to Max Values of Coefficients with $\alpha = 0.05$ at 1000Hz

<table>
<thead>
<tr>
<th>Circuit Parameters (Resistance in ohm, Capacitance in farad)</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1 = 19.6k$</td>
<td>18.6k</td>
<td>18.6k</td>
<td>20.5k</td>
<td>20.5k</td>
<td>20.5k</td>
<td>18.6k</td>
</tr>
<tr>
<td>$R_2 = 196k$</td>
<td>205k</td>
<td>205k</td>
<td>205k</td>
<td>205k</td>
<td>186k</td>
<td>186k</td>
</tr>
<tr>
<td>$R_3 = 147k$</td>
<td>139k</td>
<td>139k</td>
<td>154k</td>
<td>139k</td>
<td>139k</td>
<td>139k</td>
</tr>
<tr>
<td>$R_4 = 1k$</td>
<td>950</td>
<td>950</td>
<td>1.05k</td>
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</tr>
<tr>
<td>$C_4 = 2.67n$</td>
<td>2.5n</td>
<td>2.8n</td>
<td>2.5n</td>
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<td>$C_6 = 2.67n$</td>
<td>2.5n</td>
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</tr>
<tr>
<td>$C_7 = 2.67n$</td>
<td>2.5n</td>
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## Results at 1000Hz – Elliptic filter

Parameter Combinations Leading to Min Values of Coefficients with $\alpha = 0.05$ at 1000Hz

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<tr>
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<td>2.8n</td>
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## Fault Detection at Multi-Tones

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<thead>
<tr>
<th>Injected fault</th>
<th>Coefficients out of Bounds at</th>
<th>Detect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DC</td>
<td>( f_1 = 100\text{Hz} )</td>
</tr>
<tr>
<td>( R_1 ) down 15%</td>
<td>( a_0 - a_4 )</td>
<td>( a_1 - a_4 )</td>
</tr>
<tr>
<td>( R_2 ) down 5%</td>
<td>( a_2, a_5 )</td>
<td>( a_1, a_3 )</td>
</tr>
<tr>
<td>( R_3 ) up 10%</td>
<td>( a_1, a_2, a_3 )</td>
<td>( a_3, a_5 )</td>
</tr>
<tr>
<td>( R_4 ) down 20%</td>
<td>( a_0 - a_3 )</td>
<td>( a_1 - a_2 )</td>
</tr>
<tr>
<td>( C_5 ) up 5%</td>
<td>–</td>
<td>( a_0, a_1 )</td>
</tr>
<tr>
<td>( C_6 ) up 15%</td>
<td>–</td>
<td>( a_3, a_4 )</td>
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Fault Diagnosis

Definition

To determine the circuit parameters responsible for deviation of circuit from its desired behavior.
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Sensitivity based diagnosis

\[ S_{p_k}^{C_i} = \frac{p_k}{C_i} \frac{\partial C_i}{\partial p_k} \]

\[ P(\delta p_k | \delta C_i) = \phi \left( \frac{S_{p_k}^{C_i} \delta p_k}{\delta C_i} \right) \]

\( \phi \) is any probability measure. We chose negative exponential function.
Fault Diagnosis

\[ P(p_k \text{ is a fault site} | C_i \text{ is out of bound}) = 1 - \prod_{j=1}^{N} (1 - P_{f_j}(\delta p_k | \delta C_i)) \]
Fault Diagnosis at Multi-Tones for Elliptic Filter

Parametric Fault Diagnosis with Confidence Levels $\approx 98.9\%$

<table>
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<tr>
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<td>DC</td>
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<tr>
<td>$R_1$ down 15%</td>
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<td>$C_7$ up 15%</td>
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<td>$C_6, C_7$</td>
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</table>
Technique for parametric fault detection in analog circuits – faults as small as 10% were uncovered in elliptic filter
Conclusion

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- Could uncover parametric deviations in reactive elements as input is swept at DC and selected frequencies.
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- Could uncover parametric deviations in reactive elements as input is swept at DC and selected frequencies
- Technique to diagnose faults through multi-frequency excitation using sensitivity as a measure
**Conclusion**

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- Could uncover parametric deviations in reactive elements as input is swept at DC and selected frequencies.
- Technique to diagnose faults through multi-frequency excitation using sensitivity as a measure.
Future Work

- Increasing the sensitivity of coefficients to parameters (to enhance coverage & fault diagnosis), using “V-transforms”
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- Techniques for optimal choice of frequencies at which CUT can be excited
Acknowledgments

- Kaushal Kumar Jha, ADI
- Pawan Kumar, IISc
- Pramod Subramanyan, IISc

Thanks for your Attention!