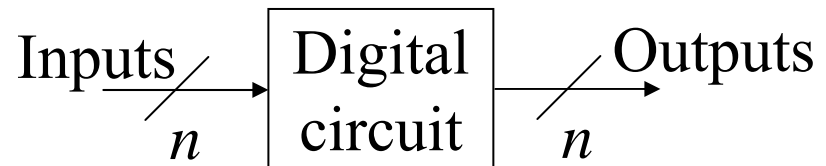


Binary Data Transmission

- Data transmission for n -bit data words

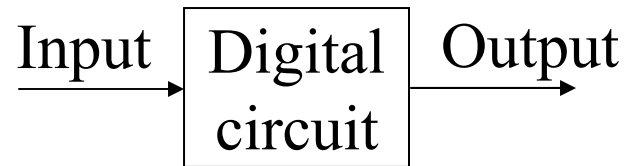
- › Parallel

- all bits at once
- 1 time step to get all data



- › Serial

- one bit at a time
- n time steps to get all data

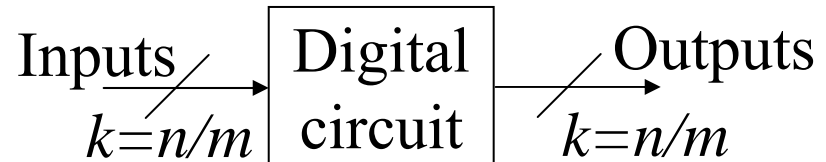


- › Serial-parallel

- both serial & parallel components
- m time steps to get all n bits, k bits at a time

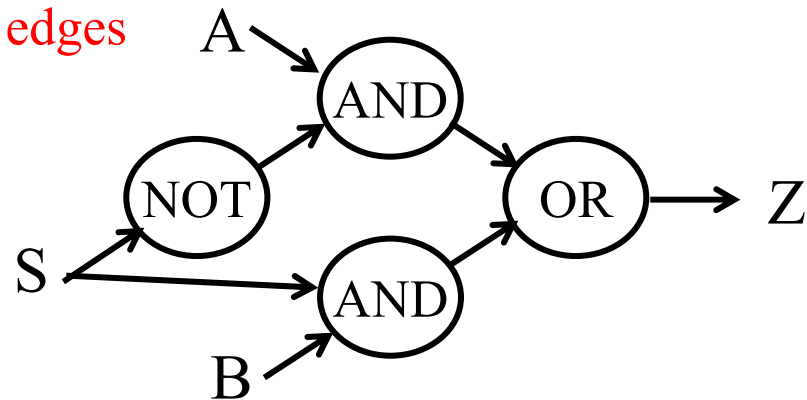
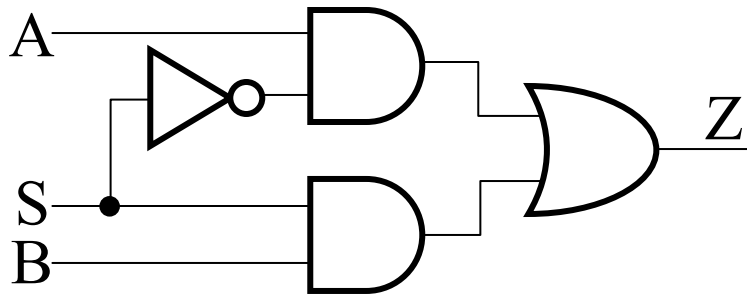
- › Trade-off:

- # inputs/outputs (I/O)
- speed of data transmission
- Combinational logic has parallel input data and output data



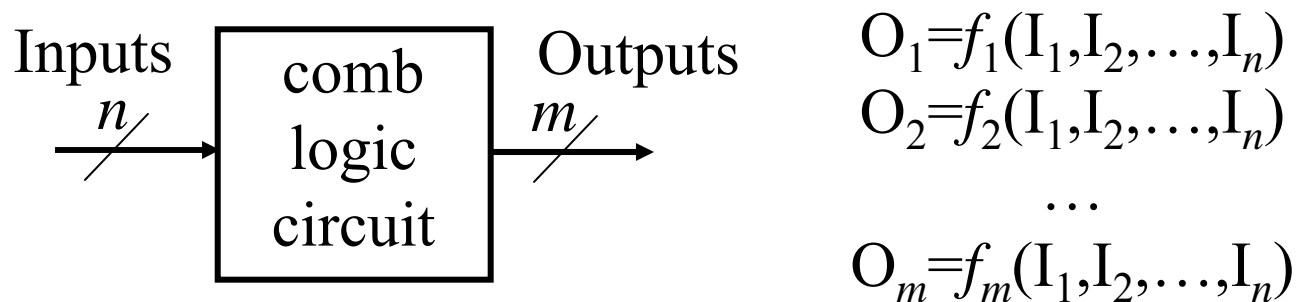
What is Combinational Logic?

- A collection of logic gates in which there are **NO** feedback loops
 - › No feedback loops means there is no path in the circuit on which you will pass through a given gate more than once
 - › Also defined as a circuit that can be represented by a directed acyclic graph (no cycles in the graph)
 - Gates represented by **vertices** (aka *nodes*)
 - Connections represented by **edges**



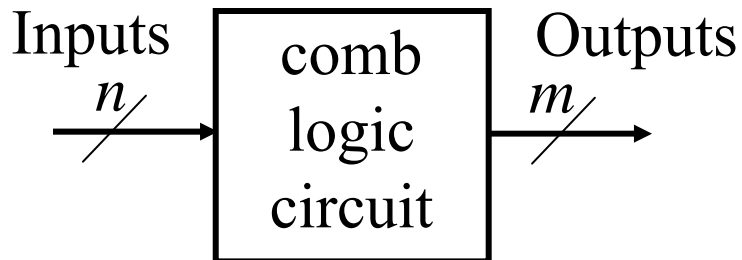
Boolean (Logic) Equations

- Any n -input, m -output combinational logic circuit can be completely described by a set of m logic equations
 - › One logic equation for each output
 - › Gives the output responses to all 2^n possible combinations of input values



Truth Tables

- Any n -input, m -output combinational logic circuit can be completely described by a truth table
 - › Gives the output responses to all 2^n possible combinations of input values
 - › Therefore, truth tables and logic equations contain the same information
- Two logic equations (or two combinational logic circuits) are *equivalent* if they produce the same truth tables



Inputs	Outputs
00...00	$V_1 \dots V_m$
00...01	$V_1 \dots V_m$
00...10	$V_1 \dots V_m$
...	...
11...11	$V_1 \dots V_m$

Representations of Logic Functions

- Truth Table
- Boolean (or logic) equations
- Sum-of-Products (SOP)

- › $Z=AB+A'C$
- › AND is product
- › OR is sum

- SOP canonical form

$$Z=A'B'C+A'BC+ABC'+\overbrace{ABC}$$

- › All literals are present in all product terms

- Minterm (a 1 in a TT row)

- › $Z=\Sigma_{A,B,C}(1,3,6,7)$

Literal – a single variable or the complement of a variable

Product term – a single literal or a logic product of multiple literals

A	B	C	Z	Row value	Minterm
0	0	0	0	0	$A'B'C'$
0	0	1	1	1	$A'B'C$
0	1	0	0	2	$A'BC'$
0	1	1	1	3	$A'BC$
1	0	0	0	4	$AB'C'$
1	0	1	0	5	$AB'C$
1	1	0	1	6	ABC'
1	1	1	1	7	ABC

Other Representations

- Product-of-Sums (POS)

- › $Z=(A+C)\cdot(A'+B)$

Sum term – a single literal or a logic sum of multiple literals

- POS canonical form

- › $Z=(A+B+C)\cdot(A+B'+C)$
 $\cdot(A'+B+C)\cdot(A'+B+C')$

- › All literals are present in all sum terms

- Maxterm (a 0 in a TT row)

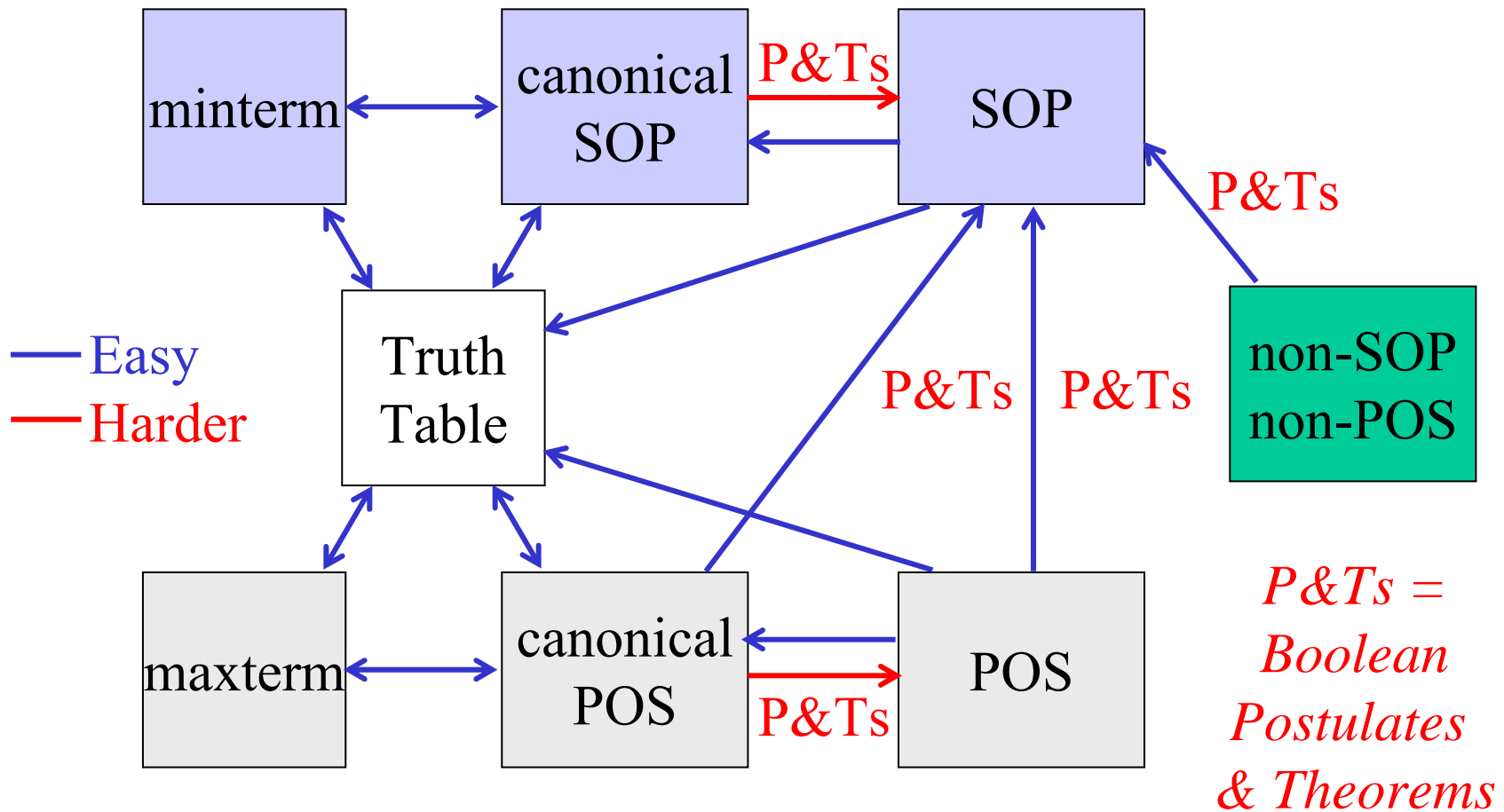
- › $Z=\Pi_{A,B,C}(0,2,4,5)$

- Note this is all TT rows not in minterm expression for this example

- POS representations are less often used than SOP

A	B	C	Z	Row value	Maxterm
0	0	0	0	0	$A+B+C$
0	0	1	1	1	$A+B+C'$
0	1	0	0	2	$A+B'+C$
0	1	1	1	3	$A+B'+C'$
1	0	0	0	4	$A'+B+C$
1	0	1	0	5	$A'+B+C'$
1	1	0	1	6	$A'+B'+C$
1	1	1	1	7	$A'+B'+C'$

Conversion Between Representations



Conversion Between Representations

- Minterm to truth table
 - › convert decimal minterm to binary
$$Z = \Sigma_{A,B,C}(1,3,6,7)$$
$$= \Sigma_{A,B,C}(001,011,110,111)$$
 - › place 1s in truth table entry for each minterm
 - Pay attention to input ordering
 - › place 0s in all other entries
- Maxterm to truth table
 - › convert decimal maxterm to binary
$$Z = \Pi_{A,B,C}(0,2,4,5)$$
$$= \Pi_{A,B,C}(000,010,100,101)$$
 - › place 0s in truth table entry for each maxterm
 - Pay attention to input ordering
 - › place 1s in all other entries

Conversion Between Representations

- Minterm to canonical SOP

- › convert decimal minterm to binary

$$\begin{aligned} Z &= \Sigma_{A,B,C}(1,3,6,7) \\ &= \Sigma_{A,B,C}(001,011,110,111) \end{aligned}$$

- › replace 1s and 0s with variable and complement of variable, respectively

$$= \Sigma_{A,B,C}(A'B'C, A'BC, ABC', ABC)$$

- Be sure to maintain input ordering

- › then sum

$$= A'B'C + A'BC + ABC' + ABC$$

- Canonical SOP to minterm

- › just reverse the procedure above

Conversion Between Representations

- Maxterm to canonical POS

- › convert decimal maxterm to binary

$$\begin{aligned} Z &= \Pi_{A,B,C}(0,2,4,5) \\ &= \Pi_{A,B,C}(000,010,100,101) \end{aligned}$$

- › replace 0s and 1s with variable and complement of variable, respectively, and sum

$$= \Pi_{A,B,C}(A+B+C, A+B'+C, A'+B+C, A'+B+C')$$

- Be sure to maintain input ordering

- › then take the product of the individual sum terms

$$= (A+B+C) \cdot (A+B'+C) \cdot (A'+B+C) \cdot (A'+B+C')$$

Note that these last 2 steps are the dual of those for minterm to canonical SOP

- Canonical POS to maxterm

- › just reverse the procedure above

Conversion Between Representations

- POS to SOP

- › multiply like in regular algebra then apply P&Ts

$$Z = (A+C) \cdot (A'+B)$$

$$= A \cdot (A'+B) + C \cdot (A'+B) \quad \text{using P5b}$$

$$= AA' + AB + CA' + CB \quad \text{using P5b (SOP but not minimal)}$$

$$= 0 + AB + A'C + CB \quad \text{using P6b}$$

$$= AB + A'C + CB \quad \text{using P2a}$$

$$= AB + A'C \quad \text{using T9a}$$

- Canonical POS to SOP

- › use same procedure

Conversion Between Representations

- SOP to canonical SOP
 - › Replace all missing variables in each product term with $X+X'$, where X is the missing variable

- Recall:

- $X+X' = 1$, *and*

- $Y \cdot 1 = Y$, so we don't change the product term

$$Z = AB + A'C$$

$$= A \cdot B \cdot 1 + A' \cdot 1 \cdot C$$

using P2b

$$= A \cdot B \cdot (C + C') + A' \cdot (B + B') \cdot C$$

using P6a

- › then multiply

$$= ABC + ABC' + A'BC + A'B'C$$

using P5b

$$= A'B'C + A'BC + ABC' + ABC$$

using P3a

Conversion Between Representations

- Canonical SOP to minimal SOP

› apply P&Ts

$$Z = A'B'C + A'BC + ABC' + ABC$$

$$= A' \cdot (B'C + BC) + A \cdot (BC' + BC) \quad \text{using P5b}$$

$$= A' \cdot (C) + A \cdot (BC' + BC) \quad \text{using T6a}$$

$$= A' \cdot (C) + A \cdot (B) \quad \text{using T6a}$$

$$= A'C + AB \quad \text{no change, just removed () and } \cdot$$

$$= AB + A'C \quad \text{using P3a}$$

- Same for SOP to minimal SOP
- ***Problem:*** How to know when it's minimal?

Conversion Between Representations

- Non-SOP/non-POS to SOP

- › apply P&Ts

$$Z = (((A'B)'C)' + D')' = \overline{\overline{\overline{\overline{\overline{A \cdot B}} \cdot C}} + \overline{D}}$$

$$= \overline{\overline{\overline{\overline{\overline{A \cdot B}} \cdot C}} \cdot \overline{D}}$$

using DeMorgan T8a

$$= \overline{\overline{\overline{A \cdot B}} \cdot C} \cdot D$$

using T3

$$= \overline{\overline{A + B}} \cdot C \cdot D$$

using DeMorgan T8b

$$= \overline{(A + \overline{B})} \cdot C \cdot D$$

using T3

Note: this is a POS

$$= (A + B') \cdot C \cdot D$$

no change, just removed ()

$$= ACD + B'CD \quad (\text{SOP}) \quad \text{using P5b}$$

Conversion Between Representations

- SOP to truth table:
 - › Place a logic 1 in each truth table output entry whose input value satisfies a given product term = 1
 - A k -variable product term will produce 2^{n-k} 1s in the truth table where n is the total number of input variables
 - › Repeat for all product terms
- POS to truth table:
 - › Place a logic 0 in each truth table output entry whose input value satisfies a given sum term = 0
 - A k -variable sum term will produce 2^{n-k} 0s in the truth table where n is the total number of input variables
 - › Repeat for all sum terms

Using Truth Tables to Prove Theorems

- Consensus Theorem

- › T9a: $X \cdot Y + X' \cdot Z + Y \cdot Z = X \cdot Y + X' \cdot Z$

- › T9b: $(X+Y) \cdot (X'+Z) \cdot (Y+Z) = (X+Y) \cdot (X'+Z)$

T9a:

X	Y	Z	Output
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

T9b:

X	Y	Z	Output
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1