

Flow in pipes (fully developed) laminar.

$$\Delta P_f \text{ (frictional losses)} = F_f = (P_1 - P_2)_f = 32 \mu \dot{V} (L_2 - L_1) / D^2$$

(laminar only)

$$\therefore F_f = 4 f \Delta L \dot{V}^2 / 2 D$$

$$F_{f, \text{lam}} = 16 / Re$$

$$F_f [=] \text{ lb} \cdot \text{lb}_f / \text{lb}$$

Turbulent Flow

Use Friction Factor charts (Moody vs Fanning f)

Problem types: 1. Given pipe info & flow, find losses ( $\Delta P_f$ ,  $h_f$ ,  $F_f$ )

trial & error { 2. Given pipe info &  $\Delta P$ , find flow rate (Q)

3. Given flow rate & losses, design & select pipe.

→ (assume f to start!)

In turbulent flow  $V_x = V_{x, \text{max}} \left( \frac{R-x}{R} \right)^n$   $n \approx 1/7$

but  $V_x = f(Re)$  different "n"s

Loss Terms

1.  $\frac{\Delta P_f}{\rho} = \Delta h \frac{g}{g_c} [=] \text{ ft} \cdot \text{lb}_f / \text{lb}$

2.  $\frac{\Delta P_f}{\gamma} = \Delta h [=] \text{ ft (or m)}$

3.  $\Delta P_f [=] \text{ atm (or Pa or psi etc)}$

4.  $\frac{g}{g_c} F [=] \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m} \left( \frac{\text{lb}_m \cdot g}{\text{lb}_f \cdot g_c} \right) [=] \text{ ft}$

Turb Flow losses:

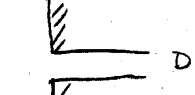
$$h_L = 32 f L g_c Q^2 / \pi^2 g D^5$$

Friction losses in Valves & Fittings (Minor losses)

Contractions



$Ke = 1.0 \dot{V}^2 / 2$



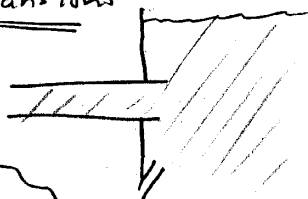
$Ke = 0.55 \dot{V}^2 / 2$



$Ke = C_1 \dot{V}^2 / 2$  where

$C_1$	$K/D$
0.55	0.0
0.12	0.1
0.04	0.2

Expansions



$$K_{ex} = 1.0 \frac{V^2}{2}$$

Other

A large variety of other fittings, valves, couplings, T's etc have tabulated coefficients (see Perry's or other fluid ref book)

Noncircular cross-sections

$\epsilon/D_e$

$$D_e = 4 R_H \quad \text{where } R_H = \frac{\text{cross-sectional area (flow)}}{\text{wetted perimeter}}$$

for circular	$D_e = D$
annular	$D_e = D_1 - D_2$
rectangular	$D_e = \frac{2ab}{a+b}$

Drag forces

$$C_D \equiv \frac{F_D / A_{proj}}{\rho V_0^2 / 2} \quad \therefore F_D = C_D A_p \rho V_0^2 / 2$$

for creeping flow (Stokes flow)

$Re < 0.1 \quad C_D = 24 / Re$

$$F_D = 3\pi \mu D \rho V_0$$

other regimes

$$C_D = 0.2 \quad Re > 5 \times 10^5$$

$$C_D = 0.44 \quad 10^3 < Re < 2 \times 10^5$$

Other  $Re$ : use  $C_D$  vs  $Re$  charts

$$A_p(\text{sphere}) = \pi D^2 / 4$$

$$A_p(\text{cyl} + flow) = LD_p$$

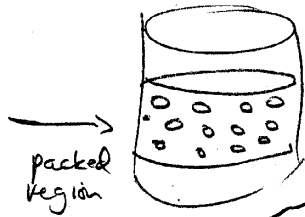
$$Re = \frac{D G}{\mu}$$

Drag on flat plates

$$F_D = C_f A_{shear} \rho V^2 / 2$$

use graphs for  $C_f$  vs  $Re$  values.

Packed Beds (not fluidization)



Defs:  $\epsilon$  = void fraction = vol of voids / vol of bed

$a_v$  = spec. area of particles

$$= \frac{S_p}{V_p} = \frac{\text{surface area sphere}}{\text{vol of particles}} \quad [\text{m}^2 \text{m}^{-3}]$$

$$a_v (\text{sphere}) = 6 / D_p \quad \text{m}^{-1}$$

$1 - \epsilon$  = Solids fraction

$$a_L = a_v (1 - \epsilon) = 6 / D_p (1 - \epsilon)$$

$\uparrow$  total area of bed

$v$  = average interstitial velocity

$v'$  = superficial velocity =  $v \epsilon$

$v' = Q / A_{\text{bed (empty)}}$

$$G' = \frac{\dot{m}}{A} = \frac{\rho v A}{A} = \rho v$$

$$K_{It} = \frac{\epsilon}{a} = \frac{\epsilon D_p}{6(1 - \epsilon)}$$

$$N_{Re} = \frac{4 D_p v' \rho}{6(1 - \epsilon) \mu}$$

Pressure drop in packed beds

$$\Delta P = \frac{32 \nu \Delta L}{(4 R_H)^2} = \frac{150 \mu v' \Delta L (1 - \epsilon)^2}{\epsilon^3 D_p}$$

$Re < 10$  (laminar)

$$\Delta P = \frac{1.75 \rho v'^2 \Delta L (1 - \epsilon)}{D_p \epsilon^3}$$

$Re \geq 1000$

All purpose

$$\frac{\Delta P \rho}{G'^2} \frac{D_p}{L} \frac{\epsilon^3}{(1 - \epsilon)} = \frac{150}{N_{Re,p}} + 1.75$$

Non-spherical particles

mixture:  
of  
different  
particles

$$\frac{x_1 / \phi_s D_{p1} + x_2 / \phi_s D_{p2} + \dots}{\text{fraction}} = D_{pm}$$

Sphericity  $\phi_s = \frac{\text{surf area of sphere with } D_p}{\text{surf area of particle}}$

$$a_v = 6 / \phi_s D_p \quad a = 6 / \phi_s D_p (1 - \epsilon)$$

Pumps, Fans, Blowers

$$\Delta P: (\text{pump}) \quad \frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + \eta h_p = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

↑ losses here  
elevation change usually small.

Use of pump curves

$$C_Q = \text{discharge coeff} = Q / n D^3$$

$$C_H = \text{head coeff} = \Delta H / (D^2 n^2 / g)$$

$$C_P = \text{power coeff} = P / \rho D^5 n^3$$

$$- w_s \text{ pump} = \left. \begin{array}{l} H g \propto \\ H g / g_c \end{array} \right\} \text{depending on work units.}$$