# LNA Test: A Polynomial Coefficient Approach

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# Outline

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# Our Idea

Taylor series expansion of circuit function in terms of magnitude of input $v_{in}$ at a frequency

$$v_{out} = f(v_{in})$$

$$v_{out} = f(0) + \frac{f'(0)}{1!} v_{in} + \frac{f''(0)}{2!} v_{in}^2 + \frac{f'''(0)}{3!} v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!} v_{in}^n + \cdots$$

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Ignoring the higher order terms we have

$$v_{out} \approx a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n$$

where every $a_i \in \Re$ and is bounded between its extreme values for

$$a_{i,\min} < a_i < a_{i,\max} \ \forall i \ 0 \leq i \leq n$$

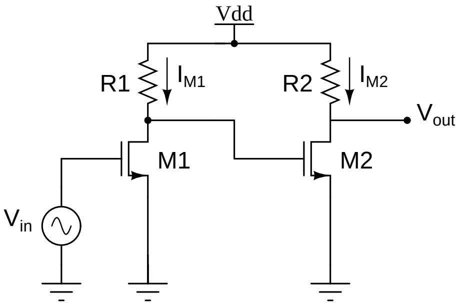
# Our Idea (Contd..)

## In a Nutshell

- Find the $V_{out}$ v/s $V_{in}$ relationship at frequencies of interest (Eg.: Cutoff, fundamental)
- Compute the coefficients of fault-free circuit
- Repeat the same for CUT by curve fitting the I/O response
- Compare each of the obtained coefficients with fault-free circuit range
- Classify CUT as Good or Bad

# Cascaded Amplifiers

## An Example




Two stage amplifier with $4^{th}$ degree non-linearity in $V_{in}$

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4$$

# Polynomial Coefficients

$$a_0 = V_{DD} - R_2K\left(\frac{W}{L}\right)_2 \left[\begin{array}{l} (V_{DD} - V_T)^2 + R_1^2K^2\left(\frac{W}{L}\right)_1^2 V_T^4 \\ -2(V_{DD} - V_T)R_1\left(\frac{W}{L}\right)_1 V_T^2 \end{array}\right]$$

$$a_1 = R_2K\left(\frac{W}{L}\right)_2 \left[4R_1^2K^2\left(\frac{W}{L}\right)_1^2 V_T^3 + 2(V_{DD} - V_T)R_1K\left(\frac{W}{L}\right)_1 V_T\right]$$

$$a_2 = R_2K\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - V_T)R_1K\left(\frac{W}{L}\right)_1 - 6R_1^2K^2\left(\frac{W}{L}\right)_1^2 V_T^2\right]$$

$$a_3 = 4V_TK^3\left(\frac{W}{L}\right)_1^2\left(\frac{W}{L}\right)_2^2 R_1^2R_2$$

$$a_4 = -K^3\left(\frac{W}{L}\right)_1^2\left(\frac{W}{L}\right)_2^2 R_1^2R_2$$

# MSDF Calculation

## Definition

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Overview of MSDF calculation of R1 with $\mathrm{V_{DD}}$=1.2V, $\mathrm{V_T}$= 400mV, $\left(\frac{W}{L}\right)_1 = \frac{1}{2}\left(\frac{W}{L}\right)_2 = 20$, and $\mathrm{K} = 100\mu\mathrm{A}/\mathrm{V}^2$

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Maximize $a_0$

$$\left\{1.2 - R_{2,nom}(1+y)\begin{pmatrix} 2.56\mathrm{x}10^{-3} + R_{1,nom}^2(1+x)^2 1.024\mathrm{x}10^{-7} \\ -5.12\mathrm{x}10^{-4} R_{1,nom}(1+x) \end{pmatrix}\right\}$$

subject to $a_1, a_2, a_3, a_4$ being in their fault free ranges and

$$-\alpha \leq x, y \leq \alpha$$

# MSDF Calculation (contd..)

Assuming single parametric faults, $\rho$ for $\mathrm{R}_1$

$$\rho = (1+\alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$

# MSDF Calculation (contd..)

Assuming single parametric faults, $\rho$ for $R_1$

$$\rho = (1+\alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$

## MSDF for Cascaded Amplifier with $\alpha$ = 0.05

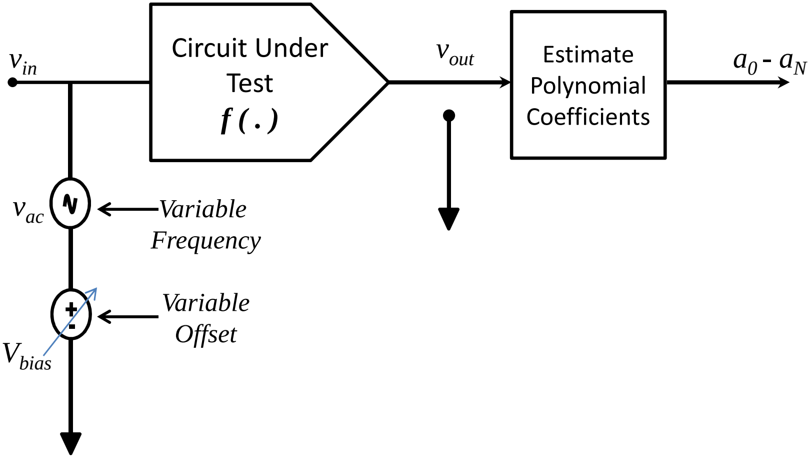
| Circuit parameter | %upside MSDF | %downside MSDF |
|---|---|---|
| Resistor $R_1$ | 10.3 | 7.4 |
| Resistor $R_2$ | 12.3 | 8.5 |

# Outline

# Generalization – Fault Simulation

1. Start
2. Choose a frequency of interest
3. Sweep bias at the input and note corresponding output voltage levels
4. Polynomial curve fit the obtained I/O data – find the coefficient values of fault free circuit
5. Simulate for all parametric faults at the simplex of hypercube
6. Find min-max values of each coefficient ($C_i$) from $i = 1 \cdots N$ across all simulations
7. Stop

# Test Setup



$v_{in}$
Circuit Under Test
$f(.)$
$v_{out}$
Estimate Polynomial Coefficients
$a_0 - a_N$
$v_{ac}$
Variable Frequency
$V_{bias}$
Variable Offset

# Generalization – Test Procedure

1. Start
2. Sweep bias at the input and note corresponding output voltage levels
3. Polynomial curve fit the obtained I/O data
4. Start with first coefficient
5. Consider next coefficient $C_{i+1}$
6. $|C_i| > |C_{i,max}|$ or $|C_i| < |C_{i,min}|$?
   If True go to step 9
7. $i < N$? If True go to step 5
8. Subject CUT to further tests. Stop
9. CUT is faulty. Stop

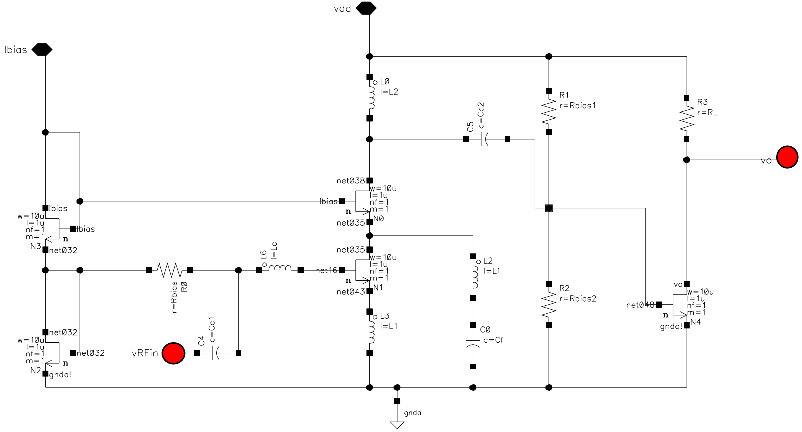
# Outline

# Low Noise Amplifier

## Specifications

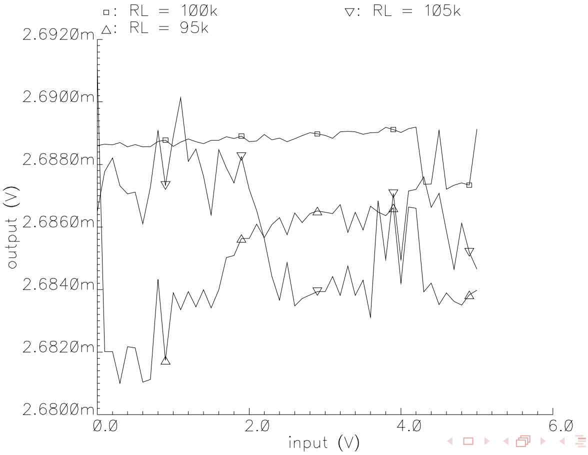
| **Performance Parameter** | **Nominal Value** |
|---|---|
| Gain (dB) | 16 |
| $IIP_3$ (dBm) | -18 |
| Noise figure (dB) | 9.1 |
| $S_{11}$ (dB) | -16.5 |

# Low Noise Amplifier – Schematic



# Results - Output Comparison @ 10GHz

## Comparison for parametric fault in $R_L = 100\text{k ohm}$



# Results – Low Noise Amplifier @ 10GHz

Parameter Combinations Leading to Max Values of Coefficients with $\alpha = 0.05$

| Component (ohm, nH, fF) | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|---|
| $R_{bias} = 10$ | 10 | 10 | 10.5 | 10.5 | 9.5 | 10.5 |
| $L_C = 1$ | 1 | 0.95 | 1.05 | 0.95 | 1.05 | 1 |
| $C_{C1} = 100$ | 95 | 95 | 95 | 95 | 95 | 105 |
| $L_1 = 1.5$ | 1.425 | 1.5 | 1.5 | 1.425 | 1.575 | 1.425 |
| $L_2 = 1.5$ | 1.5 | 1.425 | 1.425 | 1.575 | 1.5 | 1.5 |
| $L_f = 1$ | 1.05 | 1.05 | 1.05 | 1 | 1.05 | 1 |
| $C_f = 100$ | 105 | 95 | 95 | 105 | 95 | 95 |
| $C_{C2} = 100$ | 95 | 100 | 105 | 95 | 95 | 95 |
| $R_{bias1} = 100k$ | 105k | 105k | 100k | 105k | 105k | 95k |
| $R_{bias2} = 100k$ | 105k | 95k | 100k | 95k | 95k | 95k |
| $R_L = 100k$ | 100k | 95k | 95k | 100k | 105k | 100k |

# Results – Low Noise Amplifier @ 10GHz

Parameter Combinations Leading to Min Values of Coefficients with $\alpha$ = 0.05

| Component (ohm, nH, fF) | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|---|
| $R_{bias} = 10$ | 10 | 9.5 | 9.5 | 10 | 10 | 10 |
| $L_C = 1$ | 1.05 | 0.95 | 0.95 | 1 | 1 | 0.95 |
| $C_{C1} = 100$ | 100 | 105 | 95 | 100 | 95 | 105 |
| $L_1 = 1.5$ | 1.425 | 1.5 | 1.575 | 1.575 | 1.575 | 1.575 |
| $L_2 = 1.5$ | 1.5 | 1.575 | 1.5 | 1.425 | 1.425 | 1.5 |
| $L_f = 1$ | 1.05 | 1.05 | 0.95 | 0.95 | 1 | 0.95 |
| $C_f = 100$ | 105 | 95 | 95 | 105 | 105 | 105 |
| $C_{C2} = 100$ | 95 | 105 | 100 | 105 | 95 | 105 |
| $R_{bias1} = 100k$ | 100k | 95k | 105k | 105k | 95k | 100k |
| $R_{bias2} = 100k$ | 100k | 105k | 95k | 95k | 105k | 95k |
| $R_L = 100k$ | 95k | 100k | 95k | 100k | 105k | 95k |

# Results – Low Noise Amplifier @ 10GHz

## Results of some Injected Faults

| Circuit Parameter | Coefficients out of bounds | Detected |
|---|---|---|
| $R_{bias}$ down 25% | $a_0 - a_4$ | Yes |
| $L_C$ down 15% | $a_2, a_5$ | Yes |
| $C_{C1}$ up 10% | $a_1, a_2, a_3$ | Yes |
| $L_1$ down 25% | $a_0 - a_4$ | Yes |
| $L_2$ up 15% | $a_0, a_4$ | Yes |
| $L_f$ up 10% | $a_1, a_2$ | Yes |
| $C_f$ up 10% | $a_4, a_5$ | Yes |
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# Fault Diagnosis

**Definition**

To determine the circuit parameters responsible for deviation of circuit from its desired behavior.

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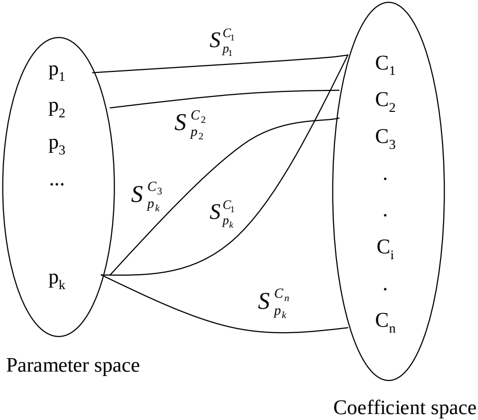
Sensitivity based diagnosis

$$S_{p_k}^{C_i} = \frac{p_k}{C_i} \frac{\partial C_i}{\partial p_k}$$

# Fault Diagnosis




Possible relation between various parameters and coefficients

# Results – Low Noise Amplifier

## Fault Diagnosis at f = 10 GHz

| Fault injected | Coefficient status | Diagnosed fault sites |
|---|---|---|
| $R_{bias}$ down 25% | $a_0 - a_4$ | $R_{bias}$ |
| $L_C$ down 15% | $a_2, a_5$ | $L_C$ or $C_{C1}$ |
| $C_{C1}$ up 10% | $a_1, a_2, a_3$ | $C_{C1}$ or $L_C$ |
| $L_1$ down 25% | $a_0 - a_4$ | $L_1$ |
| $L_2$ up 15% | $a_0, a_4$ | $L_2$ |
| $L_f$ up 10% | $a_1, a_2$ | $L_f$ or $C_f$ |
| $C_f$ up 10% | $a_4, a_5$ | $L_f$ |
| $C_{C2}$ down 10% | $a_4, a_5$ | $C_{C2}$ |

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# Conclusions and Future Work

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- Technique for parametric fault detection in analog circuits – faults as small as 10% were uncovered for LNA example
- Diagnosis based on Sensitivity of Polynomial Coefficients to circuit parameters
- Limitation – Extensive fault simulations required to cover all corner cases

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## In Future

- Neural models to map specifications to polynomial coefficients
- To implement proposed test scheme as BIST by storing polynomial coefficients on chip

# Acknowledgments


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- Virendra Singh, Indian Institute of Science, Bangalore

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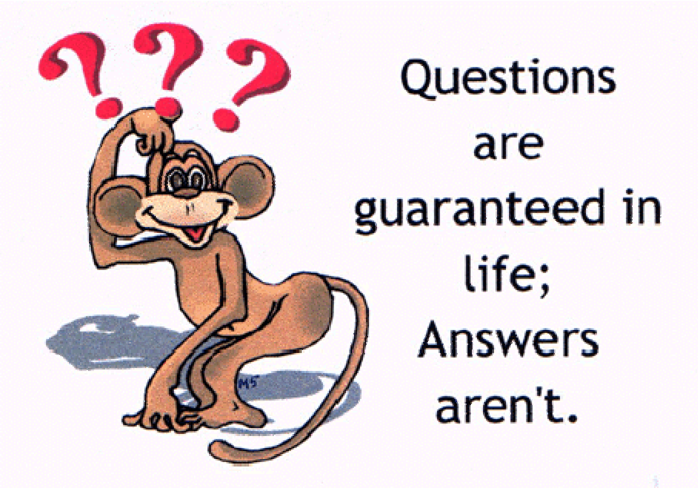
Thanks for your Attention!


???
Questions
are
guaranteed in
life;
Answers
aren't.