

Integrated Facilities Design using a Contour Distance Metric

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Why This Paper is Important

This paper demonstrates how block layout (department shapes and locations) can be concurrently optimized with the detailed layout steps of identifying the optimal number of input/output (I/O) sites and then locating them and implying the aisle (flowpath) structure. The distance metric used is rectilinear, but allows material travel only along the contours (boundaries) of the departments or work centers. This contour distance metric has been used occasionally in the literature and it better reflects the material transport paths in a majority of actual applications, however it has not yet been used for block layout optimization. Furthermore, most previous work in I/O location prespecifies the number of I/Os per department. This work ties these important aspects together into a single optimization methodology that is tractable even for large problems, and its effectiveness is demonstrated on a number of problems from the literature.

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Abstract

The unequal area facility design problem has been studied in the literature generally by considering block layout separate from detailed design. This paper shows a tractable method for concurrently optimizing department shape and location within the facility, and selection of the appropriate number of input/output (I/O) stations per department and then optimally locating them. The distance metric used is the contour distance that measures transport between I/O stations along the perimeters of departmental boundaries. This distance metric better reflects allowable material flow for many industrial applications and it directly implies the optimal aisle structure for the facility.

1. Introduction

By any monetary measure, facilities design is an important problem and one that has assumed even greater importance as industry strives to become more agile and responsive (Tompkins, 1997). For U.S. manufacturers, between 20% to 50% of total operating expenses are spent on material handling and an appropriate facilities design can reduce these costs by at least 10% to 30% (Meller and Gau, 1996). Tompkins (1997) wrote, "Since 1955, approximately 8 percent of the U.S. GNP has been spent annually on new facilities. In addition, existing facilities must be continually modified...These issues represent more than \$250 billion per year attributed to the design of facility systems, layouts, handling systems, and facilities locations...". Altering facility designs due to incorrect assumptions or inadequate formulations usually involves considerable cost, time and disruption of activities. On the other hand, good designs can reap economic and

operational benefits for a long time period. Therefore, computational time is not of primary importance; instead the critical aspects are designs that translate readily into physical reality to minimize material handling costs. The problem has been primarily studied in the literature by separating block layout from detailed design to allow for computationally tractable formulations.

The problem was originally formalized by Armour and Buffa (1963) as follows. There is a rectangular region, R , with fixed dimensions H and W , and a collection of n required departments, each of specified area a_j and dimensions (if rectangular) of h_j and w_j , whose total area, $\sum_j a_j = A = H \times W$. There is a material flow $F(j,k)$ associated with each pair of departments (j,k) which generally includes a traffic volume in addition to a unit cost to transport that volume. There may also be fixed costs between departments j and k . The objective is to partition R into n subregions representing each of the n departments, of appropriate area, in order to:

$$\min \quad Z = \sum_{j=1}^n \sum_{\substack{k=1 \\ j \neq k}}^n F(j,k) d(j,k, \Pi) \quad (1)$$

where $d(j,k,\Pi)$ is the distance (using a pre-specified metric) between the centroid of department j and the centroid of department k in the partition Π . The centroidal distance is easy to calculate for rectangular departments and it is intuitive in that the mass of material is considered to move from center to center of departments along the shortest rectilinear (Manhattan) or Euclidean distance. However, the centroid distance measure is not realistic for many applications, as it allows for material transport through departments and does not imply a reasonable aisle structure.

Chhajed et al. (1992) present a comprehensive discussion on flow networks and distinguish between free flow (shortest distance paths) and flow which follows departmental contours. The authors then present both exact and heuristic algorithms to optimize a free flow

path for a given block layout with pre-determined input/output (I/O) points. The idea of contour flow has been used in the literature related to discrete material handling systems such as Automated Guided Vehicles (AGVs), as found in the Segmented Flow Topology (SFT) work of Sinriech and Tanchoco (1995, 1997). In this work, I/Os and the resulting flow paths are chosen based on both distance-based transport costs and on fixed costs for building an I/O. In Sinriech and Tanchoco (1997), it is further allowed that material flow between two departments may be heuristically split among different paths if it is cost effective. A complete review of approaches to discrete flow systems can be found in Sinriech (1995).

A number of papers have dealt with optimally choosing the I/O locations and/or aisles for a given block layout (see for example, Montreuil and Ratliff (1988) who use free flow and Sinriech and Edouard (1996) who use the SFT flow along contours). Palliyil and Goetschalckx (1994) present exact and heuristic methods for optimally locating I/O stations and their flowpaths that can be applied to unidirectional or bidirectional flowpaths.

Integrating block layout with I/O placement has rarely been considered in the literature, probably because of the computational difficulties. Montreuil (Montreuil 1987, Montreuil and Ratliff 1988) optimally complete the block layout and place a pre-specified number of I/Os per department given a design skeleton. Luxhoj (1991) presents a two-part approach where a constructive heuristic creates a block layout that is then improved upon by pairwise interchange and material flows along a spine. Benson and Foote (1997) consider the placement of aisles and I/O points after the relative location of the departments and the general aisle structure have been selected. In the work of Nagi and others (Harhalakis et al. 1996, Kane and Nagi 1997), pre-defined departmental shapes are set on a grid covering the facility space, which is larger than the sum of the department areas. In Harhalakis et al. (1996) Dijkstra's shortest path algorithm is

used to calculate the rectilinear distance to and from pre-specified I/O points. In Kane and Nagi (1997) I/O points are placed during the optimization and a corner constraint is imposed to encourage paths that are straight. Both papers use a simulated annealing heuristic to alter departmental placement. Banerjee et al. (1997) uses a genetic algorithm to search over design skeletons¹ that are then optimized using a subordinate mathematical programming routine. The number of I/Os per department is pre-specified (either one or two per department) and then optimally located with the departmental placement. The shortest rectilinear distance between I/O points (free flow paths) is used in the objective function.

This paper seeks to integrate block layout (including both placement and shaping of the departments) with the choice of I/O stations (both number and location) and identification of aisle structure assuming uncapacitated, bidirectional aisles of negligible area. The contour distance metric enables this integrated design to translate well into physical reality while allowing computational feasibility. This still does not achieve the ideal situation where a true aisle structure will also be optimally designed concurrently. This simplification, instead, assumes that all departmental perimeters are potentially legitimate aisles.

2. Problem Formulation and Solution Methodology

The basic assumption is that the facility is comprised of rectangular departments of specified area within a rectangular bounding facility that is equal to, or larger than, the sum of the departmental areas. The formulation used is “flexbay”², or flexible bay, by Tate and Smith (1993, 1995, Tong 1991) (Figure 1). Flexbay makes cuts in a single direction to establish a set of bays that can vary in area. The bays are then subdivided into departments. The flexbay

¹ A design skeleton was originally defined by Montreuil and Ratliff (1989) and consists of the relative location of the departments within the facility.

² The term “flexbay” was first given in the review paper by Meller and Gau (1996).

encoding can strictly enforce both departmental areas and departmental shapes, through use of a maximum aspect ratio constraint³ or a minimum departmental side length constraint for a stated departmental area.

INSERT FIGURE 1 HERE

2.1 Optimization Methodology

To find the optimal or near-optimal block layout, a genetic algorithm (GA) is used with the flexbay formulation. The genetic algorithm works with a variable length encoding of the layout where there is a one to one correspondence between each encoding and each layout. The encoding is a permutation of departments that specifies their order within the layout, with a concatenated string indicating where the bay breaks within the permutation occur. For example, the flexbay layout of Figure 1 would be represented with the following encoding:

G A F H B E K C L M I J D H K I

where the last three characters indicate bay breaks after departments H, K and I.

Crossover is accomplished through a variant of uniform crossover (Goldberg 1989), where two parents create one offspring. The department sequence is subject to uniform crossover with repair to ensure feasible permutations. The bay structure is taken directly from one parent or another with equal probability. Mutation consists of permutation altering (50%), or adding (25%) or deleting (25%) a bay. The permutation mutation is inversion between two randomly selected departments. Crossover and mutation are performed independently of each other, with all solutions (parents and offspring) currently available equally likely to be mutated. Solutions are selected for crossover using a rank-based quadratic method and the worst solutions are deleted during each generation to maintain a constant population size. Tate and Smith (1995) includes the details of these.

³ Aspect ratio is defined as the quotient of the longer side divided by the shorter side.

2.2 I/O Location and Distance Measure in the Objective Function

The version of I/O location that is considered in this paper is where the allowable number of I/Os per department is not constrained. This might seem ungainly, but due to the contour measure the set of candidate I/O points for a department can be limited to the locations where that department intersects the corner of any of its adjacent departments, as will be proven below. Using the example of Figure 1, the candidate I/O points would be as shown on the top. To clarify the contour distance measure, if the I/Os used were as shown on the bottom, the material will traverse over the perimeters shown in the dashed line.

The set of departmental intersection points represents a dominant set of locations to consider for the departmental I/O points. Consider Figure 2 where x represents the location for the I/O point for department F which is currently not at an intersection point and a and b represent the locations where department F intersects with department E. It will be shown that x can be moved to one of the two intersection points a or b without increasing the overall travel distance. A related result for automated guided vehicle paths was proven in Tanchoco and Sinriech (1992) and De Guzman et al (1997). The discussion is broken into two cases.

INSERT FIGURE 2 HERE

Case 1: Assume that department F has no flow relationship with department E.

Because the flow paths go around the contour of departments all of the flow through x must pass through either a or b prior to reaching x . If more than 50% of the total flow through x passes through a then invoking the median condition (Francis and White 1974) from location theory it is clear that moving x to a would reduce the overall travel distance. Similarly, x should be placed at b if more than 50% of the total flow through

department F comes through b . If exactly 50% of the total flow through department F comes through both a and b then x can be placed at either a or b .

Case 2: Assume department F has a flow relationship with department E.

If department E's I/O location is not on the line segment connecting a and b then the result from case 1 holds. Otherwise, let y represent the location of department E's I/O point. The median condition (Francis and White 1974) can be applied to the location problem for x on the line segment joining a and b considering the flows through a , b and y . If the result is to move x to either a or b , these are intersection points. Or, if the result is to move x to y then it must be shown that the location for x and y together will be at an intersection point. Let ϕ represent the magnitude of the total flow through departments E and F that comes from relationships with departments other than E and F. Similar to case 1, all of the flows through x and y must flow through either location a or b . Utilizing the median condition, if the flow through a is $> 0.5\phi$ then place x and y together at a , if the flow through $a < 0.5\phi$ place x and y together at b , otherwise place x and y together at either a or b .

Thus the set of intersection points represent a dominant set for locating the I/O points of each department. The results for cases 1 and 2 readily extend to the case of multiple I/O points or the case where a department has several departments along one edge, for example, department B, J or D in Figure 2.

Furthermore, the set of intersection points can be upper bounded at $2n-2$ using the following argument.

1. Consider a flexible bay facility with n departments and b bays. Each department can cause at most 4 intersection points, located at each of its four corners, giving a total possible number of intersection points of $4n$.
2. However, many of the intersection points will be located concurrently. Wherever an interior bay division intersects the facility perimeter, two intersection points will be concurrent. This eliminates $2(b-1)$ intersection points.
3. For bays that contain more than one department, at each interior departmental boundary, two more intersection points will be concurrent. This eliminates $2(n-b)$ intersection points.
4. Finally, due to the objective function of minimizing flows, intersection points along the exterior bays (the first and last bays) will never be chosen. Assuming the worst case of 1 department per each bay along the exterior (i.e., the first and last bays), this eliminates 4 intersection points (the 4 corner points of the facility).
5. Combining the quantities yields $4n - 2(b-1) - 2(n-b) - 4$ which reduces to $2n-2$.

Therefore, the number of potential intersection sites depends only on n , and is $O(n)$.

When each department can have an unconstrained number of I/O stations the interdepartmental aisle travel distances can be found by formulating the problem as one of finding the shortest path on a network. All of the arc lengths in the resulting shortest path problem will be positive since they represent physical distances. The shortest path problem with positive arc lengths has been well studied in the network optimization literature and efficient algorithms exist for solving this problem (Ahuja et al. 1993). This makes it possible to quickly evaluate the actual aisle travel distance for each layout that is generated during the search process. The objective function of the integrated optimization is:

$$Z(\Pi) = \sum_{j=1}^n \sum_{\substack{k=1 \\ j \neq k}}^n F_{j,k} d_{j,k} + m^3 (Z_{feas} - Z_{all}) \quad (2)$$

where m is the number of departments in layout Π which violate the aspect ratio constraint, Z_{feas} is the objective function value for the best feasible solution found so far, and Z_{all} is the unpenalized objective function value for the best solution found so far. The penalty function is a variation of the adaptive one by Smith and others (Coit et al. 1996, Smith and Tate 1993, Smith and Coit 1997). In this case $d_{j,k}$ is defined as the shortest rectilinear distance along departmental contours between the I/O stations of departments j and k as given by:

$$d_{j,k} = \text{Min} \{d_{j,l,k,m} : l \in \text{Loc}_j, m \in \text{Loc}_k\} \quad (3)$$

where Loc_j and Loc_k are the set of candidate I/O locations for departments j and k , respectively, and $d_{j,l,k,m}$ is the shortest distance, following the department contours, from candidate I/O location l of department j to candidate I/O location m of department k .

The flow of the algorithm is shown below:

1. Initialize the population of chromosomes
2. For $j = 1$ to the maximum number of generations
 - i. Create one child solution
 - a. Select two parent chromosomes based on ranked fitness and perform crossover to produce one child
 - b. Call *evaluation routine* for child
 - c. Eliminate the worst solution from the current population and add the child solution
 - ii. Create the mutants

- a. Uniform randomly select chromosomes for mutation and perform mutation
- b. Call *evaluation routine* for mutants
- c. Eliminate the worst solutions from the current population and add the mutants

Evaluation Routine

- 1) Determine the current facility design from the chromosome
- 2) Calculate the number of infeasible departments
- 3) Construct a network of nodes corresponding to each department in the design and its candidate I/O locations
- 4) Find the shortest path between each pair of nodes in the network
- 5) Sum the flow costs along the shortest path according to equation 2

3. Test Problems and Results

Several unequal area problems from the literature were solved in the manner described in Section 2. While the material flows, departmental areas and constraints are identical to those previously studied, results cannot be compared directly as the distance measure used previously was the centroid to centroid. The problems are from Bazaraa (1975) and Hassan et al. (1986) (14 departments) and Armour and Buffa (1963) (20 departments). The ten department problem of van Camp et al. (1991) was also studied, however there is a solution (optimal) where equation 2 is equal to zero. In this design, all departments are located adjacent to the departments with which they have material flow, and since there is no limitation on the number of I/Os, the distance traveled over the entire facility is zero. The GA settings were the same as in Tate and

Smith (1995): population size of 10, mutation rate of 50% (5 mutants per generation) and number of generations generated = 100,000.

Objective function values from the contour measure are in Table 1, where the best, median, worst and standard deviation over ten runs using different random number seeds are shown. The twenty department Armour and Buffa (A&B) was studied with maximum aspect ratios of 10, 7, 5, 4, 3 and 2. At a maximum aspect ratio of 10, the problem is nearly unconstrained, while a maximum aspect ratio of 2 is extremely constrained. The Bazaraa problem used a minimum side length of one as the shape constraint, as done by previous authors. This minimum side length requirement loosely constrains the Bazaraa 14 problem.

INSERT TABLE 1 HERE

For comparison, Figure 3 shows the best layout using the contour measure compared to the best layout from Tate and Smith (1995) using the rectilinear centroid to centroid distance measure for the Bazaraa 14 problem. Also shown are the assumed I/O points and the assumed material flow paths inherent in each formulation. Note that department 14 is a “dummy” department with no flows, hence the lack of an I/O. It appears that the contour measure with I/O location on the boundaries of the departments is a better reflection of the physical movement of material for most manufacturing and service scenarios. The centroid method not only traverses through the interior of intervening departments, it assumes the minimum rectilinear distance between pairs of departments, creating nearby parallel paths as seen in departments 5 and 6. The aisle corners can also be placed inconveniently, as in departments 5 and 6. Layouts where the centroids were not located along the same line, as in departments 1 through 4 of Figure 3, would create even more paths, as shown in Figure 4. The best Tate and Smith (1995) layout for the Armour and Buffa problem using an aspect ratio constraint of 3 is compared to the best of this

paper. The centroid to centroid aisles again create parallel paths that traverse departments, and traffic through departments would normally be unacceptable in actual facilities. The appendix includes the best feasible designs identified for the other test problems.

INSERT FIGURE 3 AND FIGURE 4 HERE

4. Conclusions

This paper puts forth a tractable and effective methodology for integrated optimization of facilities designs that will translate well into physical reality. The solution methodology, based on a network algorithm that exploits the flexible bay structure within a genetic algorithm meta-heuristic framework, makes it possible to concurrently optimize five facets of the facility design problem: departmental location within the facility, departmental shape within certain constraints, selection of the proper number of I/Os, their placement, and flow paths along the departmental contours. This last item may be regarded as a precursor to development of a true aisle structure, where contour travel is further constrained. It is shown that although the number of I/Os per department is not constrained, the candidate I/O locations must be at departmental intersection points, and this number is bounded above by $2n-2$. Further work should consider incorporation of a fixed cost per I/O in the objective function so that the initial costs of building an I/O can be weighed with the subsequent reduction in material flow distance. It would also be interesting to consider aisle capacities and directional flow in future work.

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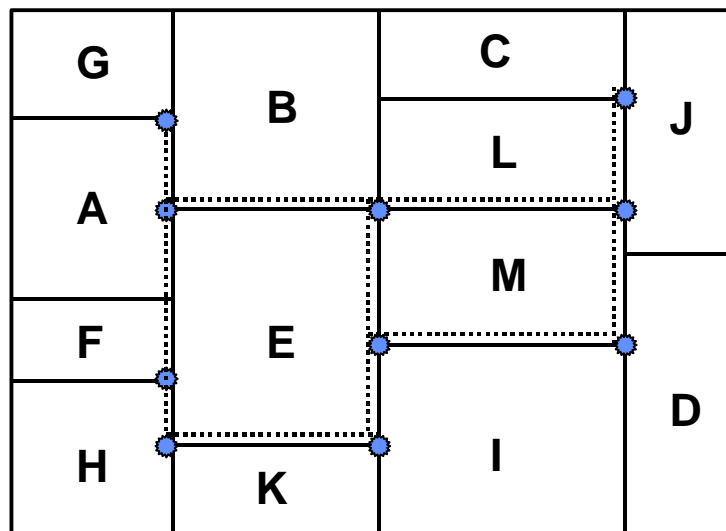
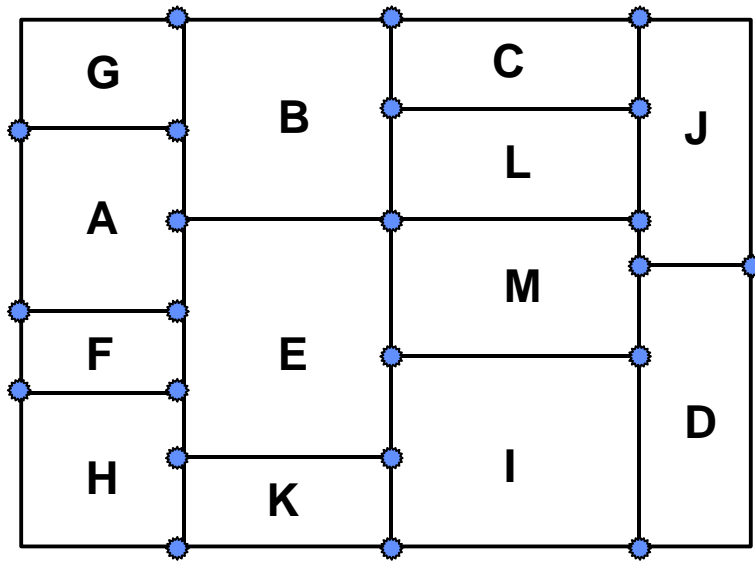


Figure 1. Possible I/O points on a flexbay layout (top) and material travel after I/O placement (bottom).

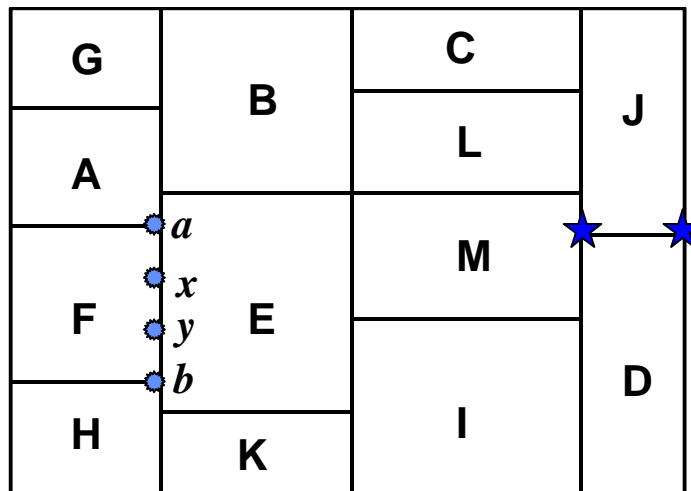
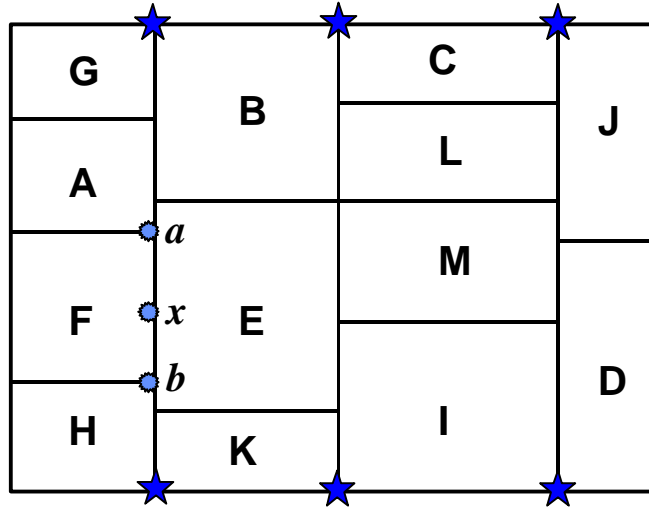


Figure 2. Possible I/O locations must be at department intersection points.

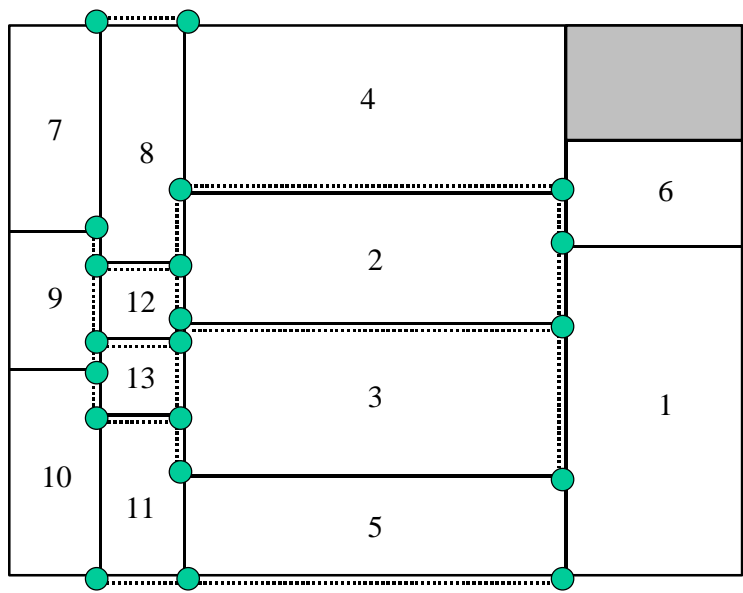
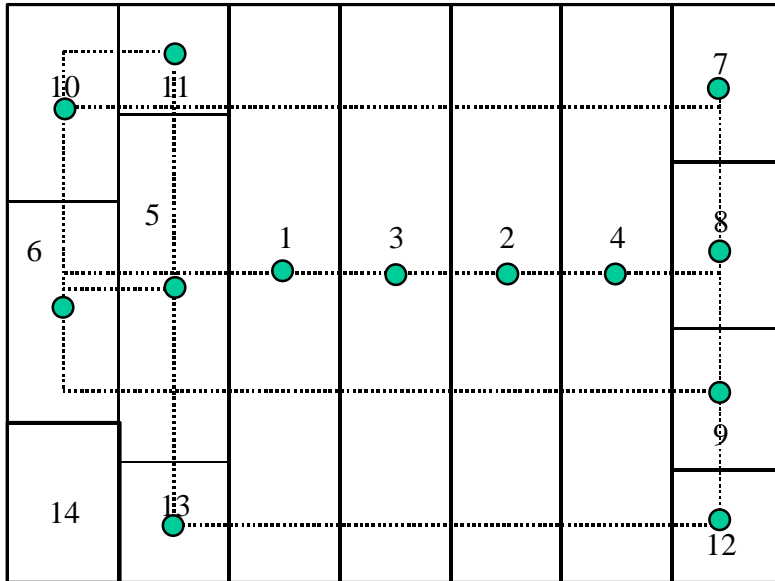


Figure 3. I/O points and flow paths (dashed) for the centroid distance measure (top) and the contour distance measure (bottom) for the Bazaraa 14 problem.

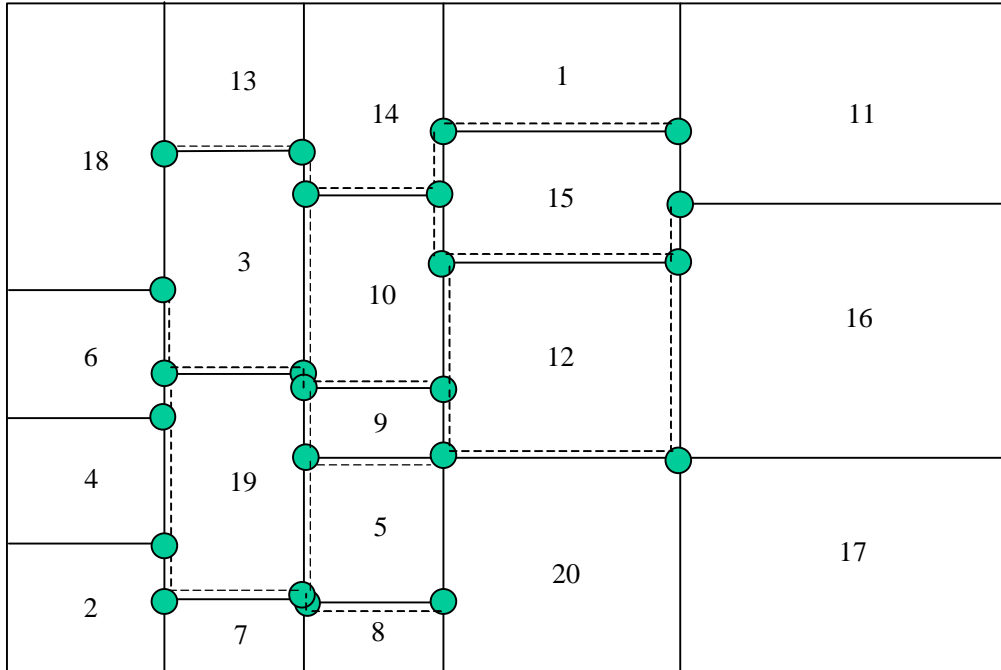
Table 1. Comparisons of Results over Ten Seeds.

Problem (# dept/aspect ratio)	Best	Median	Worst	Standard Deviation
Bazaraa 14	1343.2	1459.2	1607.9	92.8
A&B 20/10	757.1	862.9	1221.0	147.2
A&B 20/7	785.8	1079.9	1267.4	155.2
A&B 20/5	949.4	1319.6	2176.4	343.0
A&B 20/4	1025.7	1189.2	1758.1	236.3
A&B 20/3	942.6	1478.1	2298.5	354.0
A&B 20/2*	1244.1	1873.6	3359.8	787.2

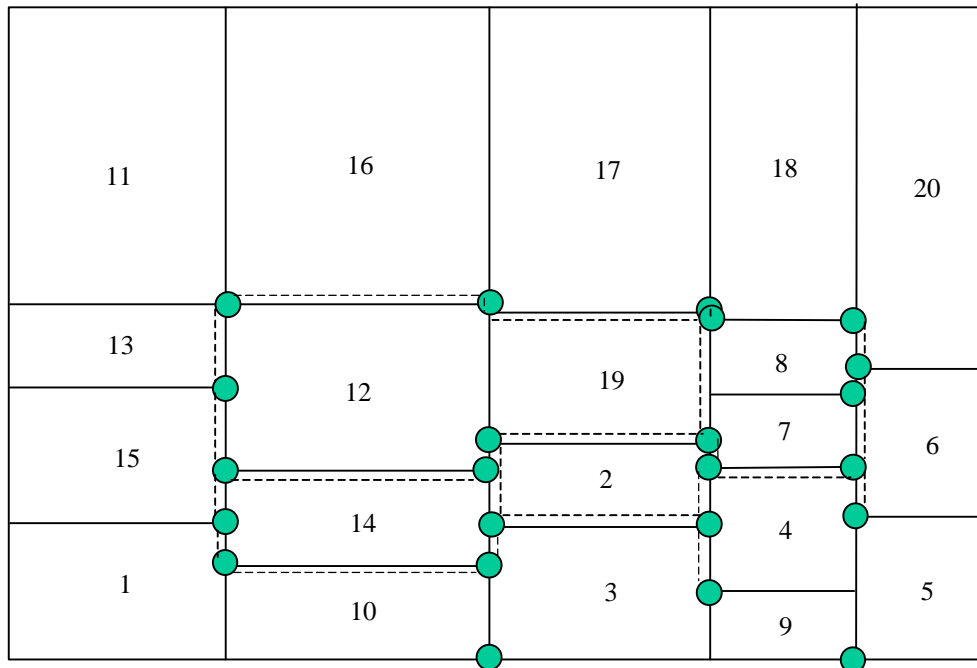
* Over the six of the ten runs that found feasible layouts.

Appendix

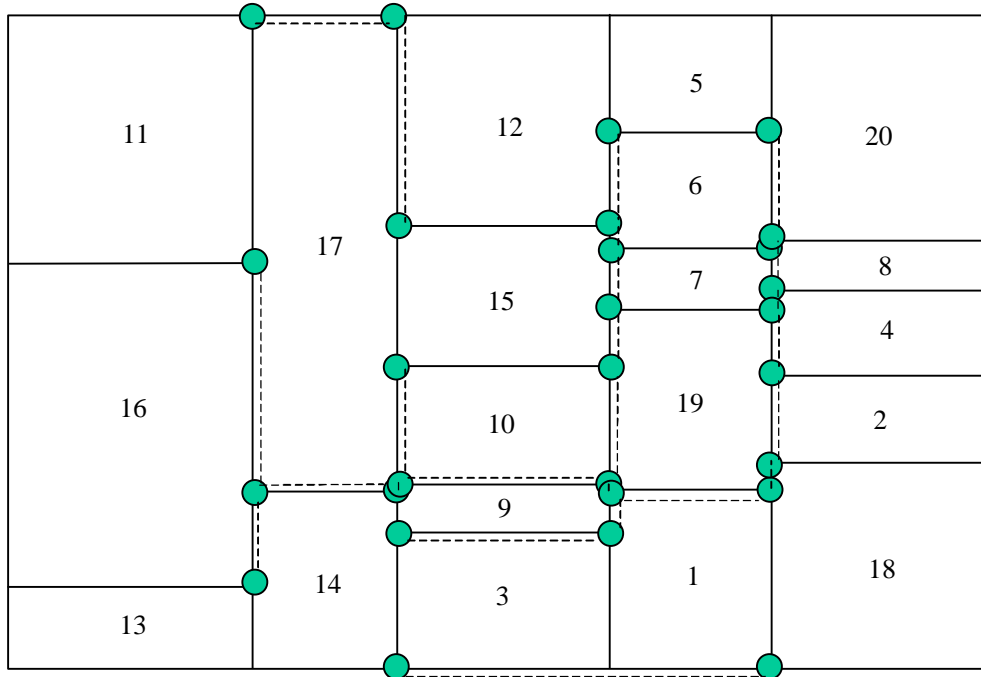
The design of the best feasible solution for each maximum aspect ratio constraint.



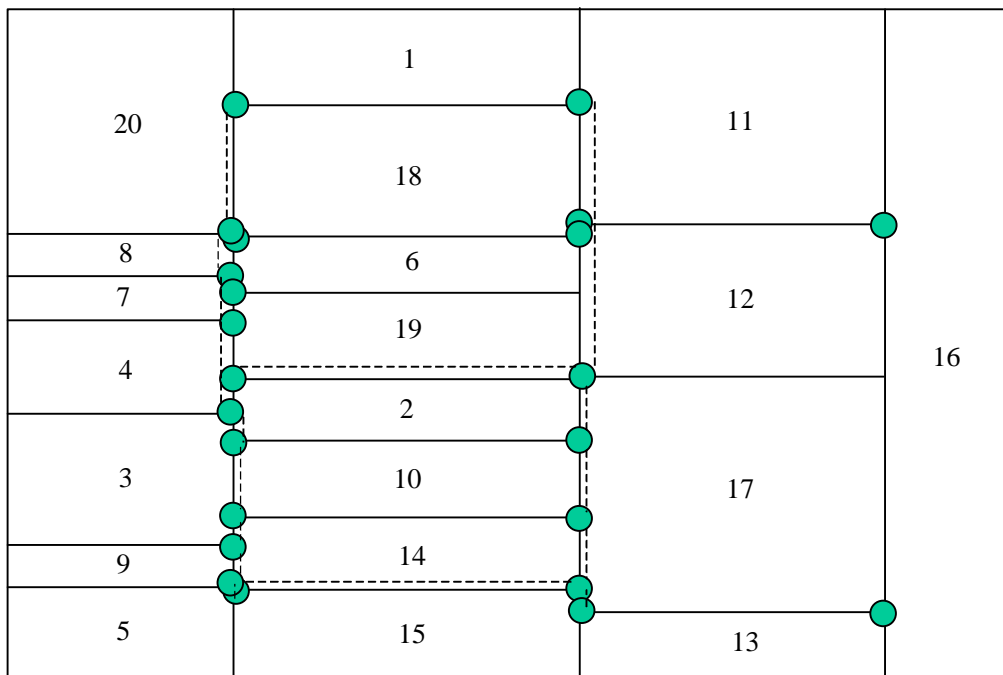
Maximum Aspect Ratio = 2



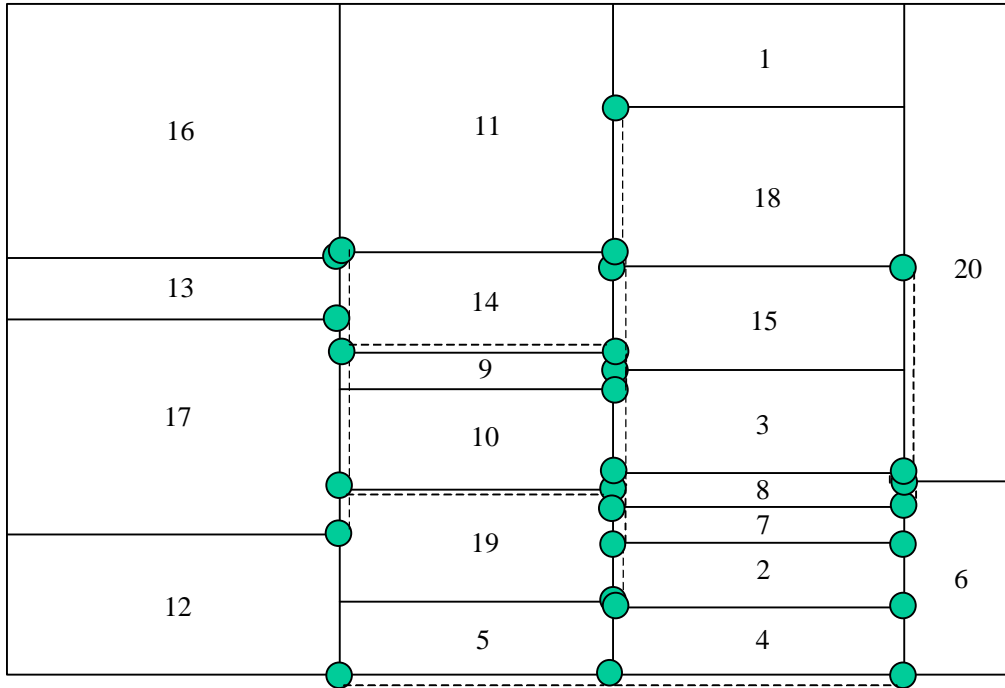
Maximum Aspect Ratio = 4



Maximum Aspect Ratio = 5



Maximum Aspect Ratio = 7



Maximum Aspect Ratio = 10