Basic Concepts (Engineering & science)

$g_c = ma/F$

SI : $1\ kg \cdot m / (N \cdot s^2)$
ENG : $32.174\ lb_m\, ft / (lb_f\, s^2)$

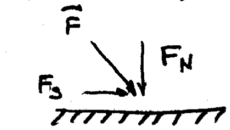
Work = $W = F \cdot d$
Power = $P = dW/dt$

$J \equiv N/m$ $W = J/s$
hp = 550 $ft \cdot lb_f / s$
btu = 778.16 $ft \cdot lb_f$

$\hat{v}$ = 22.4 l/g·mole @ 0°C, 1 atm (101.3 kPa)
= 359 ft^3/lb mole @ 32°F, 14.69 lb_f/in^2

Circle : $\pi D^2/4$ (area) πD (circum)
Sphere : πD^2 (surf area) $\pi D^3/6$ (vol)

Fluid Statics



N = normal ⊥   S = shear ∥

$p \equiv F_N/A$
$\tau \equiv F_s/A$

IGL . $PV = nRT$ $V/n = v$

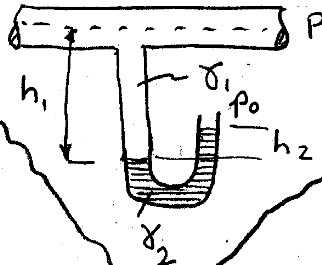
Hydrostatic Eq. $\Delta p = \rho g \Delta h$ $p = f(z)$ only

(spec. gravity) $\gamma = \frac{\rho g}{g_c}$ ∴ $\Delta p = \gamma \Delta h$

(density) ρ

Water @ 4°C ρ = 62.39 lb/ft^3 (1000 kg/m^3)
γ = 62.39 lb_f/ft^3 (9860 N/m^3)

Manometry

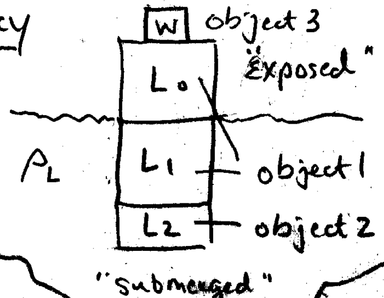


$p_0 = p_1 + \gamma_1 h_1 - \gamma_2 h_2$

$P_{abs} = P_{gage} + P_{ref}$ ← usually 1 atm

$h = \frac{\Delta P}{\gamma} = \frac{\Delta P g_c}{\rho g}$ (head)

Bouyancy

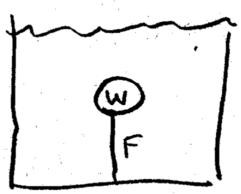


$\Delta V = \Delta h A$ (vol disp)

$W + \gamma_2 L_2 A + A(L_1 + L_0)\gamma_1 = (L_1 + L_2) A \gamma_L$

Wt obj = W_{H_2O} displaced

Tethered objects and Submerged



$W_{H_2O\,disp} = W_{obj} + F_{chain}$
$W_{app} = W_{obj} - W_{H_2O}$

apparent wt

Statics con't — kinematic viscosity $\nu = \mu / \rho$

Properties of matter: $\mu_{gas} \uparrow$ as $T \uparrow$

$\mu_{liq} \downarrow$ as $T \uparrow$

Dimensional Analysis

"Buckingham Pi Thrm"

n = # of quantities of interest

m = # of indep. variables (T, P etc)

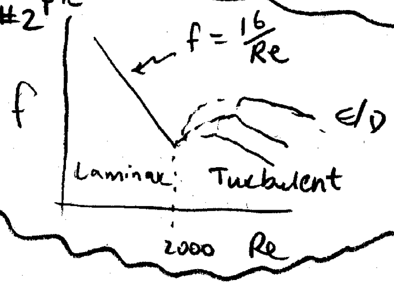
$$\#\ \pi\text{'s} = n - m \qquad (m = 3 \text{ no } T \text{ dependence}; \; m = 4 \text{ w/ } T \text{ dependence})$$

Example #1: Hagen Pouiselle Eq. (Laminar)

$$Q = (\pi/128) \frac{\Delta P}{L} \left(D^4/\mu \right) \qquad n = 4 \; (\Delta P/L, D, \mu, Q)$$

$\therefore n - m = 4 - 3 = 1$ Pi group.

Example #2:



$n = 6$ — $f, \Delta P/L, D, \rho, V^2, \epsilon$

$m = 3$ — $6 - 3 = 3$

$\pi_1 = f \quad \pi_2 = N_{Re} \quad \pi_3 = \epsilon/D$

Modeling & Similitude

Form dimensionless numbers from ratios of forces.

F_I = inertia $\rho L^2 V^2$

F_g = gravity $\rho g L^3$

F_V = viscous $\mu L V$

F_P = pressure $\Delta P (L^3)$

F_{ST} = surf tension σL

F_c = compressibility $k L^2$

$F_I / F_V = N_{Re} = DV\rho/\mu$

$N_{Fr} = V / \sqrt{Lg}$

$N_{Mach} = V / \sqrt{KP}$

$N_{Weber} = V / \sqrt{\sigma / \rho L}$

Phenomenological Models

Case I - $N_{Re} = DV\rho/\mu = LV\rho/\mu$ — "Enclosed Flow", pipes, conduits, flow notes, fans, pumps, turbines, "Fully immersed flow" Autos, submarines, buildings.

Case II — Gravity waves $N_{Fr} = V/\sqrt{Lg}$ or V^2/Lg — ships hulls or open channel flow

Case III — $N_{We} = V / \sqrt{\sigma/\rho L}$ — surface tension, capillary tubes, wicking, heat pipes

Case IV — Drag forces $C_D = F_D / (\rho V^2 / 2A)$ includ. parachutes.

Common dimensions

$Q = L^3/\theta^{-1}$ $\mu = ML^{-1}\theta^{-1}$

$\Delta p = ML^{-1}\theta^{-2}$ $F = ML\theta^{-2}$

$n = \theta^{-1}$ $\sigma = M\theta^{-2}$

$\gamma = ML^{-3}$ $\nu = L^2\theta^{-1}$

Reynolds Transport Thm

$$\left.\frac{dB}{dt}\right)_{sys} = \frac{d}{dt}\int_{CV} \beta\rho\, dv + \int_{CS}\beta\,\rho\vec{V}\cdot d\vec{A}$$

Mass Continuity $B = m$ $\beta = B/m = 1$

Yields; $\dot m = \int d\dot m = \int_A \rho u\, dA$

$\bar V = \dot m/\rho A$ $Q = \bar V A = \dot m \rho$

$\therefore\ \dot m = \rho \bar V A = \text{const}$

also $G = \bar V \rho = \dfrac{\dot m}{A} = \text{mass flux}$

$\Leftrightarrow$ At SS, $\dot m_{in} = \dot m_{out}$

Energy Eq. $B = E$ $\beta = E/m = e$

Yields: $\left.\frac{dE}{dt}\right)_{sys} = \frac{d}{dt}\int_{CV} e\rho\, dv + \int_{CS} e\rho\vec V\cdot d\vec A$ w/ $e = e_u + e_{KE} + e_{pe}$

and note: $\dot W = \vec F \cdot V$

(1) $\therefore\ \dot Q - \dot W_s = \dot m\left[\alpha\frac{\hat V^2}{2} + gz + h\right]_{out} - \dot m\left[\alpha\frac{\hat V^2}{2} + gz + h\right]_{in}$ (@ SS)

$\Uparrow$ work units

where $\alpha \cong 1.05$ turb (cir pipe)
$\alpha = 2.00$ lam (")

Divide (1) by $\dot m g$

(2) $$\frac{P_1}{\rho} + \alpha_1\frac{\hat V_1^2}{2} + z_1 g + h_p = \frac{P_2}{\rho} + \alpha_2\frac{\hat V_2^2}{2} + z_2 g + h_t + h_L$$

$\Uparrow$ KE units pump turbine

where $h_p = \dfrac{\dot W}{\dot m g} = \dfrac{\dot W}{Q\rho g} = \dfrac{\dot W}{Q\gamma}$

(3)
or also $$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p(\eta) = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

head units

where $W_p = W_s(\text{shaft})/\eta$

Conversion Factors & Phys Properties:

Power = $h_{pump}(\dot m)$ $W = J/s$ $J/s(\rho) = J/kg$ (sp energy)

7.48 lb = 1 gallon 1 atm = 33.90 ft H_2O Hp = 550 ft·lbf/s

$J = N\cdot m = kg\, m^2/s^2$ Btu = 778.16 ft·lbf $h = u + pv$

$6.895\times10^{+3}\ N/m^2 = PSI$ 1 ft·lbf = 1.356 J Btu = 1055.06 J

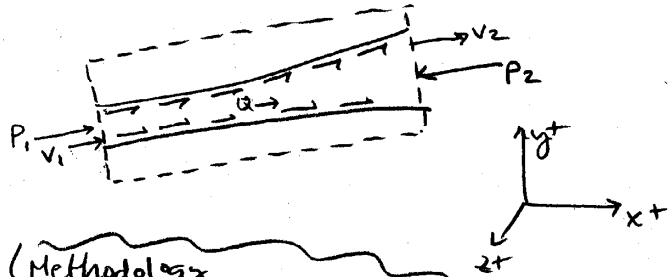
Hp = 0.7457 kW 1 ft·lbf/lbm = 2.989 J/kg

Momentum Eq $B = m\vec{v} \quad \beta = \vec{v}$

yields: $\left.\dfrac{d(m\vec{v})}{dt}\right)_{sys} = \Sigma \vec{F} = \dfrac{d}{dt}\int_{CV} \vec{v}\rho\, dV + \int_{CS} \vec{v}\rho\vec{v}\cdot d\vec{A}$

where $\Sigma F = F_N + F_B + F_S + F_e = \Sigma\, \overrightarrow{mom}_{out} - \Sigma\, \overrightarrow{mom}_{in}$

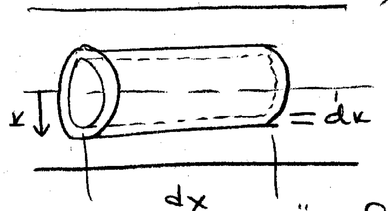
$\Sigma\, m\vec{v}_{out} - \Sigma\, m\vec{v}_{in}$



F_N = normal = $P \cdot A$ pressure
F_S = shear = τA shear
F_b = body = mg grav (wt)
F_e = external (all other)

Methodology
1. Always draw CV. Stay outside to avoid shear.
2. Specify coord system
3. Use gage pressures
4. Pick ext force direction arbitrarily (+ = right − = wrong assumption)
5. Remember trig & keep all angles $\leq 90°$

Circular pipe (laminar)



"neg force"

$F_p = P_x A_x + P_{x+dx} A_{x+dx}$ (trig)
$F_e = 0$
$F_S = \tau_{rx}(2\pi r)dx - \tau_{(r+dr)x}(2\pi r)dx$

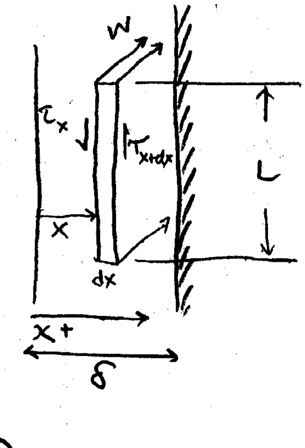
cir. pipe results

$\tau_{Rx} = \dfrac{\Delta P}{L}\dfrac{r}{2} \qquad \tau_{rx} = -\mu \dfrac{\partial v_x}{\partial r}$

$\therefore\ \bar{V}_x = (P_0 - P_L)D^2 / 32\mu L$

$\therefore\ V_{max} = (P_0 - P_L)R^2 / 4\mu L$

Falling Film (laminar)
[fully developed]



$dx\,W$

$F_p = P_0 A_0 - P_L A_L$

$F_S = \tau_x LW - \lambda_{x+dx} LW - \dfrac{\partial \tau}{\partial x} dx\, LW$

$F_B = (\delta\, x L W)\rho g$

$F_E = 0$

Falling Film Results

$\Gamma = m/t\cdot l = \dot{m}/l$

$N_{Re} = \dfrac{4\Gamma}{\mu} = 4\rho\delta V_z/\mu$

$\hat{V}_z = \rho g \delta^2 / 3\mu$
$\hat{V}_z = \tfrac{2}{3} V_{z\,max}$