Active and Passive Filter Synthesis using MATLAB*

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Analog filters play a significant role in every electronic system. There are two different ways to implement analog filters. The first one uses cascade connection of second-order circuits. These types of filters are easy to design, but they are relatively sensitive to the tolerance of elements. The second method uses passive ladder prototypes and practical VLSI implementation needs a series of integrators. This type of design is much less sensitive to the tolerance of elements, but also it is more difficult to design. The second method is rarely taught in basic analog filter courses. Both cascade as well as ladder-based filter implementation designs are complex and computationally intensive. Therefore, computer software could be very helpful to assist students and engineers in their design effort. This paper describes an educational MATLAB tool which simplifies the process of analog filter design. The software allows the use of four approximation methods: Butterworth, Chebyshev, Inverse Chebyshev and Cauer. In addition, several types of filters can be designed such as lowpass, highpass, bandpass and bandstop. In comparison to other tools this software also generates actual circuit implementation.

INTRODUCTION

In the ANALOG filter course we have to deal not only with transfer functions and their frequency and phase responses but we have to find practical circuit implementations. The complexity of the design process and tedious calculations makes teaching analog filters a difficult course. With massive computation effort, students are losing focus on the design principle and getting busy with lengthy computations. As a result, only limited parts of the filter course can be covered since students are already overloaded with their design assignments. The motivation of the development of the filter design software in this effort was twofold: (1) to release student effort from mechanical computation so they can focus on the understanding of the design algorithms (2) to increase the amount of the material which can be covered in the course.

The MATLAB software is an adequate tool to implement these goals. MATLAB is a widely used software for teaching purposes because of its high computational power and superior graphics. Analog filter circuits are designed in two ways: active filter (cascade type) and passive filter (ladder type). Traditional design of analog filters either in active form or passive form requires familiarity with the use of filter design tables [1]. Teaching as well as learning using tables is a tedious and time-consuming task. The range of values available in design tables is also limited. With MATLAB there is no need to use tables since all required values can be computed using numerical algorithms. We used MATLAB programming to design an independent software package for analog filter design. Both the active filter and passive filter design are integrated in a single menu. This software package is user friendly and simple to navigate in the design process.

There are several software tools already developed to enhance the filter design process: FILTER [2], LADDER [3], FILTERD [4], FIESTA [5]. These deal with either active filter design or passive filter design. FILTER, FIESTA and LADDER can operate only in the DOS environment and therefore they have limited applications. FILTER and FIESTA handle only cascade type of design while LADDER can produce only passive ladder prototypes. A number of software packages are available for analog filter design but MATLAB creates a unique environment for helping students to understand the design process. FILTERD is a MathCAD-based software package and the major limitation of this package is that it cannot design highpass, bandpass and bandstop Cauer filters. These limitations were overcome in this software package.

DESCRIPTION OF THE PROGRAM

In order to speed up computation the described software uses only specialized routines and it is not relying on the standard MATLAB routines for calculation of transfer function coefficients and filter circuit design. At first, a user has to choose ‘Type of filter’ and ‘Approximation method for filter design’. This software package can design...
lowpass, highpass, bandpass, bandstop filters with Butterworth, Chebyshev, Inverse Chebyshev and Cauer types of approximations methods. These approximation techniques are used because these are some of the most commonly used in textbooks dedicated to analog filter design and applications in industry.

The second stage of the design is to give ‘brick wall’ specifications of filter. As in Fig. 1, the brick wall specifications depend upon the type of filter we choose. For example, if it is a lowpass filter, specifications include pass band attenuation, stop band attenuation, pass band frequency and stop band frequency. Once specifications are given, the design process automatically gives magnitude plot and values of poles and zeros. As in Fig. 2, pole zero map and phase plots are available for analysis. The next stage is either circuit design, which can be active filter design or passive filter design. Active filter design or cascaded design gives stages of filter circuit and normalized values of capacitors and resistors. In ladder filter design, you have the choice to specify input and output resistances and design gives either a minimum inductance or minimum capacitance circuit with values of inductors and capacitors. Figure 3 shows a sample design of a 14th order Butterworth filter.

**DESCRIPTION OF ALGORITHMS FOR LOW PASS FILTER APPROXIMATIONS**

For all filters at first the lowpass prototype is designed where four input parameters are required:

- $\alpha_p$—attenuation in pass band
- $\alpha_s$—attenuation in stop band
- $\omega_p$—pass band frequency
- $\omega_s$—stop band frequency
Butterworth filters
The filter order is calculated using Equation (1):

\[ n = \log \left( \frac{10^n}{10^{-n}} \right) \]

and then rounded up to the nearest integer number. The magnitude of the transfer function is given by:

\[ |T(j\omega)|^2 = \frac{1}{1 + \left( \frac{\omega}{\omega_0} \right)^{2n}} \]

(2)

Attenuation as a function of frequency is given by:

\[ \alpha = 10 \log \left( 1 + \left( \frac{\omega}{\omega_0} \right)^{2n} \right) \]

(3)

In the next step angles for all poles are found using:

\[ \theta_k = \frac{\pi}{2} \left[ 1 + \frac{1}{n} (2k - 1) \right] \]

(4)

Real and imaginary coordinates of poles are calculated using:

\[ \alpha_k = -\cos(\theta_k) \quad b_k = \pm \sin(\theta_k) \]

while quality factor Q is evaluated using:

\[ Q_k = -\frac{\omega_0}{2a_k} = \frac{1}{2 \cos(\theta_k)} \]

(5)

Chebyshev filters
The order of the Chebyshev filter is calculated using Equation (6):

\[ n = \frac{\cosh^{-1} \sqrt{\frac{\omega_0}{10^{-n}}}}{\cosh^{-1} \left( \frac{\omega_0}{\omega_p} \right)} \]

(6)

and then rounded up to the nearest integer number.

Typical characteristics of Chebyshev filter are shown in Fig. 4 where:

\[ \varepsilon = \sqrt{10^n - 1} \]

(7)

\[ \gamma = \frac{1}{n} \sinh \left( \frac{1}{\varepsilon} \right) = \frac{1}{n} \sinh^{-1} \left( \frac{1}{\sqrt{10^n - 1}} \right) \]

(8)

The transfer function for Chebyshev filters is given by:

\[ |T(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 C_n^2(\omega)} \]

(9)

where \( C_n(\omega) \) is a Chebyshev polynomial of \( n \)-th order. Attenuation as a function of frequency is given by:

\[ \alpha = 10 \log \left( 1 + \varepsilon^2 C_n^2(\omega) \right) \]

(10)

Angles are found in the same way as for Butterworth filter using:

\[ \theta_k = \frac{\pi}{2} \left[ 1 + \frac{1}{n} (2k - 1) \right] \]

(11)

Real and imaginary coordinates of poles are calculated using: \(-a_k = \cos(\theta_k) \sinh(\gamma) \quad \pm b_k = \sin(\theta_k) \cosh(\gamma)\) while quality factor Q is evaluated using:

\[ Q_k = -\frac{\omega_0}{2a_k} = \frac{1}{2 \cos(\theta_k)} \]

Inverse Chebyshev filters
The order of the Inverse Chebyshev filter is calculated using the same formula as for the Chebyshev filters:

\[ n = \frac{\cosh^{-1} \sqrt{\frac{10^n - 1}{10^{-n}}}}{\cosh^{-1} \left( \frac{\omega_0}{\omega_p} \right)} \]

(12)

and then rounded up to the nearest integer number.
Typical characteristics of Inverse Chebyshev filter are shown in Fig. 7 where:

\[ \varepsilon = \frac{1}{\sqrt{10^{\frac{\omega}{2\pi}}} - 1} \]  

\[ \gamma = \frac{1}{n} \sinh \left( \frac{1}{\varepsilon} \right) = \frac{1}{n} \sinh^{-1} \left( \sqrt{10^{\frac{\omega}{2\pi}}} - 1 \right) \]  

The transfer function for Inverse Chebyshev filters is given by:

\[ |T(j\omega)|^2 = \frac{\varepsilon^2 C_n^2(\omega)}{1 + \varepsilon^2 C_n^2(\omega)} \]  

(15)

Where \( C_n(\omega) \) is a Chebyshev polynomial of \( n \)-th order. Attenuation as a function of frequency is given by:

\[ \alpha = 10 \log \left( 1 + \varepsilon^2 C_n^2(\omega) \right) \]  

(16)

Attenuation as a function of frequency is given by:

\[ \alpha = 10 \log \left( 1 + \left( \frac{\omega}{\omega_0} \right)^{2n} \right) \]  

(17)

Angles are found in the same way as for Butterworth filter using:

\[ \theta_k = \frac{\pi}{2} \left[ 1 + \frac{1}{n} (2k - 1) \right] \]  

(18)

Real and imaginary coordinates of poles primitives are calculated using:

\[ -a_k = \cos(\theta_k) \sinh(\gamma) \pm b_k = \sin(\theta_k) \cosh(\gamma) \]

\[ \omega_k = \sqrt{a_k^2 + b_k^2} \]
then actual pole locations are found using:

\[
p_{1C} = \frac{1}{a_k + jb_k} = \frac{a_k - jb_k}{a_k^2 + b_k^2}
\]

\[
= \frac{a_k - jb_k}{\omega_k}
\]

(19)

Zeros are located on imaginary axis and its location can be found using:

\[
\omega_i = \frac{1}{\cos \left( \frac{\pi i}{2n} \right)} = \sec \left( \frac{\pi i}{2n} \right)
\]

(20)

\[i = 1, 3, 5, \ldots I = 0 \text{ to } i < np \text{ number of poles}\]

**Cauer elliptic filters**

Cauer elliptic filters synthesis formulas are too complex to describe them here. The algorithm is however implemented in the MATLAB filter tool. The order of Cauer filter is lower than in the case of Chebyshev filters, but it has ripples in the pass band and the stop band. Typical characteristics of the Cauer filter are shown in Fig. 10. The transfer function for Cauer elliptic filters is given by:

\[
|T(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 R_0^2(\omega, L)}
\]

(21)

**CIRCUIT IMPLEMENTATION OF FILTERS**

The implementation of analog filter design requires calculation of poles, zeros and transfer
function for that filter. For each type of approximation, poles and zeros are placed differently in s-plane. For high pass, bandpass and band stop filters, given specifications are transformed to lowpass and lowpass prototype filter is again transformed to its original type. Quality factor and natural frequency of each complex conjugate poles are calculated.

This software package implements analog filters in two different ways:

- active filter circuit implementation;
- ladder circuit implementation.

**Active filter circuit implementation**

Once poles and zeros are calculated, depending on type of filter, complex conjugate poles and zeros are selected to form a second-order transfer function, which can be implemented using biquads. A wide variety of biquads like Lowpass Sallen-Key, Bandpass Sallen-Key, Notch circuit, Lpiocot circuit and center loaded Twin-T circuit are used in active filter design. The type of biquad used for design depends on the type of filter, approximation methods used and order of filter. Using design equations of biquad, elements of biquad can be found. The program automatically pairs poles and zeros.

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(a) First-order lowpass filter

(b) First-order highpass filter

(c) Second-order lowpass Sallen-Key filter

(d) Second-order highpass Sallen-Key filter and associated equations

(e) Sallen-Key bandpass filter

(f) Single operational-amplifier bandstop filter

Fig. 12. Active form implementation of filter blocks.
Figure 13 shows 12th order bandpass Inverse Chebyshev filter with center loaded Twin-T circuits. These designed biquads are shown as stages since it is not possible to draw all the blocks in the same window. Detailed circuit diagram of each circuit is obtained by choosing the stages and when these stages are cascaded, it produces required response.

**Ladder circuit implementation**

Cascade realizations are much easier to design and without regard to load and source impedances but ladder filters are less sensitive to element variations. When these ladder circuits are implemented in microelectronic form, inductors can be replaced by integrated circuit blocks like the General Impedance Converter (GIC). The design of the ladder circuit depends on presence of zeros. In absence of zeros in the transfer function continued fraction of expansion method is used and in presence of zeros, a unique algorithm [6] is used. For highpass, bandpass and bandstop circuits, frequency specifications are transformed to lowpass region and a lowpass prototype of ladder circuit is designed. Inductors and capacitors of the lowpass prototype are transformed and replaced with new elements using the Fig. 14 [7]. Further work on this ladder circuit replaces inductors with IC blocks like switched capacitors or General impedance converters.

**LADDER SYNTHESIS**

Active circuit design of filters gives transfer function of the required filter. Let \( T(s) \) be the transfer function. Auxiliary function \([A(s)]\) found from transfer function and following sequence of steps gives impedance function \([Z(s)]\) of the circuit:

\[
T(s) = A(s)A(-s) / T_d(-s)
\]

1. Find \( T_d(s) \) \( T_d(-s) \) and \( T_n(s) \) \( T_n(-s) \)
2. Find Auxiliary function \( A(s)A(-s) \)

\[
A(s)A(-s) = 1 - 4 \frac{R_{in}R_{out}}{(R_{in} + R_{out})^2} T(s)T(-s)
\]

- \( R_{in} \) = input resistance
- \( R_{out} \) = load resistance
3. Find \( A(s) \) from \( A(s)A(-s) \)

\[
A(s) = \frac{A_n(s)}{T_d(s)}
\]

\( A_n(s) \) is found by taking roots \( A(s)A(-s) \) which lie in left half of plane.
4. Find $Z(s)$

$$Z(s) = R_m \frac{1 - A(s)}{1 + A(s)}$$

Here we have two ways of designing the ladder circuit depending on whether zeros are present in the transfer function or not.

**Design method of lowpass prototype without zeros**

For a transfer function without zeros standard continued fraction of expansion method is followed. The values obtained in this division are element values of the ladder circuit. The choice of minimum inductance circuit or minimum capacitance circuit determines assignment of element values to the ladder circuit. For example an 8th order bandstop elliptic filter with brick-wall specifications, $\alpha_p = 2\text{dB}, \alpha_s = 40\text{dB}, \omega_c = 1000, B_p = 1000, B_s = 2000$ is chosen for ladder design. The transfer function obtained using the above procedure is $z(s)$. If minimum capacitance circuit is chosen element values obtained for a bandstop elliptic ladder circuit are as shown in Fig. 15.

**Design method of lowpass prototype ladder filter with zeros**

The design algorithm for a ladder circuit with zeros is unique. There is no other software available for design of ladder circuits in presence of zeros (Inverse Chebyshev and Cauer filter ladder circuit) in the transfer function. In case of even-order Chebyshev, inverse Chebyshev and Cauer filters, the above ladder synthesis procedure will not yield practically feasible values for ladder elements. Special transformations [6] are performed on poles and zeros of the transfer functions. A special frequency transformation (22) will transform the largest conjugate zero pair, $C_{ai} = 1/sZ_1(s)$ for $s = j\omega$ to infinity, thus reducing the order of the numerator by 2, while still maintaining the same basic response. The normalization frequency $\omega_N$ maps to the same frequency on both the $\omega$-axis and the $\Omega$-axis;

$$\Omega^2 = \frac{\omega_N^2 - \omega_0^2}{\omega^2 - \omega_0^2} \quad (22)$$

In the case of even-order Cauer and Chebyshev filters, this transformation, Equation (23) moves the frequency of the first maximum ($\omega_0$) to zero and allows the design of even-order Cauer and Chebyshev filters with equal terminating resistances:

$$\Omega^2 = \frac{\omega^2 - \omega_0^2}{\omega_N^2 - \omega_0^2} \quad (23)$$

A generalized ladder circuit in presence of zeros is shown in Fig. 16. Each zero pair ($z$) is represented by a resonant circuit consisting of inductor and capacitor in parallel. Resonant circuit with shunt capacitor ($C_{ai}$) constitutes complex conjugate pole and conjugate zero pair. Two different methods are analyzed for extracting ladder circuit elements.

**Method 1**

Following procedure is used to calculate ladder elements from the impedance function ($Z_1$):

1. Choose a value of zero ($z$) to be eliminated from impedance function.
2. Find value of $C_{a1}$ solving equations (25) and (26).

$$Z_B(s) = \frac{N_A}{D_A - sC_{a1}N_A}$$

$$= \frac{a_N s^{N-1} + a_N s^{N-2} + \cdots + a_1 s + a_0}{k_N s^{N} + k_N s^{N-1} + k_N s^{N-2} + \cdots + k_1 s + k_0}$$

$$k_i = b_i - C_{a1}a_i, \quad (1 \leq i \leq N) \quad (k_0 = b_0)$$

$$C_{a1} = \frac{b_1 s^{-1} - b_1 a^{-1} + b_2 s^{-2} - a_2 s^{-3} + \cdots}{a_0 s^{-1} - a_2 s^{-3} + a_2 s^{-3} - \cdots}$$

$$z^2 = \left| L_{b1} C_{b1} \right|^{-1}$$

3. Choose a different zero and repeat steps 1 and 2 if the value of $C_{a1}$ is not practically feasible.

4. Remove shunt capacitor and obtain $Z_B$ using equation (24).

5. Determine value of $D_C$ and $L_{b1}$ from Equations (29) and (30).

$$Z_B = \frac{N_B}{D_B} = \frac{N_C}{D_C} + \frac{L_{b1}s}{z^2 s^2 + 1}$$

$$D_B = D_C(z^2 s^2 + 1)$$

$$L_{b1} = \frac{N_B}{D_C} \left| s^{-1} \right|$$

6. Find value of $C_{b1}$ and $Z_C$ using equation (27) and equation (28) respectively.

7. Above steps are repeated for all resonant circuits (all zeros) and remaining circuit elements (like $C_{aN+1}$) are found using classical continued fraction method.

**Method 2**

This method [8] uses the same principle used in classical continued fraction of expansion. The value of shunt capacitor is obtained by removing part of the pole at infinity from $1/Z_A$ to produce a pair of zeros at:

$$z = \pm \frac{j \sqrt{L_{b1} C_{b1}}}{\sqrt{L_{b1} C_{b1}}}$$

Next, the resonant circuit corresponding pair of poles is removed. The term to be removed can be written as:

$$\frac{ks}{s^2 + z^2} \left( \frac{k/C_{b1}}{s + 1/L_{b1} C_{b1}} \right)$$

The value of $C_{b1}$ can be found as:

$$k \left( \frac{1}{C_{b1}} \right) = Z_B(s^2 + z^2) |s = jz$$

Values of $L_{b1}$ and $Z_C$ are found using Equations (27) and (33) respectively:

$$Z_C = Z_B - \frac{ks}{s^2 + z^2}$$

The above steps are repeated for all zeros and remaining elements are found using continued fraction method. For a given impedance function there will be multiple solutions of ladder circuits. However, proper selection of zeros only results in practically feasible values of inductors and capacitors. In this procedure, selection of zeros is very important. Zeros are selected on a trial and error basis until a correct response is obtained. For example a lowpass inverse Chebyshev 9th order filter with $\omega_p = 0.03$dB, $\omega_s = 70$dB, $\omega_p = 1000$, $\omega_s = 2000$ has 4 zeros. Selection of zeros $(\pm 5.8476j, \pm 3.1114j, \pm 2.0308j, \pm 2.3096j)$ gives negative value of shunt capacitor whereas proper selection of zeros $(\pm 3.1114j, \pm 2.0308j, \pm 2.3096j, \pm 5.8476j)$ gives ladder elements as shown in Fig. 17. Compared to the first method, this method gives ladder circuit elements for higher orders of odd ordered inverse Chebyshev and elliptic filters.

LC ladders are the basic building blocks in many current mode signal processing applications [9, 10]. Low sensitivity to component tolerances is the advantage of LC ladders, which can be further implemented in active form [7]. Element replacement method (General impedance converter, Frequency dependent negative resistances) and functional simulation (Leapfrog realization, Gring Good form) are common methods used for active circuit implementation of ladder circuits. This software will be very useful in this regard. Further work on this program involves implementation of above transformations.

**IMPLEMENTATION OF THE SOFTWARE IN CLASSROOM**

The analysis of the above software program leads to design of many lessons for students, which allow students to understand principles of analog filters both in cascade form and ladder form. Basic theoretical principles can be given using any standard book on analog filters [7]. These principles help students to analyze the lessons designed using this MATLAB software program.
As an exercise, the student can be asked to design a filter to meet the specifications with a low order and which has a good gain and phase margin.

**Cascaded filter analysis**

Different types of active circuit blocks used in implementation of cascaded filter are shown in Fig. 12 [11]. In cascade realizations, the transfer function obtained from poles and zeros is factored into first and second-order parts. A lesson can be made on designing active circuit blocks for first and second-order transfer functions and cascading of active circuit blocks to realize the filter. The design values can be verified using MATLAB filter program.

**Ladder filter analysis**

Teaching ladder circuit design as a classroom course is a complex task because of the highly computational algorithm used in extracting ladder elements. However, this program simplifies such tasks by giving ladder filter elements directly. Using this software, students can be given an exercise on the effect of terminating resistances on filter performance, transformations of ladder elements from low pass to different regions. Integration of cascaded and ladder filter realizations in a single menu allows comparative design and performance of two methods.

**CONCLUSION**

This paper describes easy to add and independent MATLAB software package. The simplicity and user-friendly interface makes it superior compared to other software packages.

The results have been verified with other software packages and filter design tables [1]. This software combines active and passive filter design methods in a single menu. This package can be used with any Windows-based machine. This package can be of great use for classroom courses. The developed software can be downloaded from: http://www.eng.auburn.edu/users/wilambm/DAF.zip

**REFERENCES**


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