Polynomial Coefficient Based Multi-Tone Testing of Analog Circuits

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Analog Circuit Testing

To determine catastrophic (open or short) faults and fractional deviations in circuit components from their nominal values.

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In this talk

To propose a method to detect fractional deviations of circuit components from their nominal values in a large class of circuits.

Outline

Motivation

To Develop an Analog Circuit test scheme

- Suitable for large class of circuits

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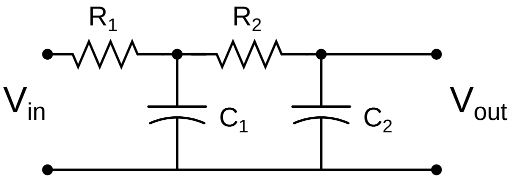
Previous Approaches

Important previous techniques

- I_{DDQ} based test – Intrusive, Area overhead is high *[Chakravarty '97]*
- Signal flow graph – Complexity order is high *[Bushnell et al. '97]*
- Transfer function based test – Valid only for LTI systems *[Savir and Guo '03]*
- Digital assisted analog test – Intrusive *[Tim Cheng et al. '06]*
- Polynomial coefficient based test – DC test *[Sindia et al. '09]*

Transfer Function Coefficient Based Test



Second order low pass filter

$$H(s) = \frac{1}{(R_1R_2C_1C_2)\,s^2 + (R_1C_1 + (R_1 + R_2)C_2)\,s + 1}$$

Our Idea

Taylor series expansion of circuit function about $v_{in} = 0$ at Multi tones

$$v_{out} = f(v_{in})$$
$$v_{out} = f(0) + \frac{f'(0)}{1!} v_{in} + \frac{f''(0)}{2!} v_{in}^2 + \frac{f'''(0)}{3!} v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!} v_{in}^n + \cdots$$

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Ignoring the higher order terms we have

$$v_{out} \approx a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n$$

where every $a_i \in \Re$ and is bounded between its extreme values for

$$a_{i,\min} < a_i < a_{i,\max}\ \forall i\ 0 \leq i \leq n$$

Our Idea (Contd..)

In a nutshell

- Find the V_{out} v/s V_{in} relationship at DC and “relevant” frequencies.

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- Find the V_{out} v/s V_{in} relationship at DC and “relevant” frequencies.
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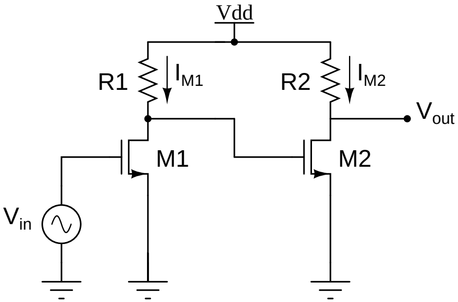
- Find the $\mathrm{V}_{\mathrm{out}}$ v/s V_{in} relationship at DC and “relevant” frequencies.
- Compute the coefficients of fault-free circuit
- Repeat the same for CUT by curve fitting the I/O response
- Compare each of the obtained coefficients with fault-free circuit range
- Classify CUT as Good or Bad

Cascaded Amplifiers



Two stage amplifier with 4th degree non-linearity in V_{in}

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4$$

Polynomial Coefficients

$$a_0 = V_{DD} - R_2K\left(\frac{W}{L}\right)_2 \left[\begin{array}{l} (V_{DD}-V_T)^2 + R_1^2K^2\left(\frac{W}{L}\right)_1^2 V_T^4 \\ -2(V_{DD}-V_T)R_1\left(\frac{W}{L}\right)_1 V_T^2 \end{array} \right]$$

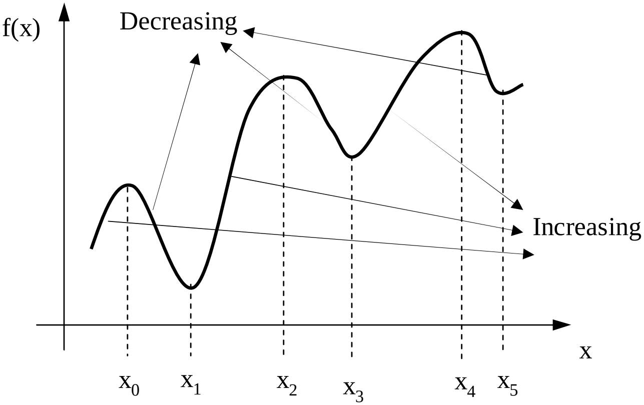
$$a_1 = R_2K\left(\frac{W}{L}\right)_2 \left[4R_1^2K^2\left(\frac{W}{L}\right)_1^2 V_T^3 + 2(V_{DD}-V_T)R_1K\left(\frac{W}{L}\right)_1 V_T\right]$$

$$a_2 = R_2K\left(\frac{W}{L}\right)_2 \left[2(V_{DD}-V_T)R_1K\left(\frac{W}{L}\right)_1 - 6R_1^2K^2\left(\frac{W}{L}\right)_1^2 V_T^2\right]$$

$$a_3 = 4V_TK^3\left(\frac{W}{L}\right)_1^2\left(\frac{W}{L}\right)_2^2 R_1^2R_2$$

$$a_4 = -K^3\left(\frac{W}{L}\right)_1^2\left(\frac{W}{L}\right)_2^2 R_1^2R_2$$

Nature of Polynomial Coefficients



Non-linear, Non-monotonic function is decomposed into piecewise monotonic functions

MSDF Calculation

Definition

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Overview of MSDF calculation of R1 with V_{DD}=1.2V, V_T= 400mV, $\left(\frac{W}{L}\right)_1 = \frac{1}{2}\left(\frac{W}{L}\right)_2 = 20$, and $K = 100\mu A/V^2$

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Maximize a_0

$$\left\{1.2 - R_{2,nom}(1+y)\begin{pmatrix} 2.56\text{x}10^{-3} + R_{1,nom}^2(1+x)^2 1.024\text{x}10^{-7} \\ -5.12\text{x}10^{-4} R_{1,nom}(1+x) \end{pmatrix}\right\}$$

subject to a_1, a_2, a_3, a_4 being in their fault free ranges and

$$-\alpha \leq x, y \leq \alpha$$

MSDF Calculation (contd..)

Assuming single parametric faults, ρ for R_1

$$\rho = (1 - \alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$

MSDF Calculation (contd..)

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MSDF for Cascaded Amplifier with α = 0.05

Circuit parameter	%upside MSDF	%downside MSDF
Resistor R_1	10.3	7.4
Resistor R_2	12.3	8.5

Let us Generalize

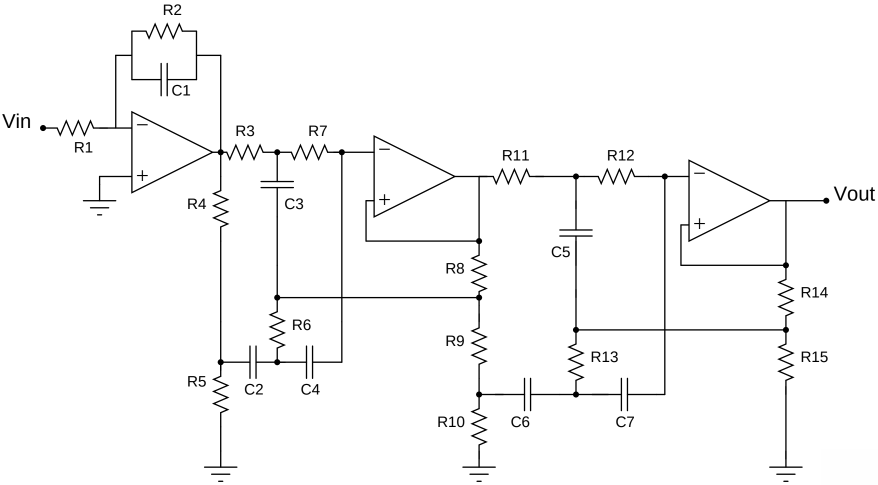
Generalization – Fault Simulation

1. Start
2. Choose frequency for fault simulation
3. Apply sweep to input and note corresponding output voltage levels
4. Polynomial Curve fit the obtained I/O data – find the coefficient values of fault free circuit
5. Simulate for all parametric faults at the simplex of hypercube
6. Find min-max values of each coefficient (C_i) from $i = 1 \cdots N$ across all simulations
7. Repeat process at all chosen frequencies
8. Stop

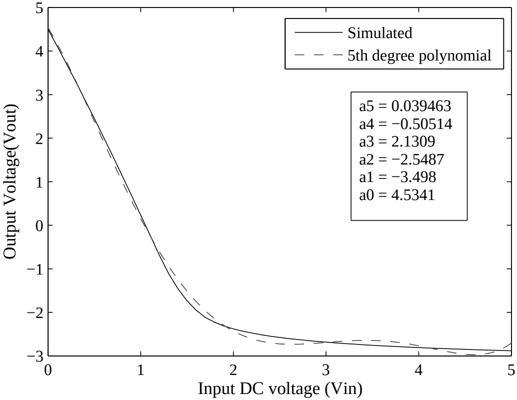
Generalization – Test Procedure

1. Start
2. Choose a frequency
3. Sweep input and note corresponding output voltage levels
4. Polynomial Curve fit the obtained I/O data. Obtain coefficients $C_i \ \forall\, i = 1 \cdots N$
5. Start with first coefficient
6. Consider next coefficient C_{i+1}
7. $|C_i| > |C_{i,max}|$ *or* $|C_i| < |C_{i,min}|$?
   If True go to step 11
8. $i < N$? If True go to step 6
9. Repeat steps 2–8 at all desired frequencies
10. Subject CUT to further tests. Stop
11. CUT is faulty. Stop

Results – Elliptic filter

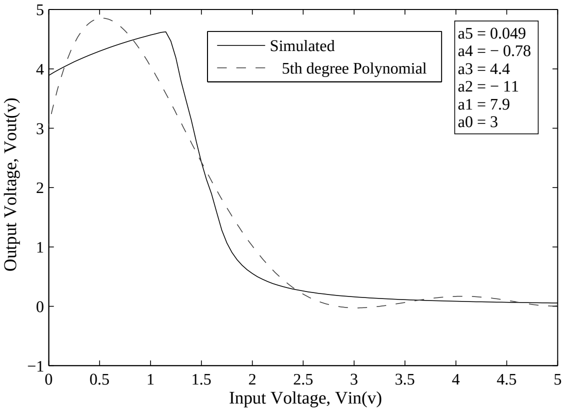


Results - Curve fitting at DC



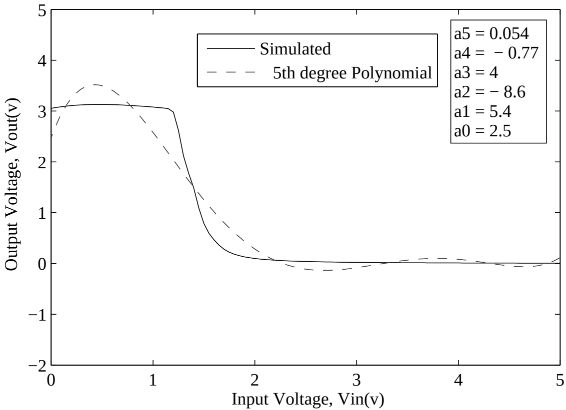
$$v_{out} = 4.5341 - 3.498 v_{in} - 2.5487 v_{in}^2 + 2.1309 v_{in}^3 - 0.50514 v_{in}^4 + 0.039463 v_{in}^5$$

Results - Curve fitting at 100Hz



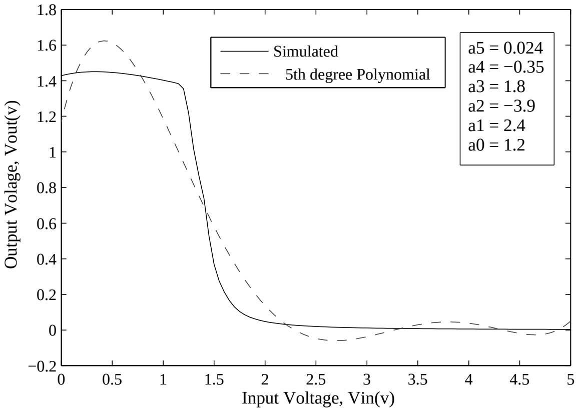
$$v_{out} = 3 - 7.9 v_{in} - 11 v_{in}^2 + 4.4 v_{in}^3 - 0.78 v_{in}^4 + 0.049 v_{in}^5$$

Results - Curve fitting at 900Hz



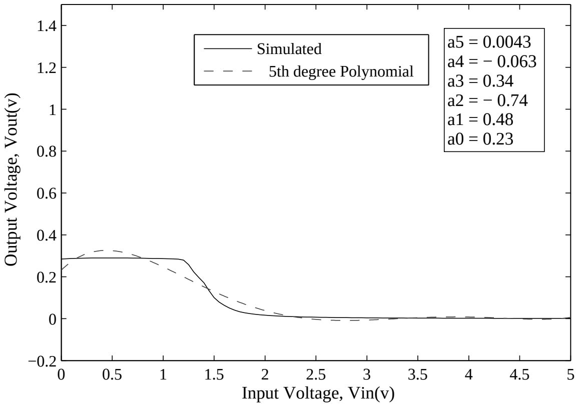
$$v_{out} = 2.5 + 5.4v_{in} - 8.6v_{in}^2 + 4v_{in}^3 - 0.77v_{in}^4 + 0.054v_{in}^5$$

Results - Curve fitting at 1000Hz



$$v_{out} = 1.2 + 2.4v_{in} - 3.9v_{in}^2 + 1.8v_{in}^3 - 0.35v_{in}^4 + 0.024v_{in}^5$$

Results - Curve fitting at 1100Hz



$$v_{out} = 0.23 - 0.48v_{in} - 0.74v_{in}^2 + 0.34v_{in}^3 - 0.063v_{in}^4 + 0.0043v_{in}^5$$

Results at 1000Hz – Elliptic filter

Parameter Combinations Leading to Max Values of Coefficients with α = 0.05 at 1000Hz

Circuit Parameters (Resistance in ohm,Capacitance in farad)						
Nominal Values	a_0	a_1	a_2	a_3	a_4	a_5
$R_1 = 19.6k$	18.6k	18.6k	20.5k	20.5k	20.5k	18.6k
$R_2 = 196k$	205k	205k	205k	205k	186k	186k
$R_3 = 147k$	139k	139k	154k	139k	139k	139k
$R_4 = 1k$	950	950	1.05k	1.05k	1.05k	1.05k
$C_4 = 2.67n$	2.5n	2.8n	2.5n	2.5n	2.5n	2.5n
$C_5 = 2.67n$	2.5n	2.5n	2.5n	2.5n	2.5n	2.8n
$C_6 = 2.67n$	2.5n	2.8n	2.5n	2.8n	2.5n	2.8n
$C_7 = 2.67n$	2.5n	2.8n	2.8n	2.8n	2.8n	2.5n

Results at 1000Hz – Elliptic filter

Parameter Combinations Leading to Min Values of Coefficients with α = 0.05 at 1000Hz

Circuit Parameters (Resistance in ohm,Capacitance in farad)						
Nominal Values	a_0	a_1	a_2	a_3	a_4	a_5
$R_1 = 19.6k$	18.6k	18.6k	18.6k	18.6k	20.5k	20.5k
$R_2 = 196k$	205k	186k	186k	205k	205k	205k
$R_3 = 147k$	139k	139k	154k	139k	139k	139k
$R_4 = 1k$	1.05k	950	1.05k	950	950	1.05k
$C_4 = 2.67n$	2.5n	2.5n	2.8n	2.5n	2.5n	2.8n
$C_5 = 2.67n$	2.8n	2.8n	2.8n	2.8n	2.8n	2.8n
$C_6 = 2.67n$	2.5n	2.5n	2.8n	2.8n	2.8n	2.5n
$C_7 = 2.67n$	2.8n	2.5n	2.5n	2.5n	2.5n	2.8n

Results at DC – Elliptic filter

Parameter Combinations Leading to Max Values of Coefficients with α = 0.05 at DC

Resistance (ohm)	a_0	a_1	a_2	a_3	a_4	a_5
$R_1 = 19.6k$	18.6k	20.5k	20.5k	20.5k	18.6k	18.6k
$R_2 = 196k$	186k	205k	186k	186k	186k	205k
$R_3 = 147k$	139k	154k	154k	154k	139k	154k
$R_4 = 1k$	950	1010	1010	1010	1010	1010
$R_5 = 71.5$	70	80	80	70	80	70
$R_6 = 37.4k$	37.4k	37.4k	37.4k	37.4k	37.4k	37.4k
$R_7 = 154k$	161k	161k	146k	161k	146k	146k
$R_{11} = 110k$	115k	115k	104k	115k	104k	104k
$R_{12} = 110k$	104k	115k	104k	104k	104k	104k

Fault Detection at Multi-Tones

Injected fault	Coefficients out of Bounds at				Detect
	DC	f_1=100Hz	f_2=900Hz	f_3=1000Hz	
R_1 down 15%	$a_0 - a_4$	$a_1 - a_4$	a_3, a_5	a_2, a_4	Yes
R_2 down 5%	a_2, a_5	a_1, a_3	a_1, a_5	a_1, a_2, a_5	Yes
R_3 up 10%	a_1, a_2, a_3	a_3, a_5	a_0, a_3, a_4	a_1, a_3, a_4	Yes
R_4 down 20%	$a_0 - a_3$	$a_1 - a_2$	a_2, a_3	a_1, a_2, a_3	Yes
C_5 up 5%	–	a_0, a_1	a_1, a_5	a_1, a_2	Yes
C_6 up 15%	–	a_3, a_4	a_1, a_2, a_4	a_3, a_4, a_5	Yes
C_7 up 15%	–	a_1, a_4	a_1, a_3, a_4	a_1, a_3, a_5	Yes

Fault Diagnosis

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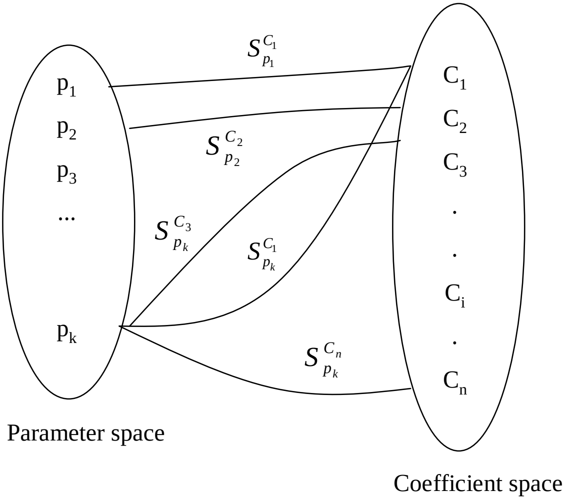
Sensitivity based diagnosis

$$S_{p_k}^{C_i} = \frac{p_k}{C_i}\frac{\partial C_i}{\partial p_k}$$

$$\mathrm{P}(\delta \mathrm{p_k} | \delta \mathrm{C_i}) = \phi\left(\frac{S_{p_k}^{C_i}\delta p_k}{\delta C_i}\right)$$

ϕ is any probability measure. We chose negative exponential function

Fault Diagnosis



Fault Diagnosis at Multi-Tones

Parametric Fault Diagnosis with Confidence Levels $\approx$ 98.9%

Injected fault	Diagnosed fault sites at				Deduction
	DC	100Hz	900Hz	1000Hz	
R_1 down 15%	R_1, R_4	R_1	R_1, R_2	R_1, R_2, C_1	R_1
R_2 down 5%	R_2	R_2, C_1	R_2, R_3, C_1	R_2, R_3	R_2
R_3 up 10%	R_1, R_3	R_3, C_3	R_3, R_4, C_3	R_3	R_3
R_4 down 20%	R_1, R_4	R_1, R_4	R_2, R_4, C_1	R_1, R_2, R_4	R_4
C_5 up 5%	–	C_5	R_{12}, C_5	C_5	C_5
C_6 up 15%	–	R_{10}, C_6	C_6, C_7	C_6, C_7	C_6
C_7 up 15%	–	C_6, C_7	C_7	C_6, C_7	C_7

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- Could uncover parametric deviations in reactive elements as input is swept at DC and selected frequencies
- Technique to diagnose faults through multi-frequency excitation using sensitivity

Future Work

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- Increasing the sensitivity of coefficients to parameters (to enhance fault coverage), using “smart” transforms
- Techniques for optimal choice of frequencies at which CUT could be excited

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Thanks for your Attention!