RFIC Design and Testing for Wireless Communications

A PragaTI (TI India Technical University) Course
July 18, 21, 22, 2008

Lecture 6: Basic Concepts
– Linearity, noise figure, dynamic range

By

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Topics

Monday, July 21, 2008

9:00 – 10:30	Introduction – Semiconductor history, RF characteristics
11:00 – 12:30	Basic Concepts – Linearity, noise figure, dynamic range
2:00 – 3:30	RF front-end design – LNA, mixer
4:00 – 5:30	Frequency synthesizer design I (PLL)

Tuesday, July 22, 2008

9:00 – 10:30	Frequency synthesizer design II (VCO)
11:00 – 12:30	RFIC design for wireless communications
2:00 – 3:30	Analog and mixed signal testing

Units for Microwave and RFIC Design

Peak-to peak voltage: V_{pp}

Root-mean-square voltage: $V_{rms} = \frac{V_{pp}}{2\sqrt{2}}$

Power in Watt :

$$Pwatt = \frac{V^2{}_{rms}}{R} = \frac{V_{pp}^2}{8R}$$

Power in dBm :

$$P_{dBm} = 10\log_{10}\left(\frac{Pwatt\,[mW]}{1mW}\right)$$

On a 50Ohm load, 0dBm=1mW=224mV_{rms}=632mV_{pp}

Noise Figure

- **In RF design, most of the front-end receiver blocks are characterized in terms of “noise figure” rather than input referred noise.**
- **Noise factor F is defined as**

$$\mathrm{F} = \frac{SNR_{in}}{SNR_{out}} = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{GN_i} = \frac{N_{o(total)}}{N_{o(source)}} = \frac{N_{o(source)} + N_{o(added)}}{N_{o(source)}} = 1 + \frac{N_{o(added)}}{N_{o(source)}}$$

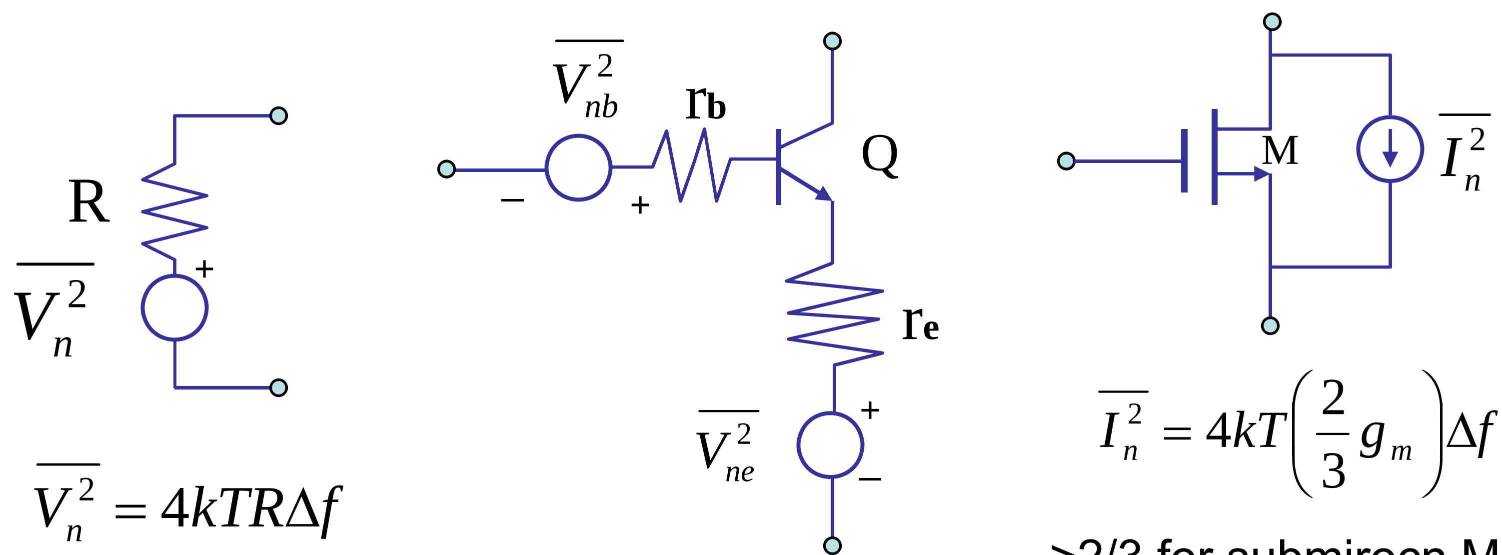
$$\text{Noise Figure, NF} = 10\log_{10} F$$

- **Noise figure measures how much the SNR degrades as the signal passes through a system.**
- **For a noiseless system, SNRin = SNRout, namely, F=1, NF=0dB, regardless of the gain. This is because both the input signal and the input noise are amplified (or attenuated) by the same factor and no additional noise is introduced.**

Thermal Noise

- **Thermal noise (Johnson noise) – due to random thermal motion of electrons and is generated by resistors, base and emitter resistance r_b,r_E,and r_c. of bipolar devices, and channel resistance of MOSFETs. Thermal noise is a white noise with Gaussian amplitude distribution.**

- **Thermal noise floor:** $$10\log\left(\frac{kT}{1mW}\right) = -174dBm/Hz \text{ at } 290^0 K$$



$$\overline{V_n^2} = 4kTR\Delta f$$

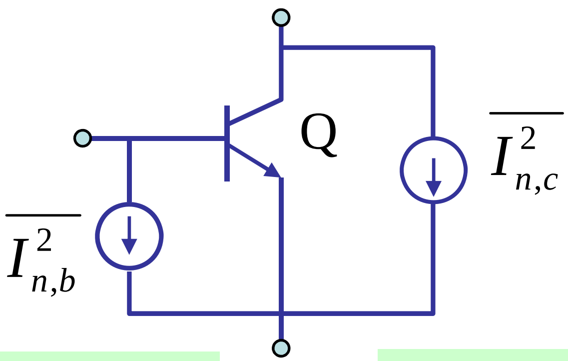
$$\overline{I_n^2} = 4kT\left(\frac{2}{3}g_m\right)\Delta f$$

>2/3 for submirocn MOS

Shot Noise

- **Shot noise (Schottky noise) – due to the particle-like nature of charge carriers. Only the time-average flow of electrons and holes appears as constant current. Any fluctuation in the number of charge carriers produces a random noise current at that instant. Shot noise is a Gaussian white process associated with the transfer of charge across an energy barrier (e.g., a p-n junction). This random process is called shot noise and is expressed in amperes per root hertz.**

$$\overline{I_n^2} = 2qI\Delta f$$



$$i_{bn} = \sqrt{2qI_B}$$

$$i_{cn} = \sqrt{2qI_C}$$

Flicker noise

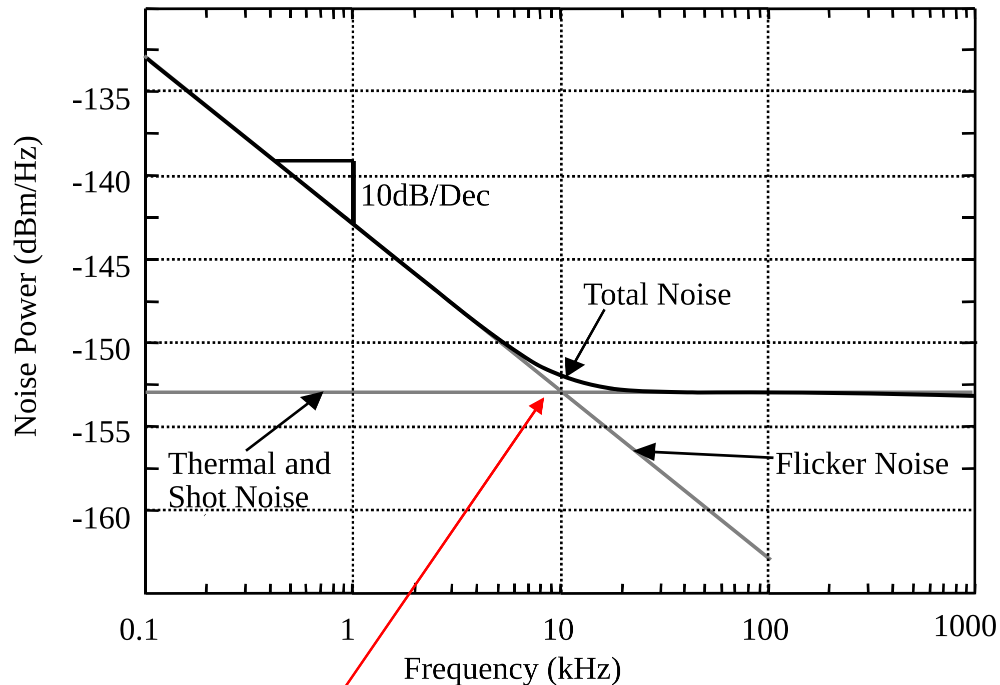
- **Flicker noise (1/f noise) – found in all active devices. In bipolar transistors, it is caused by traps associated with contamination and crystal defects in the emitter-base depletion layer. These traps capture and release carriers in a random fashion with noise energy concentrated in low frequency. K depends on processing and may vary by order of magnitude.**

$$\overline{I_n^2} = K \frac{{I_C}^a}{f} \Delta f, \; a \approx 0.5\text{\textasciitilde}2$$

- **In MOSFETs, 1/f noise arises from random trapping of charge at the oxide-silicon interfaces. Represented as a voltage source in series with the gate, the noise spectral density is given by**

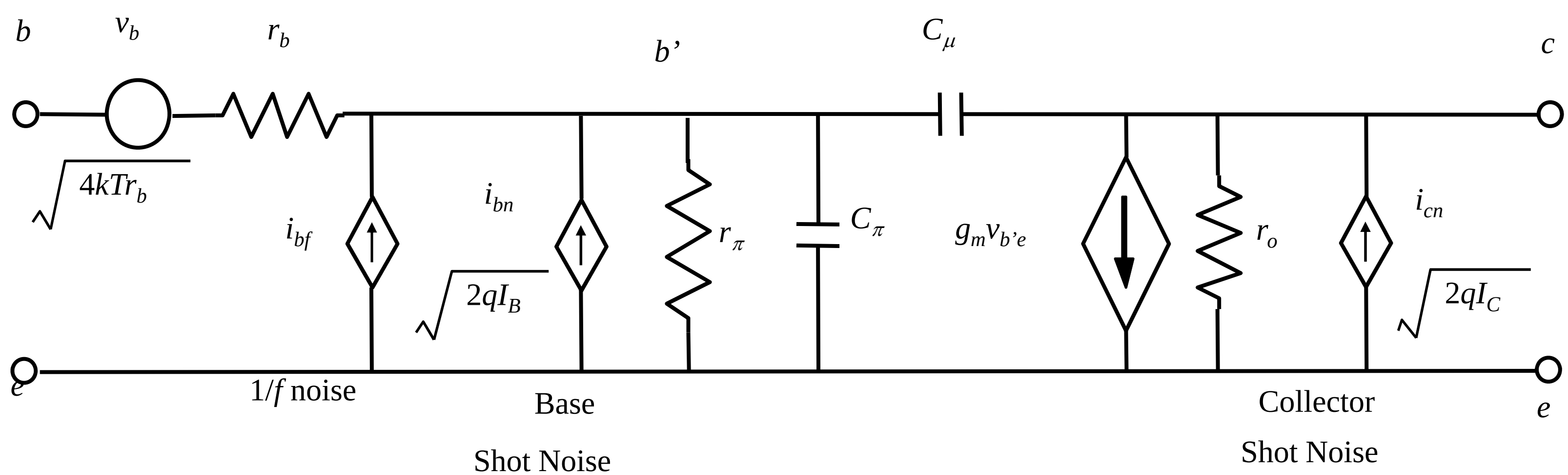
$$\overline{V_n^2} = \frac{K}{WLCox} \frac{1}{f}$$

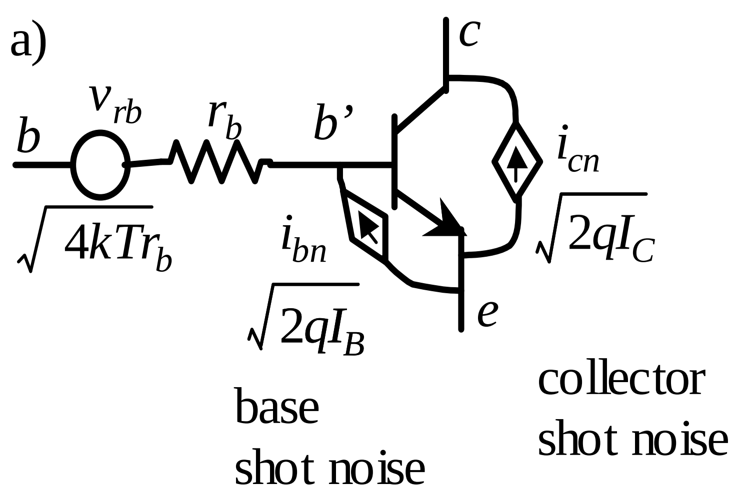
Noise Power Spectral Density

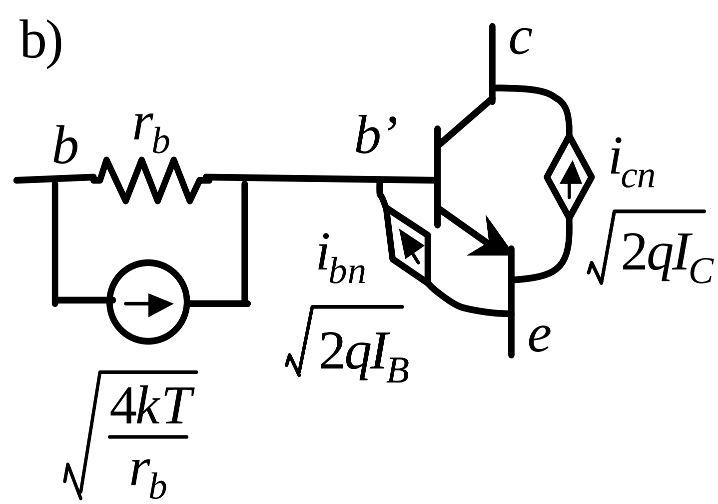


The 1/f corner frequency can be significantly higher for MOSFET

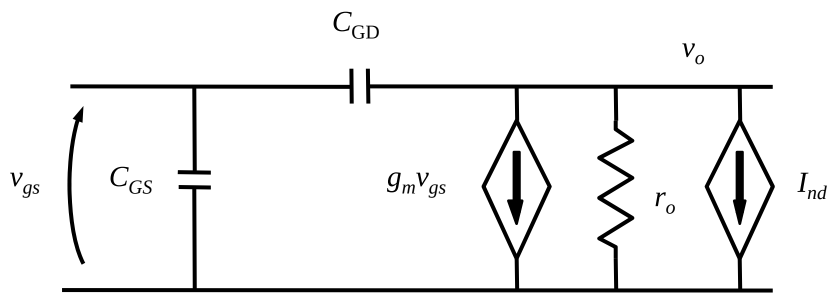
BJT Model with Noise Sources

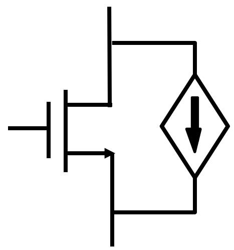






CMOS Model with Noise Sources

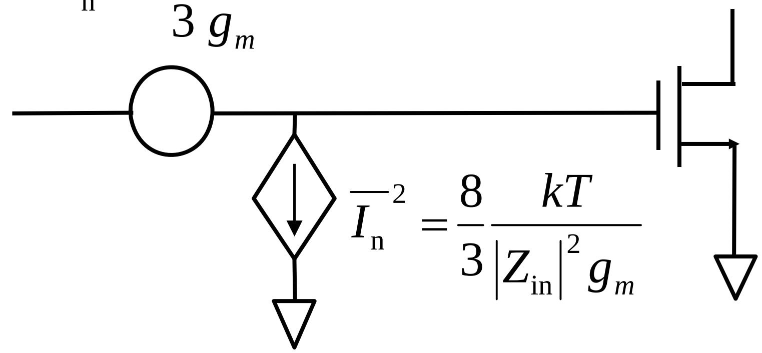




$$\overline{I_{nd}}^2 = 4kT\left(\frac{2}{3}g_m\right)$$

Input-referred noise

$$\overline{V_n}^2 = \frac{8}{3}\frac{kT}{g_m}$$



$$\overline{I_n}^2 = \frac{8}{3}\frac{kT}{|Z_{in}|^2 g_m}$$

- Gate resistance can be added to the noise model with gate resistivity ρ

$$R_{GATE} = \frac{1}{3}\rho\frac{W}{L}$$

Linear vs. Nonlinear Systems

- A system is linear if for any inputs $x_1(t)$ and $x_2(t)$, $x_1(t) \rightarrow y_2(t)$, $x_2(t) \rightarrow y_2(t)$ and for all values of constants a and b, it satisfies

$$a\,x_1(t)+bx_2(t) \rightarrow ay_1(t)+by_2(t)$$

- A system is nonlinear if it does not satisfy the superposition law.

Effects of Nonlinearity

- Harmonic Distortion
- Gain Compression
- Desensitization
- Intermodulation

- For simplicity, we limit our analysis to memoryless, time invariant system. Thus,

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \dots \qquad (3.1)$$

Effects of Nonlinearity -- Harmonics

If a single tone signal is applied to a nonlinear system, the output generally exhibits fundamental and harmonic frequencies with respect to the input frequency. In Eq. (3.1), if $x(t) = A\cos\omega t$, then

$$
\begin{aligned}
y(t) &= \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t \\
&= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t) \\
&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t
\end{aligned}
$$

Observations:

1. even order harmonics result from α_j with even j and vanish if the system has odd symmetry, i.e., differential circuits.
2. For large *A*, the *n*th harmonic grows approximately in proportion to A^n.

Effects of Nonlinearity -- Gain Compression

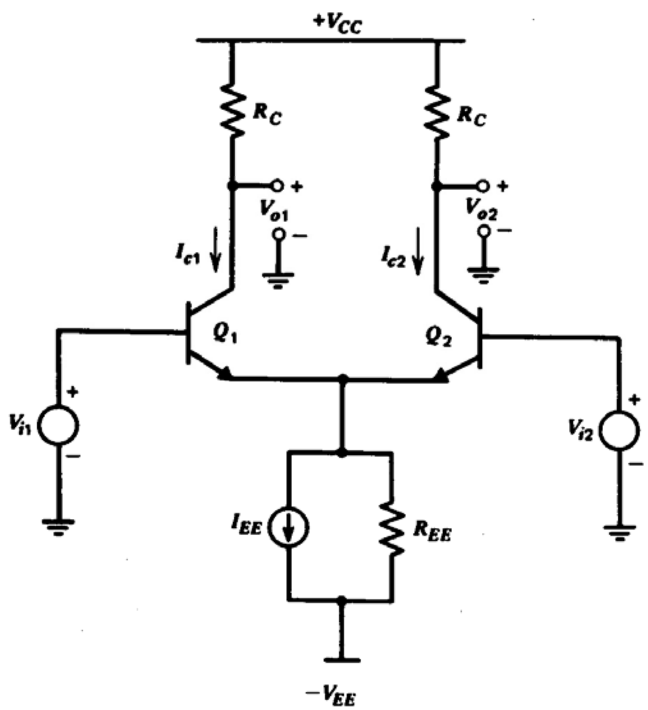
$$y(t) = \frac{\alpha_2 A^2}{2} + (\alpha_1 A + \frac{3\alpha_3 A^3}{4})\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t \quad (3.2)$$

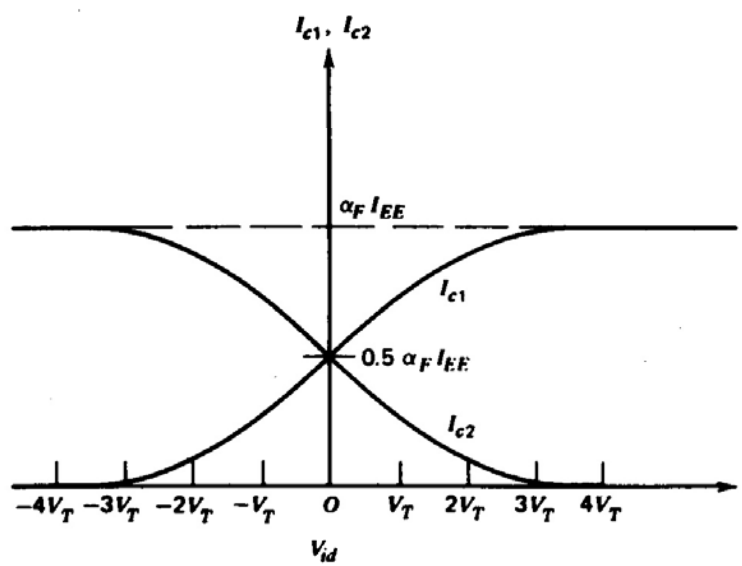
- **Under small-signal assumption, the system is normally linear and harmonics are negligible. Thus, $\alpha_1 A$ dominates → small-signal gain = α_1.**
- **For large signal, nonlinearity becomes evident. large-signal gain = $\alpha_1 + 3\alpha_3 A^3 / 4$. The gain varies when input level changes.**
- **If $\alpha_3 < 0$, the output is a “compressive” or “saturating” function of the input → the gain is compressed when A increases.**

Output of Bipolar Differential Pair

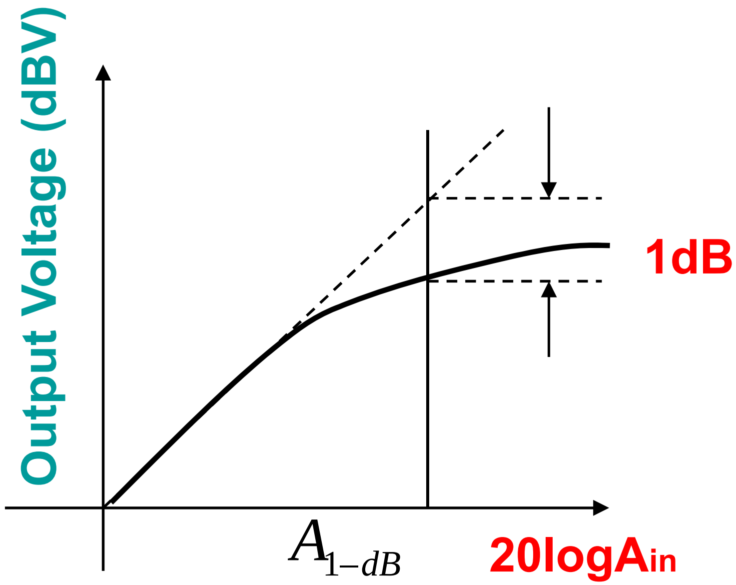
$$\tanh(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{7}{315}x^7 + \ldots$$

$$\frac{V_{od}}{V_{id}} = \alpha_F I_{EE} R_C \tanh\left(\frac{-V_{id}}{2V_T}\right)$$





Effects of Nonlinearity – 1dB Compression Point



$$20\log\left|\frac{\alpha_1 A_{1-dB}}{\alpha_1 A_{1-dB} + \frac{3}{4}\alpha_3 A_{1-dB}{}^3}\right| = 1dB$$

$$A_{1-dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|} = 0.3808\sqrt{\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$dBm = 10\log\frac{V_{pp}{}^2/8}{50\Omega \times 1mW}$$

- **1-dB compression point is defined as the input signal level that causes small-signal gain to drop 1 dB. It's a measure of the maximum input range.**

- **1-dB compression point occurs around -20 to -25 dBm (63.2 to 35.6mVpp in a 50-Ω system) in typical frond-end RF amplifiers.**

Effects of Nonlinearity – Desensitization (Blocking)

• **Desensitization -- small signal experiences a vanishingly small gain when co-exists with a large signal, even if the small signal itself does not drive the system into nonlinear range.**

Applying two-tone inputs *$x(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t$* to Eq.(3.1), we have

$$y(t) = \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2 \right)\cos\omega_1 t + \cdots,$$

gain of desired signal

For *$A_1 \ll A_2$*, it reduces to

$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2 \right) A_1 \cos\omega_1 t + \cdots,$$

Observations:

• **Weak signal's gain decreases as a function of A_2 if $\alpha_3 < 0$. For sufficiently large A_2, the gain drops to zero → the weak signal is "blocked" by the strong signal. (Why cannot we see stars during day?)**

• **Many RF receivers must be able to withstand blocking signals 60 to 70 dB greater than the wanted signals.**

Effects of Nonlinearity – Intermodulation

- Harmonic distortion is due to self-mixing of a single-tone signal. It can be suppressed by low-pass filtering the higher order harmonics.
- However, there is another type of nonlinearity -- intermodulation (IM) distortion, which is normally determined by a "two tone test".
- When two signals with different frequencies applied to a nonlinear system, the output in general exhibits some components that are not harmonics of the input frequencies. This phenomenon arises from cross-mixing (multiplication) of the two signals.

Effects of Nonlinearity – Intermodulation

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2$$

$$y(t) = \frac{1}{2}\alpha_2 (A_1{}^2 + A_2{}^2) +$$

DC Term

$$\left[\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1 (A_1^2 + 2A_2^2)\right]\cos \omega_1 t +$$

$$\left[\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2 (2A_1^2 + A_2^2)\right]\cos \omega_2 t +$$

1st Order Terms

$$\frac{1}{2}\alpha_2 \left[A_1{}^2 \cos 2\omega_1 t + A_2{}^2 \cos 2\omega_2 t\right] +$$

$$\alpha_2 A_1 A_2 \left[\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t\right] +$$

2nd Order Terms

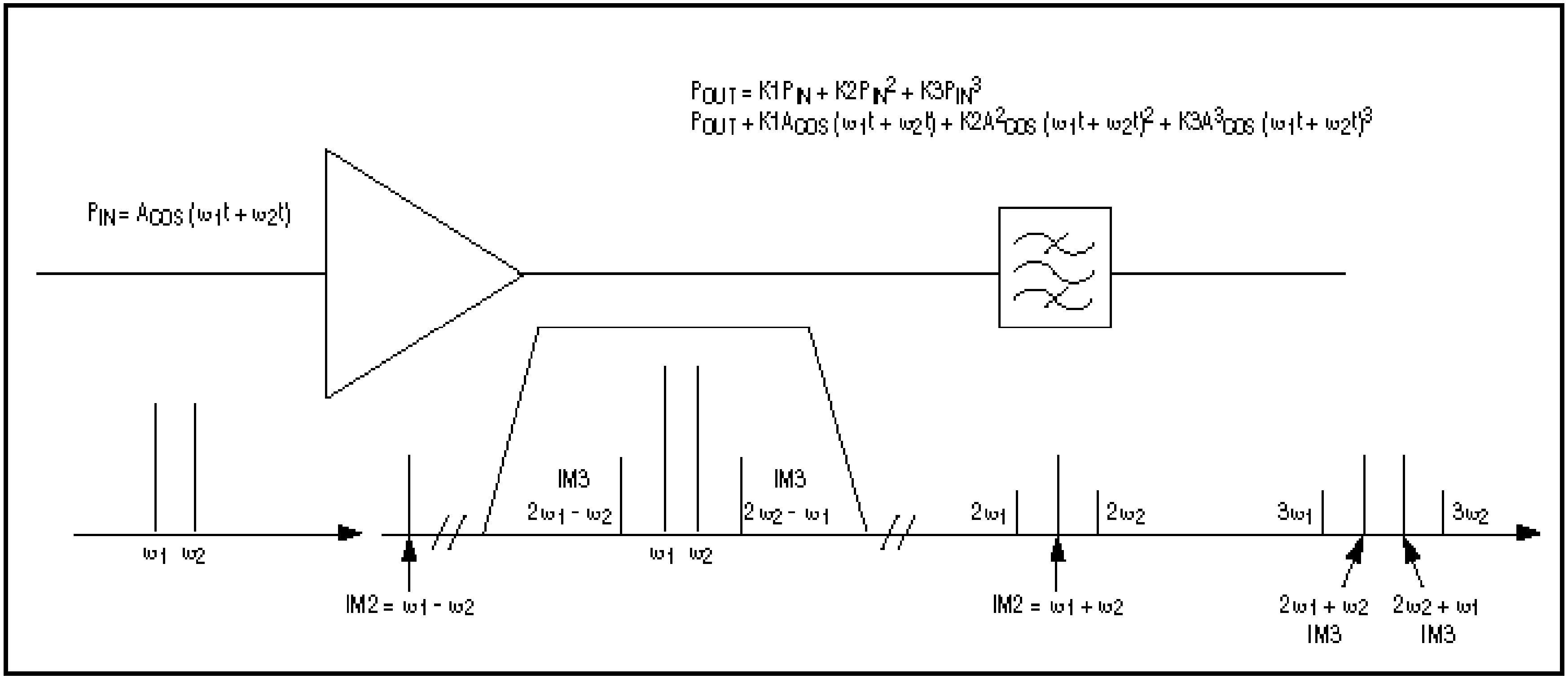
$$\frac{1}{4}\alpha_3 \left[A_1{}^3 \cos 3\omega_1 t + A_2{}^3 \cos 3\omega_2 t\right] +$$

$$\frac{3}{4}\alpha_3 \begin{Bmatrix} A_1{}^2 A_2 \left[\cos(2\omega_1 + \omega_2)t + \cos(2\omega_1 - \omega_2)t\right] + \\ A_1 A_2{}^2 \left[\cos(2\omega_2 + \omega_1)t + \cos(2\omega_2 - \omega_1)t\right] \end{Bmatrix}$$

3rd Order terms

Intermodulation – Why do we care about IM3 mostly?

- **In wireless communication system such as cellular handsets with narrow-band operating frequencies (i.e., a few tens of MHz), only the IM3 spurious signals (2w1 - w2) and (2w2 - w1) fall within the filter passband.**



Intermodulation -- Third Order Intercept Point (IP3)

- Two-tone test: $A_1 = A_2 = A$ and A is sufficiently small so that higher-order nonlinear terms are negligible and the gain is relatively constant and equal to α_1.
- As A increases, the fundamentals increases in proportion to A, whereas IM3 products increases in proportion to A^3.

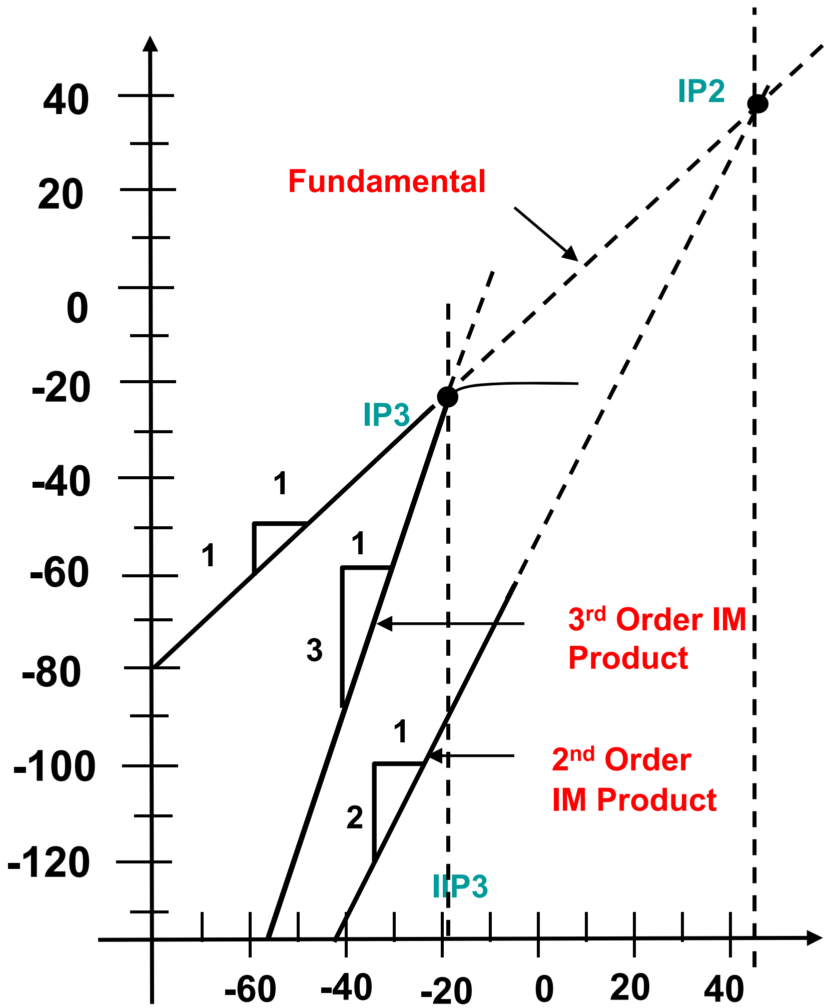
$$\alpha_1 >> \frac{9}{4}\alpha_3 A^2 \rightarrow IIP_3 = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} \text{ and } OIP_3 = \alpha_1 IIP_3$$

$$\frac{A_{1dB}}{A_{IP3}} = \sqrt{\frac{0.145}{4/3}} \approx -9.6dB$$

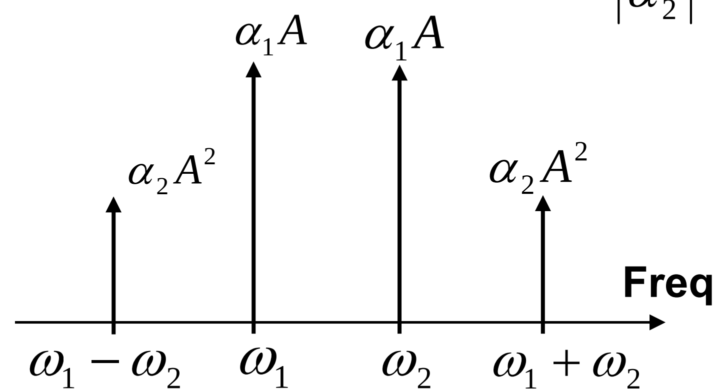
$$x(t) = A\cos\omega_1 t + A\cos\omega_2$$

$$\begin{aligned} y(t) = {} & \alpha_2 A^2 + \\ & A\left[\alpha_1 + \frac{9}{4}\alpha_3 A^2\right]\cos\omega_1 t + A\left[\alpha_1 + \frac{9}{4}\alpha_3 A^2\right]\cos\omega_2 t + \\ & \frac{1}{2}\alpha_2 A^2\left[\cos 2\omega_1 t + \cos 2\omega_2 t\right] + \alpha_2 A^2\left[\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t\right] + \\ & \frac{1}{4}\alpha_3 A^3\left[\cos 3\omega_1 t + \cos 3\omega_2 t\right] + \frac{3}{4}\alpha_3 A^3 \begin{Bmatrix} \left[\cos(2\omega_1 + \omega_2)t + \cos(2\omega_1 - \omega_2)t\right] + \\ \left[\cos(2\omega_1 + \omega_2)t + \cos(2\omega_1 - \omega_2)t\right] \end{Bmatrix} \end{aligned}$$

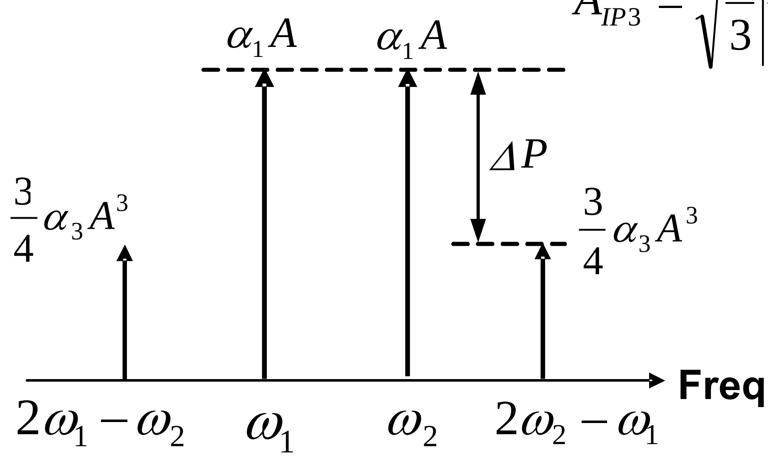
Intermodulation – IP2 vs. IP3



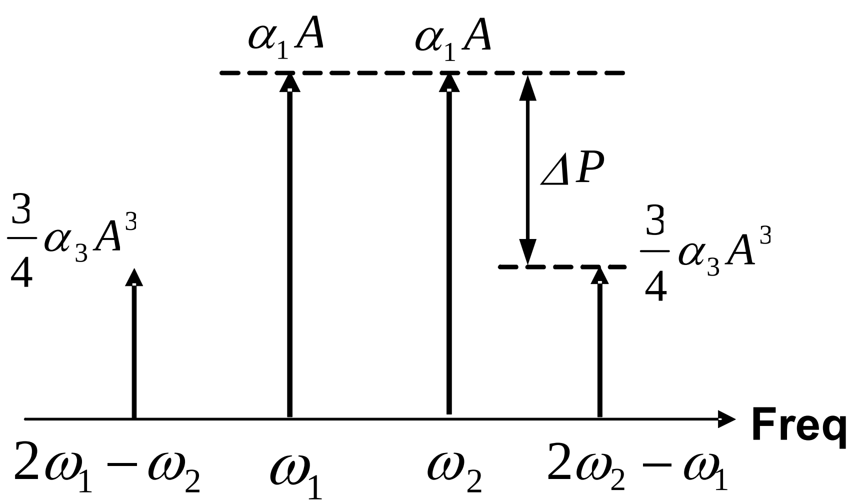
- $A_1 = A_2 = A$
- 2^{ND} Order IM Product: $A_{IP2} = \left|\frac{\alpha_1}{\alpha_2}\right|$

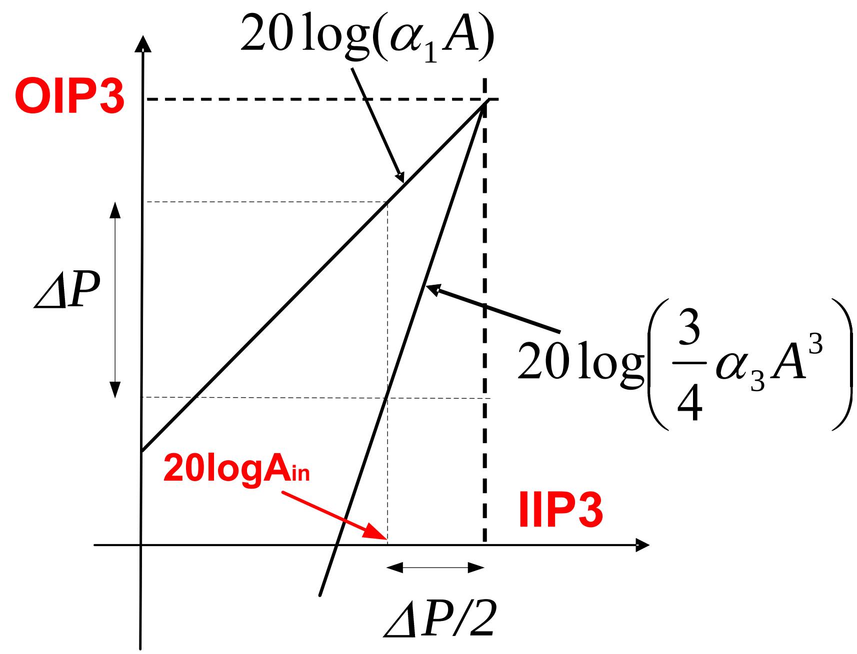


- 3^{RD} Order IM Product $A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$



Calculate IIP3 without Extrapolation





$$IIP_3[dBm] = \frac{\Delta P[dB]}{2} + P_{in}[dBm]$$

Relationship Between 1-dB Compression and IP3

1-dB compression point with single tone applied:

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} \qquad A_{1-dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|} \qquad \frac{A_{IP3}}{A_{1dB}} = 3.04 = 9.66dB$$

1-dB compression point with two tones applied:

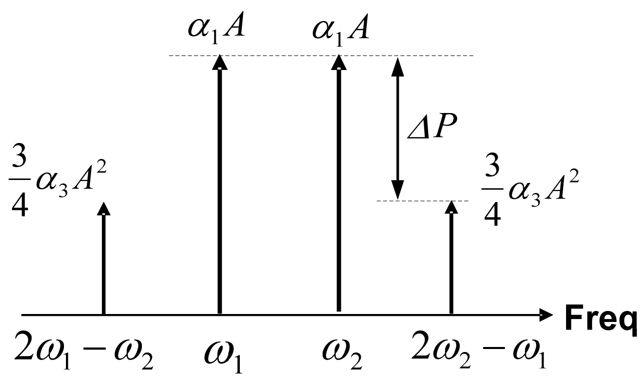
$$\frac{A_{IP3}}{A_{1dB}} = \frac{2\sqrt{\frac{\alpha_1}{3\alpha_3}}}{0.22\sqrt{\frac{\alpha_1}{\alpha_3}}} = 5.25 = 14.4dB$$

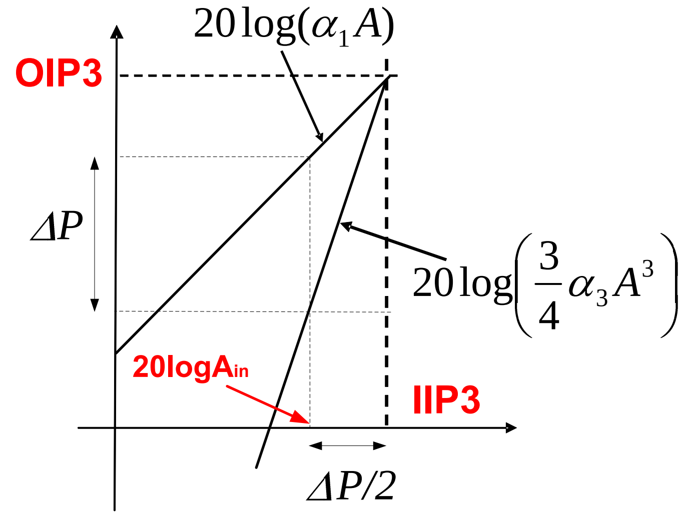
Determine IIP3 and 1-dB Compression Point from Measurement

- An amplifier operates at 2 GHz with a gain of 10dB. Two-tone test with equal power applied at the input, one is at 2.01 GHz. At the output, four tones are observed at 1.99, 2.0, 2.01, and 2.02GHz. The power levels of the tones are -70,-20,-20, and -70dBm. Determine the IIP3 and 1-dB compression point for this amplifier.
- Solution: 1.99 and 2.02 GHz are the IP3 tones.

$$IIP3 = (P_1 - G) + \frac{1}{2}[P_1 - P_3] = -20 - 10 + \frac{1}{2}[-20 + 70] = -5dBm$$

$$P_{1dB} = -5 - 9.66 = -14.66dBm$$





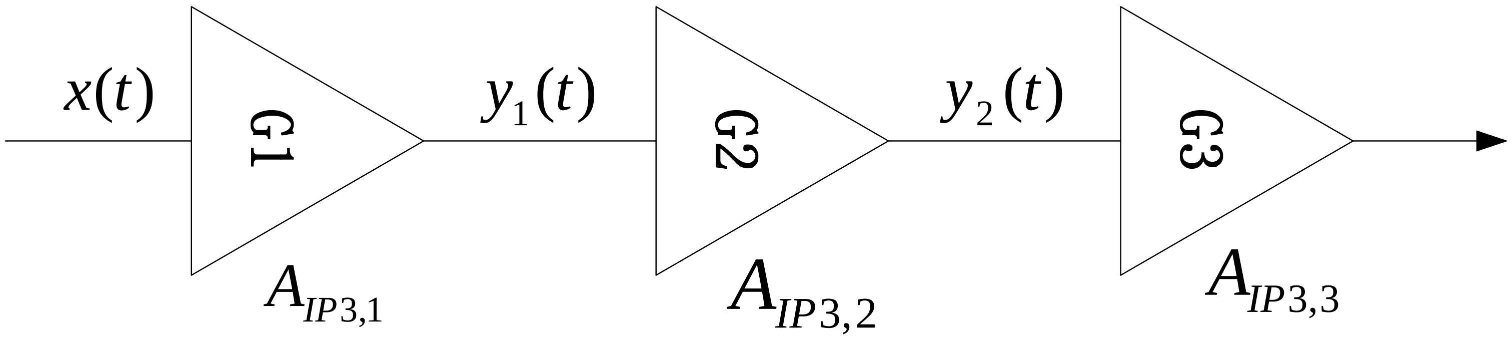
$$IIP_3[dBm] = \frac{\Delta P[dB]}{2} + P_{in}[dBm]$$

Intermodulation of Cascade Nonlinear Stages

$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \ldots$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t) + \ldots$$

$$\ldots$$



Linearity is more important for back-end stages.

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2 \beta_1^2}{A_{IP3,3}^2} + \ldots$$

Noise Figure of Cascade Stages

$$NF_{tot} = 1 + \left(NF_1 - 1\right) + \frac{NF_2 - 1}{G_{p1}} + \frac{NF_3 - 1}{G_{p1} G_{p1}} + ... + \frac{NF_m - 1}{G_{p1} G_{p2.....} G_{p(m-1)}}$$

NF_{tot} – total equivalent Noise Figure

NF_m – Noise Figure of m^{th} stage

G_{pm} – Available power gain of m^{th} stage

Noise figure is more important for front-end stages.

Sensitivity

- **Sensitivity -- defined as the minimum signal level that the system can detect with acceptable SNR.**

$$NF = \frac{SNR_{in}}{SNR_{OUT}} = \frac{P_{sig} / P_{RS}}{SNR_{OUT}}$$

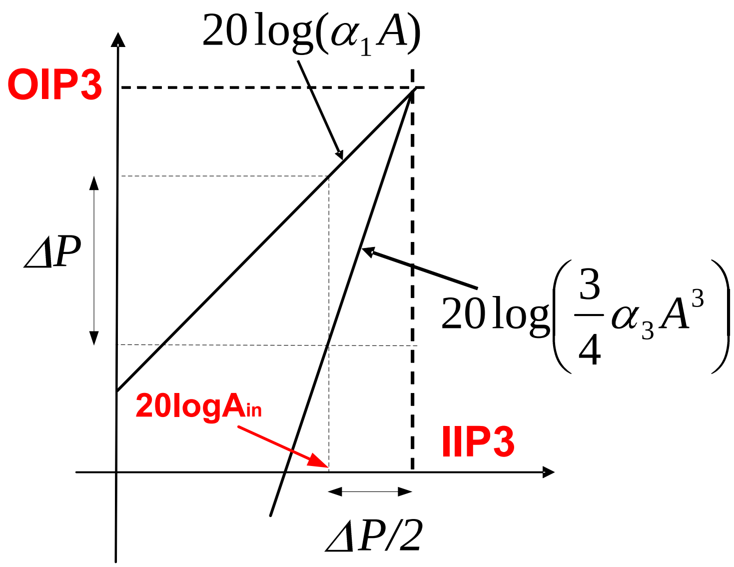
- **The overall signal power is distributed across the channel bandwidth, B, integrating over the bandwidth to obtain total mean square power**

$$P_{sig,tot} = P_{RS} \bullet NF \bullet SNR_{OUT} \bullet B$$

$$P_{in,\min}\Big|_{dBm} = P_{RS}\Big|_{dBm/Hz} + NF\Big|_{dB} + SNR_{\min}\Big|_{dB} + 10\log B$$

where P_{RS} is the source resistance noise power.

Maximum Input Power



$$P_{IIP3} = P_{in} + \frac{P_{out} - P_{IM,out}}{2}$$
$$= P_{in} + \frac{P_{in} - P_{IM,in}}{2} = \frac{3P_{in} - P_{IM,in}}{2}$$

$$P_{in} = \frac{2P_{IIP3} + P_{IM,in}}{3}$$

where $P_{IM,out}$ denotes output-referred power of IM_3 products, P_{out}=P_{in}+G, $P_{IM,OUT}$= $P_{IM,in}$+G. The input level for which the IM products become equal to the noise floor F is thus given by

$$P_{in} = \frac{2P_{IIP3} + F}{3} = \frac{2P_{IIP3} - 174dBm + NF + 10\log B}{3}$$

Dynamic Range

- **Dynamic Range (DR) -- defined as the ratio of the maximum to minimum input levels that the circuit provides a reasonable signal quality.**
- **Spurious-Free Dynamic Range (SFDR) -- determine the upper end of dynamic range on the intermodulation behavior and the lower end on sensitivity.**
- **The upper end of the dynamic range is defined as the maximum input power in a two tone test for which the 3rd IM products do not exceed the noise floor F=-174dBm+NF+10logB.**
- **The SFDR is thus given by**

$$SFDR = P_{in,\max} - P_{in,mim} = \left(\frac{2P_{IIP3} + F}{3}\right) - (F + SNR_{\min}) = \frac{2(P_{IIP3} - F)}{3} - SNR_{\min}$$

- **Example: NF=9dB, P_{IIP3}=-15dBm, B=100kHz, SNR_{min}=12dB → SFDR=(-15-(-174+9+50))/1.5-12=54.7dB.**