

Bending Member Design

- Check:
 - Flexure
 - Shear
 - Deflection
 - Bearing

Bending Member Design

- Check:
 - Flexure

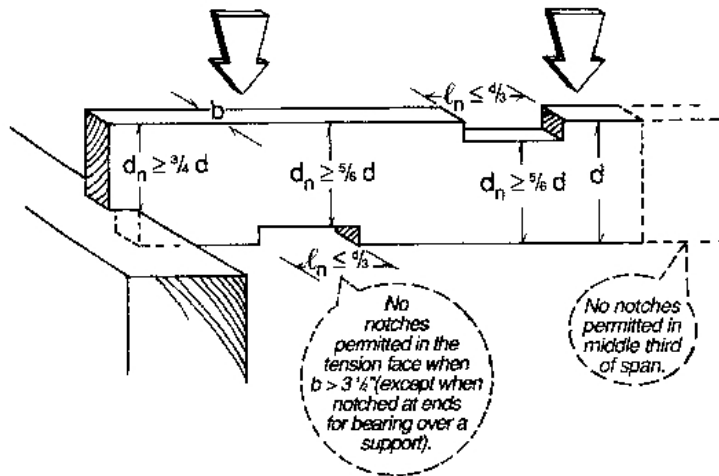
Bending Member Design

- Avoid notching whenever possible - especially on tension faces of beams
- NDS states “bending members shall not be notched except as permitted by 4.4.3, 5.4.4, 7.4.4, and 8.4.1.”
- Use gradually tapered notches
- Square cut notches result in greater problems from stress concentrations

Bending Member Design

- See Figure 4A for SAWN LUMBER notch recommendations
 - No notches in middle third of span
 - Interior notches located in outer thirds of span are permitted; depth can't exceed $1/6$ beam depth
 - Interior notches are not permitted on tension side of nominal 4 in. thick members
 - Notches at ends of member over a support can be larger – not greater than $1/4$ beam depth

Figure 4A Notch Limitations for Sawn Lumber Beams



Bending Member Design

- Notching requirements for GLUED-LAMINATED TIMBERS:
 - Don't notch tension side of glulam beams
 - Notches are permitted on tension side at ends of beams over a support
 - Notch depth can't exceed lesser of 1/10 depth or 3 in.
 - Notches are permitted on compression side of beam only at the ends, and notch depth can't exceed 2/5 depth of beam

Bending Member Design

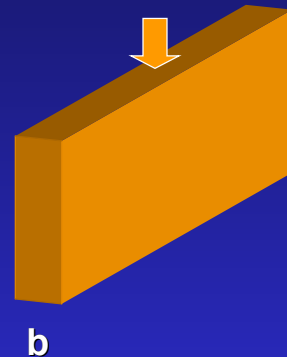
- Notching requirements for Wood I-JOISTS:
 - In general, don't notch I-joists.
 - Consult with manufacturer before notching
- Notching for STRUCTURAL COMPOSITE LUMBER
 - Use same recommendations as glulam

- Flexural design equations (NDS 3.3.2):

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$

for rectangular beams :

$$f_b = \frac{M}{S} = \frac{6M}{bd^2}$$



$$f_b \leq F'_b = F_b C_D C_M C_t C_L C_F C_V C_{fu} C_i C_r C_c C_f$$

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Form Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor
$F'_b = F_b$	x	C_D	C_M	C_t	C_L	C_F	C_{fu}	C_i	C_r	C_f	-	-	-
$F'_t = F_t$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	-	-	-	-
$F'_v = F_v$	x	C_D	C_M	C_t	-	-	-	C_i	-	-	-	-	-
$F'_{c\perp} = F_{c\perp}$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	-	C_b
$F'_c = F_c$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	-	C_P	-	-
$E' = E$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	C_T	-

Example Test Beam

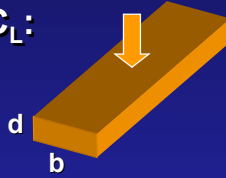
Sample beam from bending test. The tension face of the beam is shown here at midspan.



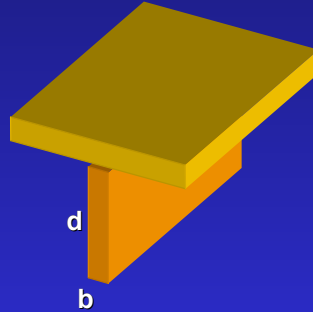
Bending failure on tension face of beam

- Beam stability factor, C_L :

- when $d \leq b$, $C_L = 1$



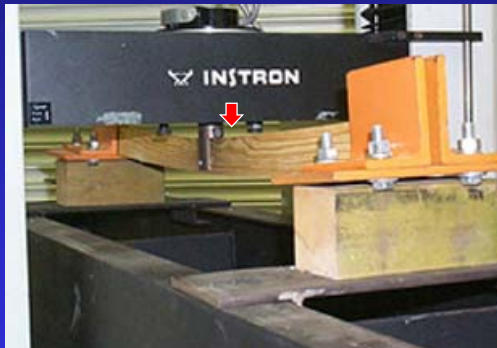
- when compression edge is fully supported to prevent lateral displacement, $C_L = 1$



- When sawn lumber bending members are designed in accordance with NDS 4.4.1, $C_L = 1$

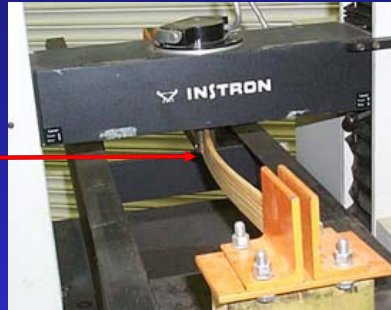
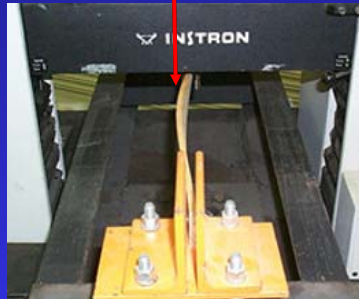
Bending Member Stability

- Test beam after load was applied



Bending Member Stability

- Lateral torsional buckling occurs on compression edge of beam



- Beam stability factor, C_L :
 - when $d > b$ and no additional lateral support provided beyond that at the bearings, C_L is calculated
 - find unsupported length, l_u
 - then find effective length, l_e , from Table 3.3.3

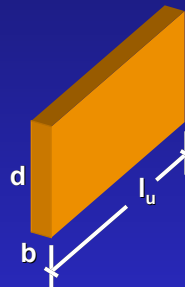


Table 3.3.3 Effective Length, ℓ_e , for Bending Members

Cantilever ¹	when $\ell_u/d < 7$	when $\ell_u/d \geq 7$
	Uniformly distributed load	$\ell_e = 1.33 \ell_u$
Concentrated load at unsupported end	$\ell_e = 1.87 \ell_u$	$\ell_e = 1.44 \ell_u + 3d$
Single Span Beam ^{1,2}	when $\ell_u/d < 7$	when $\ell_u/d \geq 7$
	Uniformly distributed load	$\ell_e = 2.06 \ell_u$
Concentrated load at center with no intermediate lateral support	$\ell_e = 1.80 \ell_u$	$\ell_e = 1.37 \ell_u + 3d$
Concentrated load at center with lateral support at center	$\ell_e = 1.11 \ell_u$	
Two equal concentrated loads at 1/3 points with lateral support at 1/3 points	$\ell_e = 1.68 \ell_u$	
Three equal concentrated loads at 1/4 points with lateral support at 1/4 points	$\ell_e = 1.54 \ell_u$	
Four equal concentrated loads at 1/5 points with lateral support at 1/5 points	$\ell_e = 1.68 \ell_u$	
Five equal concentrated loads at 1/6 points with lateral support at 1/6 points	$\ell_e = 1.73 \ell_u$	
Six equal concentrated loads at 1/7 points with lateral support at 1/7 points	$\ell_e = 1.78 \ell_u$	
Seven or more equal concentrated loads, evenly spaced, with lateral support at points of load application	$\ell_e = 1.84 \ell_u$	
Equal end moments	$\ell_e = 1.84 \ell_u$	

1. For single span or cantilever bending members with loading conditions not specified in Table 3.3.3:

$\ell_e = 2.06 \ell_u$ when $\ell_u/d < 7$

$\ell_e = 1.63 \ell_u + 3d$ when $7 \leq \ell_u/d \leq 14.3$

$\ell_e = 1.84 \ell_u$ when $\ell_u/d > 14.3$

2. Multiple span applications shall be based on table values or engineering analysis.

- **Beam stability factor, C_L :**

- find I_e

- find slenderness ratio, R_B

$$R_B = \sqrt{\frac{I_e d}{b^2}} \leq 50$$

- calculate stability factor, C_L :

$$C_L = \frac{1 + \left(\frac{F_{bE}}{F_b} \right)^*}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{F_{bE}}{F_b} \right)^*}{1.9} \right]^2 - \left(\frac{F_{bE}}{F_b} \right)^*} - 0.95$$

- Beam stability factor, C_L :

$$C_L = \frac{1 + \left(\frac{F_{bE}}{F_b^*} \right)}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{F_{bE}}{F_b^*} \right)}{1.9} \right]^2 - \frac{\left(\frac{F_{bE}}{F_b^*} \right)}{0.95}}$$

- F_b^* is tabulated bending design value multiplied by all adjustments except C_{fu} , C_V , and C_L

$$F_b^* = F_b C_D C_M C_t C_F C_i C_r C_C C_f$$

- Beam stability factor, C_L :

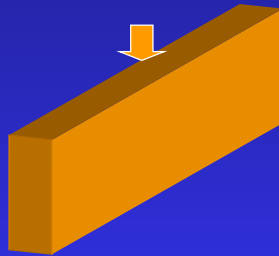
$$C_L = \frac{1 + \left(\frac{F_{bE}}{F_b^*} \right)}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{F_{bE}}{F_b^*} \right)}{1.9} \right]^2 - \frac{\left(\frac{F_{bE}}{F_b^*} \right)}{0.95}}$$

$$F_{bE} = \frac{K_{bE} E'}{R_B^2} \quad E' = E C_M C_t C_i C_T$$

- $K_{bE} = 0.439$ for visually graded lumber
- $K_{bE} = 0.561$ for MEL
- $K_{bE} = 0.610$ for MSR, glulam

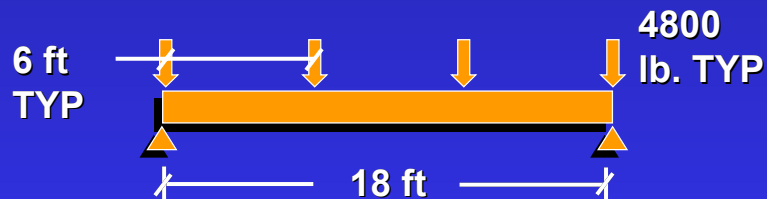
- Flexural design equations:

$$f_b = \frac{M}{S} \leq F'_b = F_b C_D C_M C_t C_L C_F C_V C_{fu} C_i C_r C_c C_f$$



Beam Design Example

- Given:
 - Beam supporting roof trusses; 18 ft Span
 - 4 point loads, 6 ft OC; 3200 lb. Snow + 1600 lb. Dead
 - Lateral restraint at supports and at each load application point



- Find:

- Southern pine glulam beam size and combination that will carry loads

- Assume:

- Dry conditions
- Normal temperatures
- Maximum live load deflection = $L / 360$



Solution:

- Max. Bending Moment = 28,800 lb.ft
- Max. Shear Force = 4,800 lb.
- Max. Reaction Force = 9,600 lb.

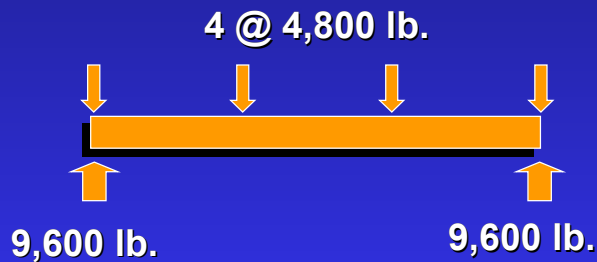
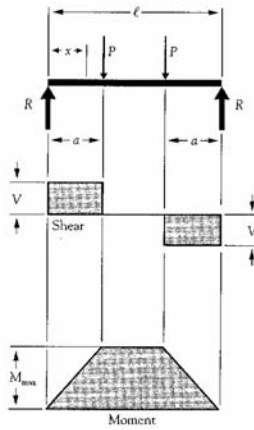


Figure 10.9 Simple Beam – Two Equal Concentrated Loads Symmetrically Placed

$$R = V \dots \dots \dots = P$$

$$M_{\max} \text{ (between loads)} \dots \dots \dots = Pa$$

$$M_x \text{ (when } x < a) \dots \dots \dots = Px$$

$$\Delta_{\max} \text{ (at center)} \dots \dots \dots = \frac{Pa}{24EI} (3\ell^2 - 4a^2)$$

$$\Delta_x \text{ (when } x < a) \dots \dots \dots = \frac{Px}{6EI} (3a - 3a^2 - x^2)$$

$$\Delta_x \text{ (when } x > a \text{ and } < (\ell - a)) \dots \dots \dots = \frac{Pa}{6EI} (3\ell x - 3x^2 - a^2)$$

- Find trial size first by assuming that we will use a 24F-1.8E stress class; initial guess for $F_b' = 2400$ psi

$$f_b = \frac{M}{S} \leq F_b' \quad \text{or} \quad \frac{M}{F_b'} \leq S$$

$$S \geq \frac{(28,800 \text{ lb.ft}) \left(12 \frac{\text{in.}}{\text{ft}} \right)}{2400 \text{ psi}} = 144 \text{ in}^3$$

Trial Size:

- Try a glulam beam with a 3.0 in. width and find size with $S \geq 144 \text{ in}^3$
- From NDS Supplement Table 1D, we can try a 3.0 x 17.875 in. beam ($S = 159.8 \text{ in}^3$)
- This is based on the assumption that we have a 24F-1.8E glulam stress class

Table 5A Design Values for Structural Glued Laminated Softwood Timber
(Members stressed primarily in bending) (Tabulated design values are for normal load duration and dry service conditions. See NDS 5.3 for a comprehensive description of design value adjustment factors.)

Use with Table 5A Adjustment Factors

Stress Class	Bending About X-X Axis Loaded Perpendicular to Wide Faces of Laminations				Bending About Y-Y Axis Loaded Parallel to Wide Faces of Laminations				Axially Loaded			Fasteners			
	Extreme Fiber in Bending		Compression Perpendicular to Grain	Shear Parallel to Grain (Horizontal)	Modulus of Elasticity	Extreme Fiber in Bending	Compression Perpendicular to Grain	Shear Parallel to Grain (Horizontal)	Modulus of Elasticity	Tension Parallel to Grain	Compression Parallel to Grain	Modulus of Elasticity	Specific Gravity for Dowel-Type Fastener Design	Species Group for Split Ring and Shear Plate Connectors	
	Tension Zone Stressed in Tension (Positive Bending)	Compression Zone Stressed in Tension (Negative Bending)											Top or Bottom Face	Side Face	
	F_{bx}^* (psi)	$F_{bx}^{(1)}$ (psi)	F_{cx} (psi)	$F_{vx}^{(6)}$ (psi)	E_x (10^6 psi)	F_{by} (psi)	F_{cy} (psi)	$F_{vy}^{(6)}$ (psi)	E_y (10^6 psi)	F_t (psi)	F_c (psi)	E_{axial} (10^6 psi)	G		
11F-1.3E	1600	925	315	175	1.3	800	315	155	1.1	625	725	1.2	0.35	C	C
20F-1.5E	2000	1100	425	190 ⁽⁶⁾	1.5	800	315	155	1.2	725	925	1.3	0.42	C	C
24F-1.7E	2400	1450	500	190 ⁽⁶⁾	1.7	850	315	160	1.3	775	1000	1.4	0.42	C	C
24F-1.8E	2400	1450 ⁽⁶⁾	650	240 ⁽⁶⁾	1.8	1450	560	210 ⁽⁶⁾	1.6	1100	1600	1.7	0.50 ⁽⁶⁾	A	B
26F-1.9E ⁽⁷⁾	2600	1950	650	240 ⁽⁶⁾	1.9	1600	560	210 ⁽⁶⁾	1.6	1150	1600	1.7	0.50 ⁽⁶⁾	A	B
28F-2.1E SP ⁽⁷⁾	2800	2300	740	270	2.1 ⁽⁶⁾	1600	850	235	1.7	1250	1750	1.7	0.55	A	A
30F-2.1E SP ⁽⁷⁾⁽⁸⁾	3000	2400	740	270	2.1 ⁽⁶⁾	1750	850	235	1.7	1250	1750	1.7	0.55	A	A

- For balanced layouts, F_{bx}^* shall be equal to $F_{bx}^{(1)}$ for the stress class. Designer shall specify when balanced layout is required.
- Negative bending stress, F_{bx}^* , is permitted to be increased to 1850 psi for Douglas Fir and to 1950 psi for Southern Pine for specific combinations. Designer shall specify when these increased stresses are required.
- For structural glued laminated timber of Southern Pine, shear stress is permitted to be increased to: $F_{vx} = 270 \text{ psi}$, $F_{vy} = 235 \text{ psi}$.
- For non-prismatic members, notched members, members subject to impact or cyclic loading, or shear design of bending members at connections (NDS 3.4.3.3), the design value for shear shall be multiplied by a factor of 0.8. For the determination of radial tension design values (NDS 5.2.2), the design value for shear shall be multiplied by a factor of 0.7 for DF-L and SP or by 0.8 for all other species.
- Design values are for timbers with laminations made from a single piece of lumber across the width or multiple pieces that have been edge bonded. For timbers manufactured from multiple piece laminations (across width) that are not edge bonded, value shall be multiplied by 0.4 for members with 5, 7, or 9 laminations or by 0.5 for all other members. This reduction shall be cumulative with the adjustment in footnote (4).
- Certain Southern Pine combinations may contain coarse grain lumber with wane. If lumber with wane is used, the design value for shear parallel to grain, F_{vx} , shall be multiplied by 0.67 if wane is allowed on both sides. If wane is limited to one side, F_{vx} shall be multiplied by 0.83. This reduction shall be cumulative with the adjustment in footnote (4).
- 26F, 28F, and 30F beams are not produced by all manufacturers, therefore, availability may be limited. Contact supplier or manufacturer for details.
- 30F combinations are restricted to a maximum 6 in. nominal width.
- For 28F and 30F members with more than 15 laminations, $E_x = 2.0$ million psi.
- For structural glued laminated timber of Southern Pine, specific gravity for fastener design is permitted to be increased to 0.55.

Design values in this table represent design values for groups of similar glued laminated timber combinations. Higher design values for some properties may be obtained by specifying a particular combination listed in AITC 117-2001 Design or APA-EWS Y117. Design values are for members with 4 or more laminations. For 2 and 3 lamination members, see Table 5B. Some stress classes are not available in all species. Contact structural glued laminated timber manufacturer for availability.

Table 1D Section Properties of Southern Pine Glued Laminated Timber

Depth d (in.)	Area A (in. ²)	X-X Axis			Y-Y Axis	
		I _x (in. ⁴)	S _x (in. ³)	r _x (in.)	I _y (in. ⁴)	S _y (in. ³)
2-1/2 in. Width (r_y = 0.722 in.)						
5-1/2	13.75	34.66	12.60	1.588	7.161	5.729
6-7/8	17.19	67.70	19.69	1.985	8.952	7.161
8-1/4	20.63	117.0	28.36	2.382	10.74	8.594
9-5/8	24.06	185.3	38.60	2.778	12.53	10.03
11	27.50	277.3	50.42	3.175	14.32	11.46
12-3/8	30.94	394.8	63.81	3.572	16.11	12.89
13-3/4	34.38	541.6	78.78	3.969	17.90	14.32
15-1/8	37.81	720.9	95.32	4.366	19.69	15.76
16-1/2	41.25	935.9	113.4	4.763	21.48	17.19
17-7/8	44.69	1190	133.1	5.160	23.27	18.62
19-1/4	48.13	1486	154.4	5.557	25.07	20.05
20-5/8	51.56	1828	177.2	5.954	26.86	21.48
22	55.00	2218	201.7	6.351	28.65	22.92
23-3/8	58.44	2661	227.7	6.748	30.44	24.35
3 in. Width (r_y = 0.866 in.)						
5-1/2	16.50	41.59	15.13	1.588	12.38	8.250
6-7/8	20.63	81.24	23.63	1.985	15.47	10.31
8-1/4	24.75	140.4	34.03	2.382	18.56	12.38
9-5/8	28.88	222.9	46.32	2.778	21.66	14.44
11	33.00	332.8	60.50	3.175	24.75	16.50
12-3/8	37.13	473.8	76.57	3.572	27.84	18.56
13-3/4	41.25	649.9	94.53	3.969	30.94	20.63
15-1/8	45.38	865.0	114.4	4.366	34.03	22.69
16-1/2	49.50	1123	136.1	4.763	37.13	24.75
17-7/8	53.63	1428	159.8	5.160	40.22	26.81
19-1/4	57.75	1783	185.3	5.557	43.31	28.88
20-5/8	61.88	2193	212.7	5.954	46.41	30.94
22	66.00	2662	242.0	6.351	49.50	33.00
23-3/8	70.13	3193	273.2	6.748	52.59	35.06
3-1/8 in. Width (r_y = 0.902 in.)						
5-1/2	17.19	17.19	43.33	15.76	13.99	8.952
6-7/8	21.48	21.48	84.62	24.62	17.48	11.19
8-1/4	25.78	25.78	146.2	35.45	20.98	13.43
9-5/8	30.08	30.08	232.2	48.25	24.48	15.67
11	34.38	34.38	346.6	63.02	27.97	17.90
12-3/8	38.67	38.67	493.5	79.76	31.47	20.14
13-3/4	42.97	42.97	677.0	98.47	34.97	22.38
15-1/8	47.27	47.27	901.1	119.1	38.46	24.62
16-1/2	51.56	51.56	1170	141.8	41.96	26.86
17-7/8	55.86	55.86	1487	166.4	45.46	29.09
19-1/4	60.16	60.16	1858	193.0	48.96	31.33
20-5/8	64.45	64.45	2285	221.6	52.45	33.57
22	68.75	68.75	2773	252.1	55.95	35.81
23-3/8	73.05	73.05	3326	284.6	59.45	38.05

Design Data: 24F-1.8E southern pine 3.0 x 17.875

A = 53.63 in²

F_{bxx} = 2400 psi

S_{xx} = 159.8 in³

F_{c_{xx}} = 650 psi

I_{xx} = 1428 in⁴

F_{vxx} = 240 psi

E_{xx} = 1.8 x 10⁶ psi

Design Data: 24F-1.8E southern pine 3.0 x 17.875

$$C_D = 1.15 \text{ for snow load} \quad C_c = \text{NA}$$

$$C_M = 1.0 \quad C_f = \text{NA}$$

$$C_t = 1.0 \quad C_H = 1.0$$

$$C_L = ? \text{ (to be determined)} \quad C_b = \text{NA}$$

$$C_V = ? \text{ (to be determined)} \quad C_T = \text{NA}$$

$$C_{fu} = \text{NA}$$

$$C_i = \text{NA}$$

$$C_r = \text{NA}$$

• Volume Factor, C_V

$$C_V = K_L \left(\frac{21}{L} \right)^{\frac{1}{20}} \left(\frac{12}{d} \right)^{\frac{1}{20}} \left(\frac{5.125}{b} \right)^{\frac{1}{20}} \leq 1.0$$

$$C_V = 1.0 \left(\frac{21}{18} \right)^{\frac{1}{20}} \left(\frac{12}{17.875} \right)^{\frac{1}{20}} \left(\frac{5.125}{3.0} \right)^{\frac{1}{20}} = 1.01$$

use $C_V = 1.0$

- Beam Stability Factor, C_L

- Effective length for 2 equal concentrated loads @ 1/3 points

$$l_e = 1.68l_u = (1.68)(72 \text{ in.}) = 120.96 \text{ in.}$$

- Slenderness Ratio, R_B

$$R_B = \sqrt{\frac{l_e d}{b^2}} = \sqrt{\frac{(120.96 \text{ in.})(17.875 \text{ in.})}{(3.0 \text{ in.})^2}}$$

$$R_B = 15.5$$

- Beam Stability Factor, C_L

$$C_L = \frac{1 + \left(\frac{F_{bE}}{F_b^*}\right)}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{F_{bE}}{F_b^*}\right)}{1.9}\right]^2 - \frac{\left(\frac{F_{bE}}{F_b^*}\right)}{0.95}}$$

$$F_{bE} = \frac{K_{bE} E'}{R_B^2}$$

$$F_b^* = F_b C_D C_M C_t C_F C_i C_r C_c C_f$$

- Beam Stability Factor, C_L

$$E' = E C_M C_t C_i C_T = 1.8 \times 10^6 \text{ psi}$$

$$F_{bE} = \frac{K_{bE} E'}{R_B^2} = \frac{(0.610)(1.8 \times 10^6 \text{ psi})}{(15.5)^2} = 4,570 \text{ psi}$$

$$F_b^* = F_b C_D = (2400 \text{ psi})(1.15) = 2760 \text{ psi}$$

- Beam Stability Factor, C_L

$$C_L = \frac{1 + \left(\frac{4,570 \text{ psi}}{2,760 \text{ psi}} \right)}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{4,570 \text{ psi}}{2,760 \text{ psi}} \right)}{1.9} \right]^2 - \frac{\left(\frac{4,570 \text{ psi}}{2,760 \text{ psi}} \right)}{0.95}}$$

$$C_L = 0.94$$

- Check Bending Stress:

$$f_b = \frac{M}{S} \leq F_b'$$

$$f_b = \frac{(28,800 \text{ lb.ft}) \left(12 \frac{\text{in.}}{\text{ft}} \right)}{(159.8 \text{ in}^3)}$$

$$f_b = 2163 \text{ psi}$$

- Check Bending Stress:

$$F_b' = F_b C_D C_L$$

$$F_b' = (2400 \text{ psi})(1.15)(0.94)$$

$$F_b' = 2594 \text{ psi}$$

- Check Bending Stress:

$$f_b = 2263 \text{ psi} < F_b' = 2594 \text{ psi}$$

Conclusion: beam size is acceptable in bending stress