

Stability and Control of a Parametrically Excited Rotating System. Part II: Controls

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Abstract. In Part-I of this paper, the stability of a parametrically excited rotating system was analyzed. In this part the design of a feedback controller and an observer for the same mechanical system is considered. First, the time periodic system equations are transformed to a time invariant form which is suitable for an application of the standard techniques of linear control theory. A full-state feedback controller is designed in the transformed domain using the pole placement technique. Next, a Luenberger observer is constructed for estimating the unmeasurable states. Robustness of the observer is tested under the assumption that white noise is present in the measured states. Simulations for several combination of excitation and rotation parameters are provided.

Keywords: control, systems with periodic coefficients, Lyapunov-Floquet transformation

1. Introduction

The control of a time periodic system is a challenging task due to the time varying nature of the coefficients. The main problem is that the time varying eigenvalues of the periodic matrix do not determine the stability of the system and the standard methods of control theory cannot be applied directly. Therefore, one possible approach to handle such problems would be to construct equivalent time invariant systems suitable for the application of conventional techniques. Among other methods, time invariant system can be obtained either by applying the averaging method [2] or the Lyapunov-Floquet (L-F) transformation [3, 4]. It is known that the averaging method can be used only for those systems where the periodic coefficients can be expressed in terms of a small parameter. The Lyapunov-Floquet transformation technique does not have such limitations and hence it can be applied to general periodic systems. Once the time invariant system is constructed, the controller for the original time varying problem can be designed using simple techniques, such as pole placement or optimal control theory [5]. In the past several studies have dealt with the problem associated with a periodic system including the iterative schemes suggested in references [6, 7]. However, a general straightforward procedure was developed only recently by Sinha and his coauthors [4, 8]. Due to its simplicity and generality, this method is used in the present work to control the motion of a planar rotating pendulum subjected to a base excitation.

An active feedback control algorithm based on Poincare maps was derived by Schmitt *et al.* [9]. The periodic control scheme was applied to simulations of helicopter rotor blade

dynamics in forward flight. Unstable periodic flap-lag dynamics were stabilized by the periodic application of very small perturbations of the blade pitch angle.

Several authors have studied the control of time-invariant inverted pendulum models. Mori *et al.* [10], Schaefer and Cannon [11] investigated the observer based control of a single inverted pendulum supported on a cart and moving on a horizontal rail where a feedforward-feedback controller was used. Furuta *et al.* [12] designed and constructed a digital controller to stabilize a double inverted pendulum in the upright position moving on an inclined rail. Sturgeon and Loscutoff [13] used an observer in the control algorithm that measured all the states of a double inverted pendulum. Next, the attitude control was considered by Furuta *et al.* [14] wherein the control was implemented by applying torques at the upper two hinges on a cart and moving on a horizontal rail. Maletinsky *et al.* designed an observer based controller where the observer measured the position of the cart and the angle between the cart and the first pendulum. Later, Mayer *et al.* [16] discussed the discrete computer control of a triple inverted pendulum hinged on a cart and moving on a horizontal rail.

In this work, the double planar pendulum has a parametrically excited motion of the base and the whole system can also rotate in the horizontal plane. Control torques are applied at each hinge. The stability of this system was studied in the first part of this paper [1]. Using Lyapunov-Floquet transformation a full state feedback controller is constructed. Because not all the states may be available for control, an observer based controller is also designed. The pole placement technique is used to determine the control gain of the original time periodic system. Due to the fact that the measured states always contain noise, the robustness of the system is tested for the case when the data is contaminated by white noise. The robustness is also studied by changing the nominal values of the system parameters.

Despite the large number of calculations required, the control systems via Lyapunov-Floquet transformation can be easily implemented in real cases, because all the matrix involved in generating the time varying gain matrices can be computed off line and stored into the computer memory.

2. The System Model

The mechanical system considered is shown in figure 1. The massless base has a known sinusoidal movement in the horizontal plane of amplitude L_1 and angular velocity ω . The two link arms are connected to a moving base. The whole system can rotate in the horizontal plane (no gravity force) with a constant angular speed Ω . Both links have the length L . Concentrated masses ($m_1 = m_2 = m$) are located at the end of the links. On each joint there are restoring spring hinges and dampers, with the stiffness constant k and the damping constant c , respectively. The control torques u_1 and u_2 are employed for system stabilization. As shown in [1], a linear system with periodic coefficient is obtained if the nonlinear equations are linearized around the equilibrium point $q_1 = 0, q_2 = 0$. Introducing the nondimensional time τ through

$$\tau = t\sqrt{\frac{k}{mL^2}}, \quad (1)$$

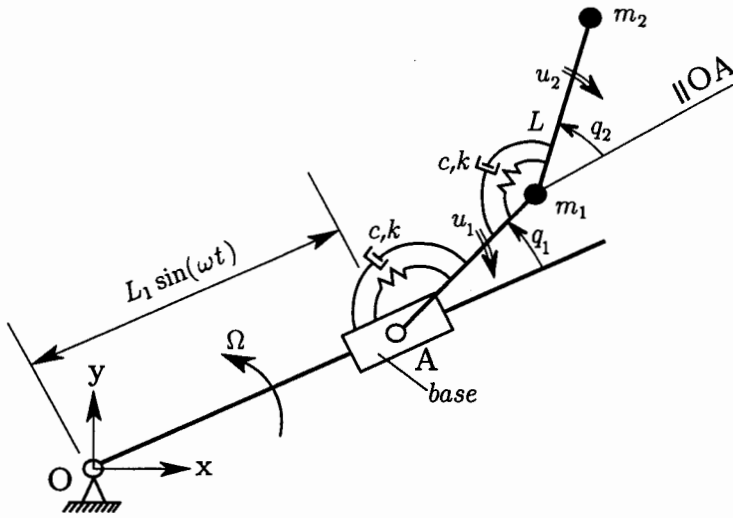


Figure 1. Planar double pendulum with vibrating base.

the original equations can be rewritten as

$$\mathbf{M}\mathbf{x}''(\tau) + \mathbf{C}(\delta)\mathbf{x}'(\tau) + \mathbf{K}(\varepsilon, \gamma, \psi, \tau)\mathbf{x}(\tau) = \mathbf{B}\mathbf{u}(\tau) + \mathbf{h}(\varepsilon, \gamma, \psi, \tau)$$

$$\mathbf{y}(\tau) = \mathbf{E}\mathbf{x}(\tau), \quad (2)$$

where $\mathbf{x}(\tau) = [q_1, q_2]^T$ and primes denote the derivatives with respect to the normalized time τ . The matrices are

$$\mathbf{M} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{C}(\delta) = \delta \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix},$$

$$\mathbf{K}(\varepsilon, \gamma, \psi, \tau) = \begin{bmatrix} 2 + \gamma & -1 - \gamma \\ -1 - \gamma & 1 + \gamma \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \varepsilon(\gamma + \psi^2) \sin \Psi \tau,$$

$$\mathbf{h}(\varepsilon, \gamma, \psi, \tau) = - \begin{bmatrix} 4 \\ 2 \end{bmatrix} \varepsilon \Psi \sqrt{\gamma} \cos \Psi \tau, \quad (3)$$

$\mathbf{B} = \mathbf{I}_2$ (the identity matrix of second order) and \mathbf{E} is the output matrix. Further, we have used the following nondimensional quantities

$$\delta = \sqrt{\frac{c^2}{mL^2k}}, \quad \gamma = \frac{mL^2\Omega^2}{k}, \quad \varepsilon = \frac{L_1}{L},$$

$$\psi = \omega \sqrt{\frac{mL^2}{k}}, \quad \mathbf{u} = \left[\frac{u_1}{k}, \frac{u_2}{k} \right]^T. \quad (4)$$

When the controller is designed, the vector $\mathbf{h}(\varepsilon, \gamma, \psi, \tau)$ is considered as a disturbance, and is not taken into account. In the state space form, equation (2) can be rewritten as

$$\begin{aligned} \theta'(\tau) &= \mathbf{A}(\tau)\theta(\tau) + \mathbf{b}\mathbf{u}(\tau) + \mathbf{d}(\tau), \\ \mathbf{y}(\tau) &= \mathbf{J}\theta(\tau), \end{aligned} \quad (5)$$

where

$$\begin{aligned} \theta(\tau) &= \begin{bmatrix} \mathbf{x}(\tau) \\ \mathbf{x}'(\tau) \end{bmatrix}, \quad \mathbf{A}(\tau) = \begin{bmatrix} 0 & \mathbf{I}_2 \\ -\mathbf{M}^{-1}\mathbf{K}(\tau) & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \\ \mathbf{b} &= \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{B} \end{bmatrix}, \quad \mathbf{d}(\tau) = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{h}(\tau) \end{bmatrix}, \quad \mathbf{J} = [\mathbf{E} \ 0]. \end{aligned} \quad (6)$$

3. The Full State Feedback Controller Design via Lyapunov-Floquet Transformation

In this section the idea of Lyapunov Floquet transformation is utilized to design a full state control law for the system given by equation (5) such that

$$\mathbf{u}(\tau) = \mathbf{F}(\tau)\theta(\tau), \quad (7)$$

where $\mathbf{F}(\tau)$ is the feedback matrix.

To determine the Lyapunov-Floquet Transformation, one must first compute the Fundamental Matrix $\Phi(\tau)$ (also called the *State Transition Matrix (STM)*). This can be done either analytically using the Chebyshev polynomials expansion approach [6], or by numerical integration of the matrix differential equation

$$\Phi'(\tau) = \mathbf{A}(\tau)\Phi(\tau), \quad 0 < \tau \leq T, \quad (8)$$

with the initial condition

$$\Phi(0) = \mathbf{I}_4. \quad (9)$$

For time $\tau_1 > T$, the state transition matrix can be computed using the expression [5]

$$\Phi(\tau_1) = \Phi(\tau + rT) = \Phi(\tau)\Phi'(T), \quad (10)$$

where r is an integer and $\Phi(T)$ is the state transition matrix evaluated at the end of the principal period. This matrix is also called *Floquet Transition Matrix (FTM)*.

Matrix $\Phi(\tau)$ can be factored as

$$\Phi(\tau) = \mathbf{Q}(\tau)e^{\mathbf{R}\tau}, \quad (11)$$

where $\mathbf{Q}(\tau)$ is a $2T$ real periodic matrix, and \mathbf{R} is a real constant matrix. Due to the periodicity of the matrix $\mathbf{Q}(\tau)$, the matrix \mathbf{R} is computed using the expression

$$\mathbf{R} = \frac{1}{2T} \ln \Phi(2T) = \frac{1}{2T} \ln \Phi^2(T) = \frac{1}{T} \ln \Phi(T). \quad (12)$$

Applying the Lyapunov-Floquet transformation

$$\theta(\tau) = \mathbf{Q}(\tau)\mathbf{z}(\tau), \quad (13)$$

to equation (5), the following system is obtained [3]

$$\mathbf{z}'(\tau) = \mathbf{R}\mathbf{z}(\tau) + \mathbf{Q}^{-1}(\tau)\mathbf{b}\mathbf{u}(\tau), \quad (14)$$

where \mathbf{R} , of course, is a constant matrix. At this point, in the spirit of reference [3], an auxiliary time invariant system of the type

$$\bar{\mathbf{z}}'(\tau) = \mathbf{R}\bar{\mathbf{z}}(\tau) + \mathbf{B}_0\mathbf{v}(\tau), \quad (15)$$

is constructed. In equation (15), \mathbf{B}_0 is a full rank constant matrix, such that the pair $(\mathbf{R}, \mathbf{B}_0)$ is controllable. The control vector $\mathbf{v}(\tau)$ for system (15) is determined by designing a full state feedback controller using either the pole placement technique or the optimal control theory. Thus one has

$$\mathbf{v}(\tau) = \mathbf{F}_0\bar{\mathbf{z}}(\tau). \quad (16)$$

where \mathbf{F}_0 is a constant gain matrix. Let $\mathbf{e}(\tau) \equiv \mathbf{z}(\tau) - \bar{\mathbf{z}}(\tau)$ be the dynamic error between the state vectors \mathbf{z} and $\bar{\mathbf{z}}$. Subtracting equation (15) from (14), one can obtain

$$\mathbf{e}'(\tau) = (\mathbf{R} + \mathbf{B}_0\mathbf{F}_0)\mathbf{e}(\tau) + \mathbf{Q}^{-1}(\tau)\mathbf{b}\mathbf{u}(\tau) - \mathbf{B}_0\mathbf{F}_0\mathbf{z}(\tau). \quad (17)$$

Since $(\mathbf{R} + \mathbf{B}_0\mathbf{F}_0)$ is a stability matrix, systems defined by equations (14) and (15) can be made equivalent if

$$\mathbf{Q}^{-1}(\tau)\mathbf{b}\mathbf{u}(\tau) = \mathbf{B}_0\mathbf{F}_0\mathbf{z}(\tau). \quad (18)$$

Because condition (18) can not be exactly satisfied, these systems can only be made equivalent in a least square sense. For this purpose, an error vector is defined as

$$\boldsymbol{\eta} = \mathbf{b}\mathbf{u}(\tau) - \mathbf{Q}(\tau)\mathbf{B}_0\mathbf{F}_0\mathbf{z}(\tau), \quad (19)$$

and $\mathbf{u}(\tau)$ is computed such that the performance index

$$\Lambda^* = \boldsymbol{\eta}^T \mathbf{W} \boldsymbol{\eta} \quad (20)$$

is minimized. Here $\boldsymbol{\eta}^T$ is the transpose of the vector $\boldsymbol{\eta}$ and \mathbf{W} is a symmetric and positive definite matrix. The minimization of Λ^* with respect to $\mathbf{u}(\tau)$ yields the weighted least

square estimation $\hat{\mathbf{u}}(\tau)$. For the case of unconstrained input systems, $\hat{\mathbf{u}}(\tau)$ is computed by

$$\frac{\partial \Lambda^*}{\partial \mathbf{u}(\tau)} = \mathbf{b}^T \mathbf{W}(\mathbf{b}\hat{\mathbf{u}}(\tau) - \mathbf{Q}(\tau)\mathbf{B}_o\mathbf{F}_o\mathbf{z}(\tau)) = 0, \quad (21)$$

or

$$\hat{\mathbf{u}}(\tau) = (\mathbf{b}^T \mathbf{W} \mathbf{b})^{-1} \mathbf{b}^T \mathbf{W} \mathbf{Q}(\tau) \mathbf{B}_o \mathbf{F}_o \mathbf{z}(\tau). \quad (22)$$

In the case when the weighting matrix \mathbf{W} is the identity matrix, one has

$$\hat{\mathbf{u}}(\tau) = \mathbf{b}^* \mathbf{Q}(\tau) \mathbf{B}_o \mathbf{F}_o \mathbf{z}(\tau), \quad (23)$$

where \mathbf{b}^* is the generalized inverse of the matrix \mathbf{b} , defined as

$$\mathbf{b}^* = (\mathbf{b}^T \mathbf{b})^{-1} \mathbf{b}^T. \quad (24)$$

Applying the inverse Lyapunov-Floquet transformation to equation (23), the following expression is obtained

$$\mathbf{u}(\tau) = \mathbf{b}^* \mathbf{Q}(\tau) \mathbf{B}_o \mathbf{F}_o \mathbf{Q}^{-1}(\tau) \boldsymbol{\theta}(\tau). \quad (25)$$

Comparing equation (25) with equation (7), the desired feedback gain matrix $\mathbf{F}(\tau)$ is

$$\mathbf{F}(\tau) = \mathbf{b}^* \mathbf{Q}(\tau) \mathbf{B}_o \mathbf{F}_o \mathbf{Q}^{-1}(\tau). \quad (26)$$

It should be observed that the feedback matrix from equation (26) can be computed off line and stored into the computer memory. This is important for a real time implementation of the control algorithm based on the L-F transformation technique.

4. Design of an Observer Based Controller

In the previous section, the feedback matrix $\mathbf{F}(\tau)$ was computed assuming that all states of the system given by equation (5) are measurable. However, in practice, not all the states may be available for control, and in this case an observer must be designed for the unmeasurable states.

Let us assume that only the angular displacements q_1 and q_2 are measurable. Then an observer must be designed to estimate the angular velocities q_1' and q_2' . The observer state equations are given by [17]

$$\begin{aligned} \boldsymbol{\theta}'_e(\tau) &= \mathbf{A}(\tau) \boldsymbol{\theta}_e(\tau) + \mathbf{b} \mathbf{u}(\tau) + \mathbf{G}(\tau) (\mathbf{J} \boldsymbol{\theta}_e(\tau) - \mathbf{y}(\tau)), \\ \mathbf{y}_e(\tau) &= \boldsymbol{\theta}_e(\tau), \end{aligned} \quad (27)$$

where $\boldsymbol{\theta}_e(\tau)$ is the estimated states vector of the system given by equation (5) and $\mathbf{y}_e(\tau)$ is the output of the observer. Here $\mathbf{G}(\tau)$ is the estimation law and the objective of this section is to determine $\mathbf{G}(\tau)$ via Lyapunov-Floquet transformation. This observer, called the Luenberger observer, has the same dimension as the original system, and estimates

all the states, including the measurable ones. Therefore, in case of a large scale system, a real time implementation of the observer can be a problem due to the large number of calculations required. In such cases, a reduced order observer should be implemented.

To design the observer, first, the following dual system of equation (5) is considered

$$\begin{aligned}\bar{\theta}'(\tau) &= \mathbf{A}^T(\tau)\bar{\theta}(\tau) + \mathbf{J}^T\bar{\mathbf{u}}(\tau), \\ \mathbf{y}(\tau) &= \mathbf{b}^T\bar{\theta}(\tau).\end{aligned}\quad (28)$$

Due to the duality, the observer design procedure is similar to the controller design procedure developed in the previous section. Thus, the Floquet transition matrix $\Phi_e(T)$ is computed from equation (28). Once again, using a factorization of the type

$$\Phi_e(\tau) = \mathbf{Q}_e(\tau)e^{\mathbf{R}_e\tau} \quad (29)$$

and the Lyapunov-Floquet transformation

$$\bar{\theta}(\tau) = \mathbf{Q}_e(\tau)\mathbf{z}(\tau), \quad (30)$$

from equation (28), we obtain

$$\mathbf{z}'(\tau) = \mathbf{R}_e\mathbf{z}(\tau) + \mathbf{Q}_e^{-1}(\tau)\mathbf{J}^T\bar{\mathbf{u}}(\tau). \quad (31)$$

The auxiliary time invariant system is selected as

$$\bar{\mathbf{z}}'(\tau) = \mathbf{R}_e\bar{\mathbf{z}}(\tau) + \mathbf{J}_o\mathbf{p}(\tau), \quad (32)$$

such that \mathbf{J}_o is a full rank constant matrix, and the pair $(\mathbf{R}_e, \mathbf{J}_o)$ is controllable.

Using either the pole placement technique or optimal control theory, we can have

$$\mathbf{p}(\tau) = \mathbf{G}_o\bar{\mathbf{z}}(\tau). \quad (33)$$

and the minimization of the error dynamics between equations (31) and (32) then yields

$$\mathbf{F}_e(\tau) = \mathbf{J}^{T*}\mathbf{Q}_e(\tau)\mathbf{J}_o\mathbf{G}_o\mathbf{Q}_e^{-1}(\tau), \quad (34)$$

where \mathbf{J}^{T*} is the generalized inverse of the matrix \mathbf{J}^T . The desired estimation law $\mathbf{G}(\tau)$ is the transposed of the matrix $\mathbf{F}_e(\tau)$, *i.e.*,

$$\mathbf{G}(\tau) = \mathbf{F}_e^T(\tau). \quad (35)$$

Once again we notice that the observer law can be computed off line and stored into the computer memory. Finally, from equations (5), (7) and (27) the observer-based closed-loop system is given by

$$\begin{bmatrix} \theta'(\tau) \\ \theta_e'(\tau) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{F}(\tau) \\ -\mathbf{G}(\tau)\mathbf{J} & \mathbf{A} + \mathbf{F}(\tau) + \mathbf{G}(\tau)\mathbf{J} \end{bmatrix} \begin{bmatrix} \theta(\tau) \\ \theta_e(\tau) \end{bmatrix}. \quad (36)$$

5. Results of the Simulations

For the system shown in figure 1 described by the normalized equation of motion (2), several simulations were performed. Here results for some typical cases are presented. In all these cases the normalized value of the excitation amplitude $\varepsilon = 0.4$, and the normalized frequency of the excitation $\psi = \omega_2$. The initial states were taken to be $q_1(0) = 2$, $q_2(0) = 2$, $w_1(0) = 0$ and $w_2(0) = 0$.

It should be pointed out that the natural frequencies ω_1 and ω_2 of the undamped system described by equation (2) were computed in [1] and they were found to be $\omega_1 = 0.414$, $\omega_2 = 2.417$ for the nonrotating case ($\gamma = 0.00$) and $\omega_1 = 0.427$, $\omega_2 = 3.025$ for the rotating case when $\gamma = 0.66$.

Case a: No Damping ($\delta = 0$) and No Rotation ($\gamma = 0$)

For this case the Floquet multipliers of the Floquet Transition Matrix $\Phi(T)$ are found to be: 2.772, $-0.024 \pm 1.00j$ and 0.361, indicating an unstable system. This was expected since $\Psi = \omega_2$, which implies dynamic resonance with ω_2 . The behavior of the uncontrolled system is shown in figure 2.a. Figure 2.b shows the controlled states of the double planar pendulum when a full state feedback controller is used. The states of the system are controlled in about 3 periods. For an observer based control, the angular velocities are estimated. Figure 2.c shows the actual ($w_{1,r}$, $w_{2,r}$) and estimated angular velocities ($w_{1,e}$, $w_{2,e}$). One can notice that the actual and estimated states are almost identical. Finally, in figure 2.d the normalized control torques are displayed.

Case b: No Damping ($\delta = 0$) and Rotation ($\gamma = 0.666$)

The results are depicted in figure 3. For this case, the system is unstable, because the Floquet multipliers are: 3.455, 0.289, $0.012 \pm j$. Similar conclusions can be made for this case as well. The system is now more oscillatory due to the rotation of the system (figure 3.b). For this rotating case the states are controlled in about 6 periods.

Case c: With Damping ($\delta = 0.0163$) and No Rotation ($\gamma = 0$)

The results are shown in figure 4. The Floquet multipliers are: 2.447, 0.319, $-0.024 \pm 0.996j$, and the system is unstable (figure 4.a). The behavior of the controlled system is shown in figure 4.b. Comparing figure 4.b with figure 2.b, one can see that, in the case of a damped system, the required time to control the system is reduced by almost 25%. It should be also noticed that the required control torques are smaller in this case (figure 4.d), than in the case of an undamped system (figure 2.d). Figure 4.c shows the actual and estimated angular velocities, for an observer based control.

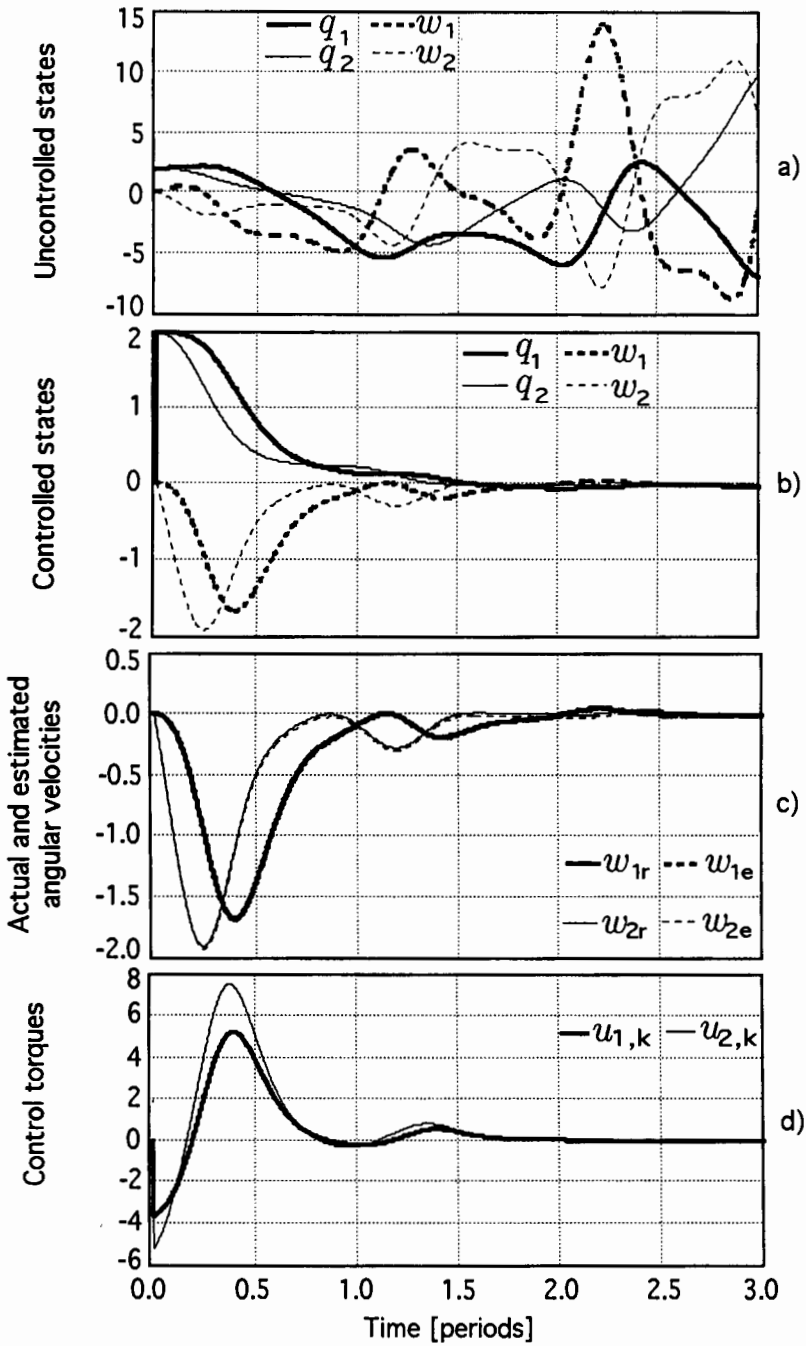


Figure 2. Undamped-nonrotating system a) uncontrolled states, b) controlled states, c) actual and estimated angular velocities, d) control torques.

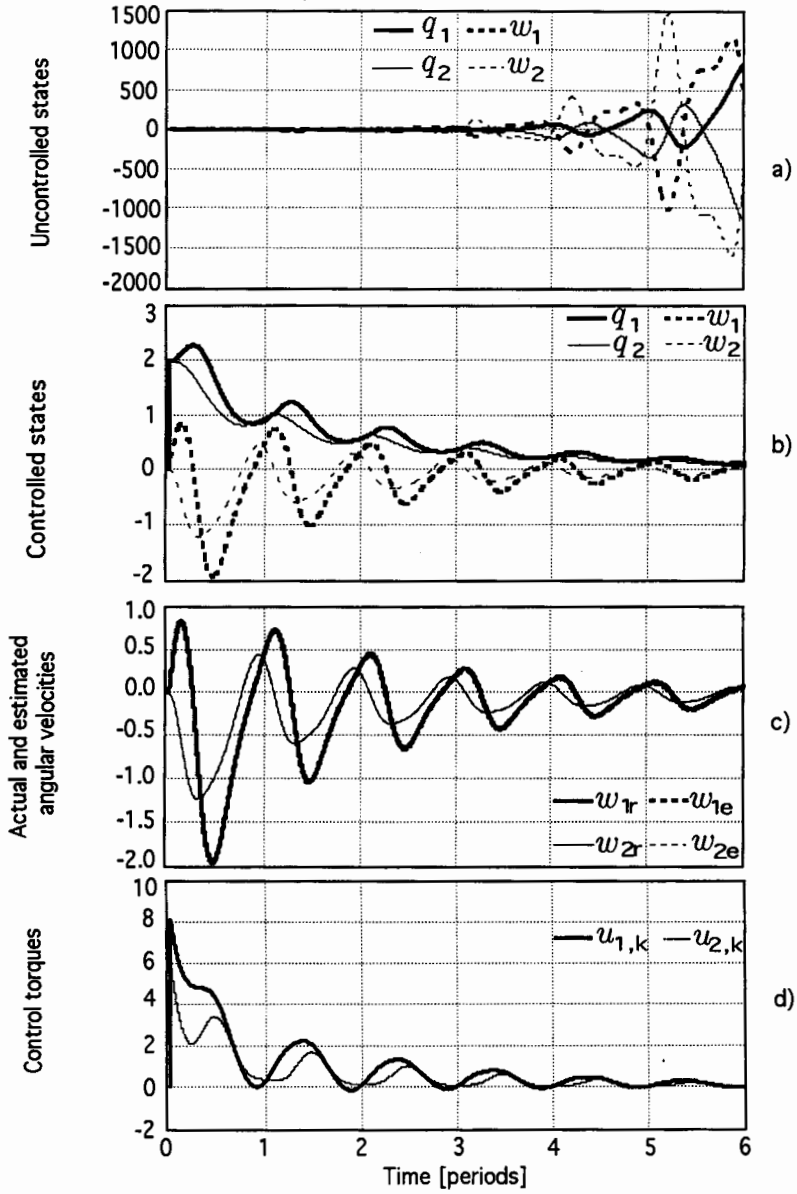


Figure 3. Undamped-rotating system a) uncontrolled states, b) controlled states, c) actual and estimated angular velocities, d) control torques.

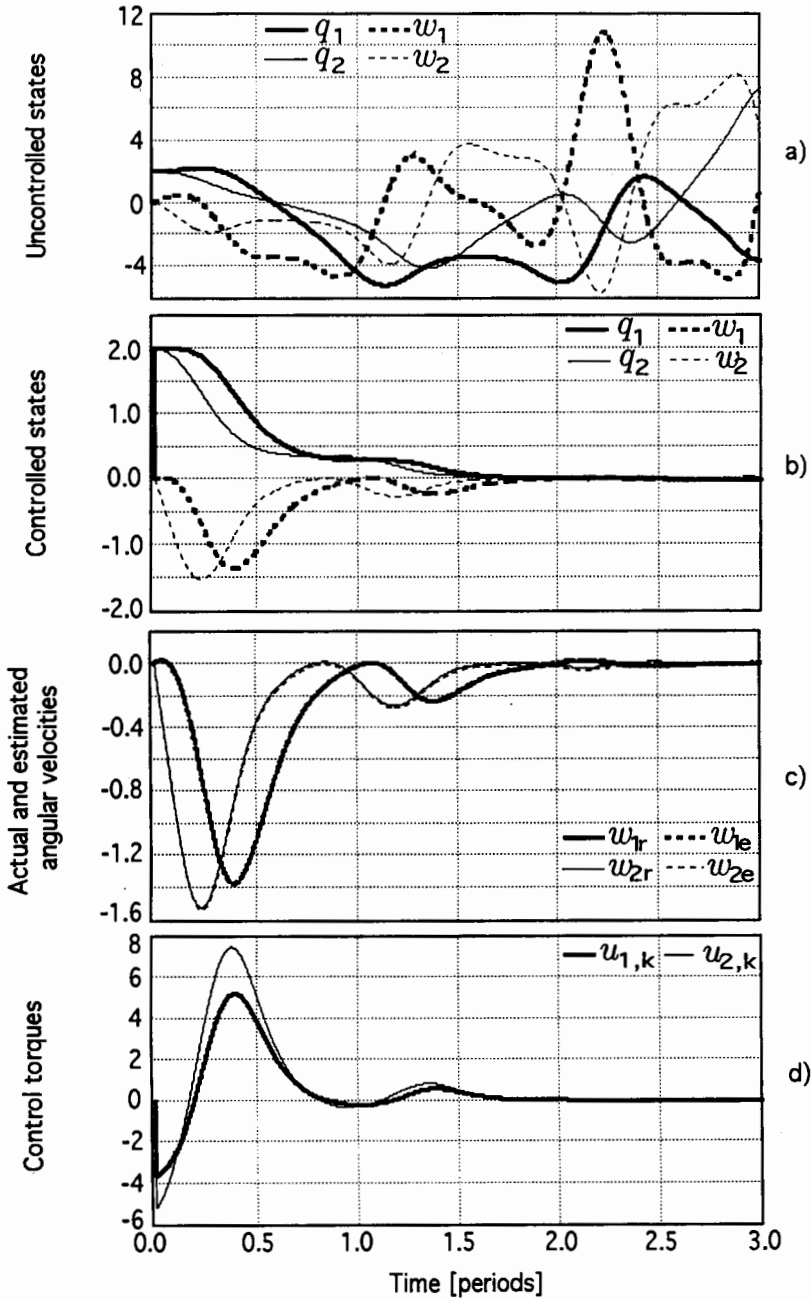


Figure 4. Damped-nonrotating system a) uncontrolled states, b) controlled states, c) actual and estimated angular velocities, d) control torques.

Case d: With Damping ($\delta = 0.0163$) and Rotation ($\gamma = 0.666$)

The results for this case are illustrated in figure 5. The system is unstable, with Floquet multipliers: 3.12, 0.261, $-0.012 \pm 0.997j$. The behavior of the unstable system is shown in figure 5.a. Conclusions similar to *Case c* can be noted. The system is controlled in a shorter time interval (figure 5.b) when compared to the case when there is no damping (figure 3.b). The required control torques are also smaller (figure 5.d). Figure 5.c shows the actual and estimated angular velocities, for an observer based control. Once again the observer response is very accurate.

6. The Observer Robustness

In this section the robustness of the designed observer is tested, because in practice noise is always present in the measured states. For this purpose the measured states are contaminated with white noise and the observer performance is evaluated. The noise, obtained by random number generation, has a zero mean value and an amplitude of 0.01 radians. Several simulations were performed and a typical one is shown in figure 6, for an undamped, rotating system. Figure 6.a shows the actual angles (q_{1r}, q_{2r}) and estimated angles (q_{1e}, q_{2e}). Figure 6.b shows the actual angular velocities (w_{1r}, w_{2r}) and the estimated ones (w_{1e}, w_{2e}). In order to see the noise level in the states, the response for $2.4 \leq \tau \leq 2.5$ in figures 6.a and 6.b are redrawn as figures 6c and 6d, respectively. One can note that the level of noise is lower in the estimated states than in the actual states. This indicates that the observer has a filtering effect. Since the mean value in the steady state regime is less than 0.1 radian, the level of the noise is 10%. The performance of the controllers in the presence of noise is not as good compared to the case when there is no noise; however, the system is still controlled. This indicates that the controller is sensitive to noise and not as robust as we would like to have.

The robustness of the observer based controller is also tested to parameters variation. After two periods, the mass attached at the end of the first link is decreased by 20% ($m_1 = 0.8m$), while the mass at the end of the second link is increased by 20% ($m_2 = 1.2m$). Figures 7 and 8 show the states and the control torques for two cases and it is observed that the motion is practically unaffected by the mass variation.

7. Discussion and Conclusions

The linear control problem associated with a rotating double inverted pendulum with a parametrically excited base is considered. The lagrangean formulation leads to a pair of second-order differential equations whose coefficients contain periodic terms and also depend on the rotation parameter Ω . Since the system is time-periodic, the stability analysis is performed via well known Floquet theory. The control system design is accomplished through an application of the Lyapunov-Floquet transformation technique recently suggested by Sinha and his coauthors [3, 4, 5]. This technique permits the designer to obtain the desired time varying gains by employing standard time invariant classical procedures. Both, full state and observer based controllers were constructed using the pole placement

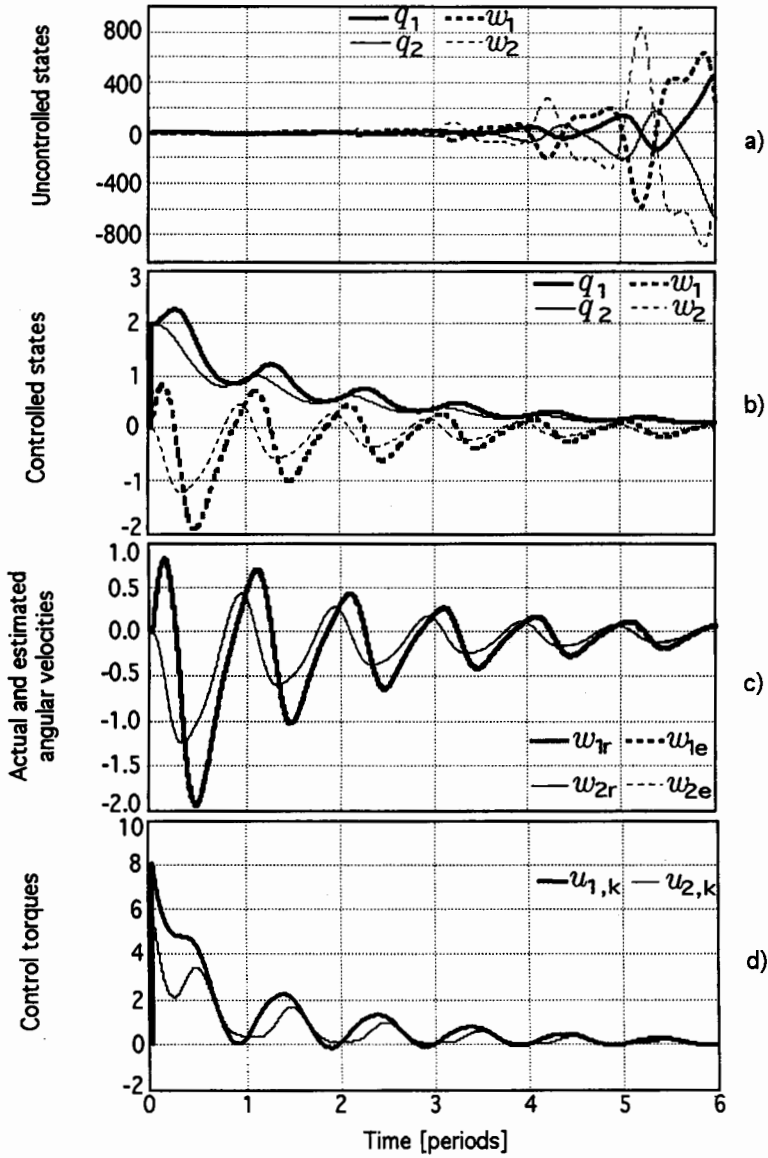


Figure 5. Damped rotating system a) uncontrolled states, b) controlled states, c) actual and estimated angular velocities, d) control torques.

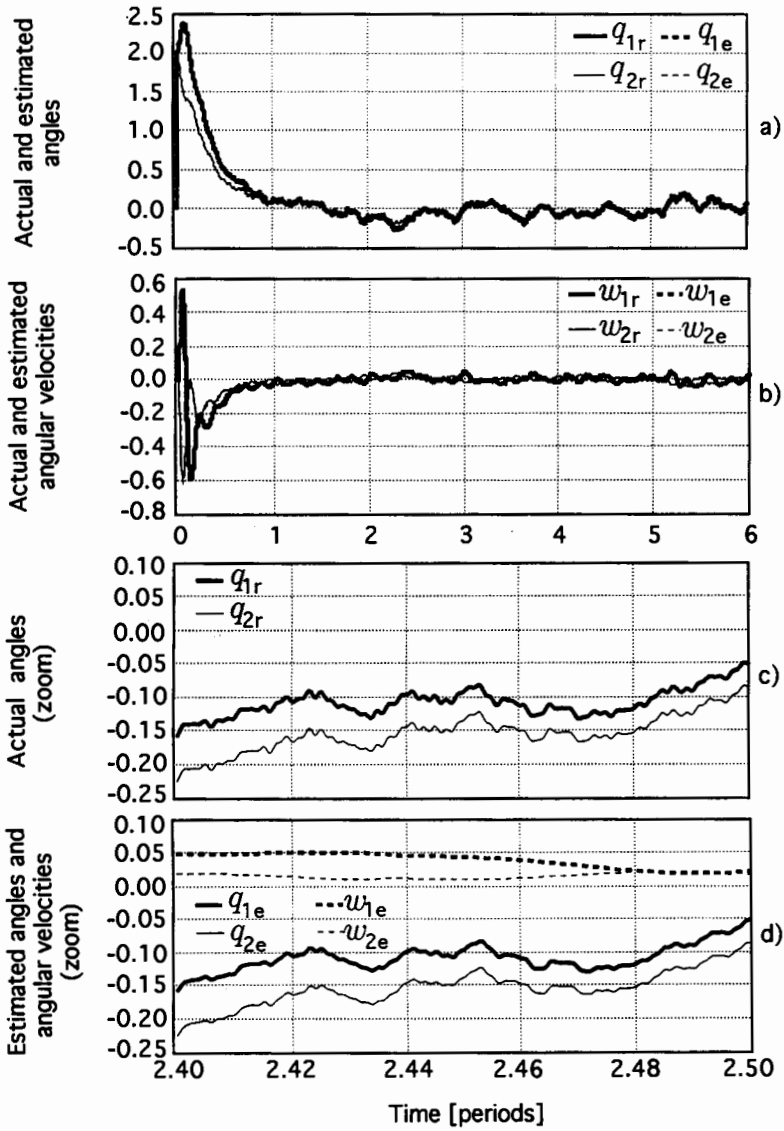


Figure 6. Undamped-rotating system with white noise a) actual and estimated angles, b) actual and estimated angular velocities, c) actual angles (zoom), d) estimated angles and angular velocities (zoom).

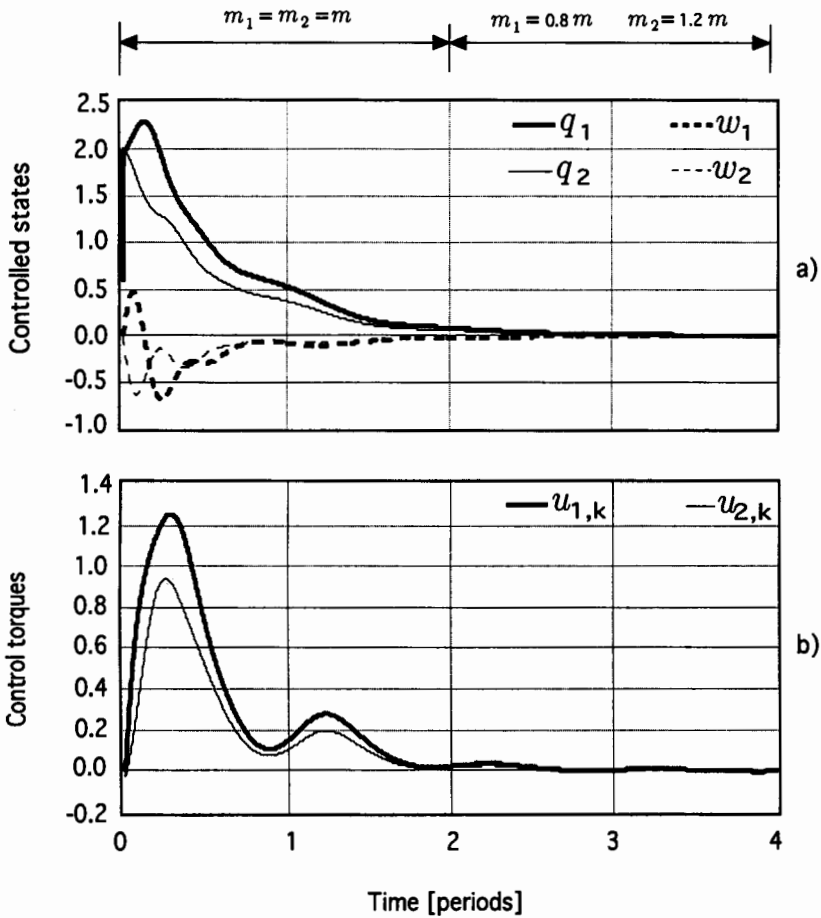


Figure 7. Damped-nonrotating system (robustness to mass variation) a) controlled states, b) control torques.

and/or optimal control strategies. Results of simulations show that this technique is simple and effective for a wide range of parameter values. Unlike some other methods [6, 7] reported in the literature, solutions are not generated by iterative schemes and one never has to worry about the canonical forms. Since all necessary computations can be performed off line and stored, the method is suitable for real time implementation.

When the states are contaminated by white noise, the observer has a smoothing effect and the estimated states are affected to a minimal extent. The controller design is still adequate if the noise level is up to 10%. However, for higher noise levels, analog/digital filters must be used to assure satisfactory performance. The controller is found to be very robust to changes in the inertia force as indicated in figures 7 and 8.

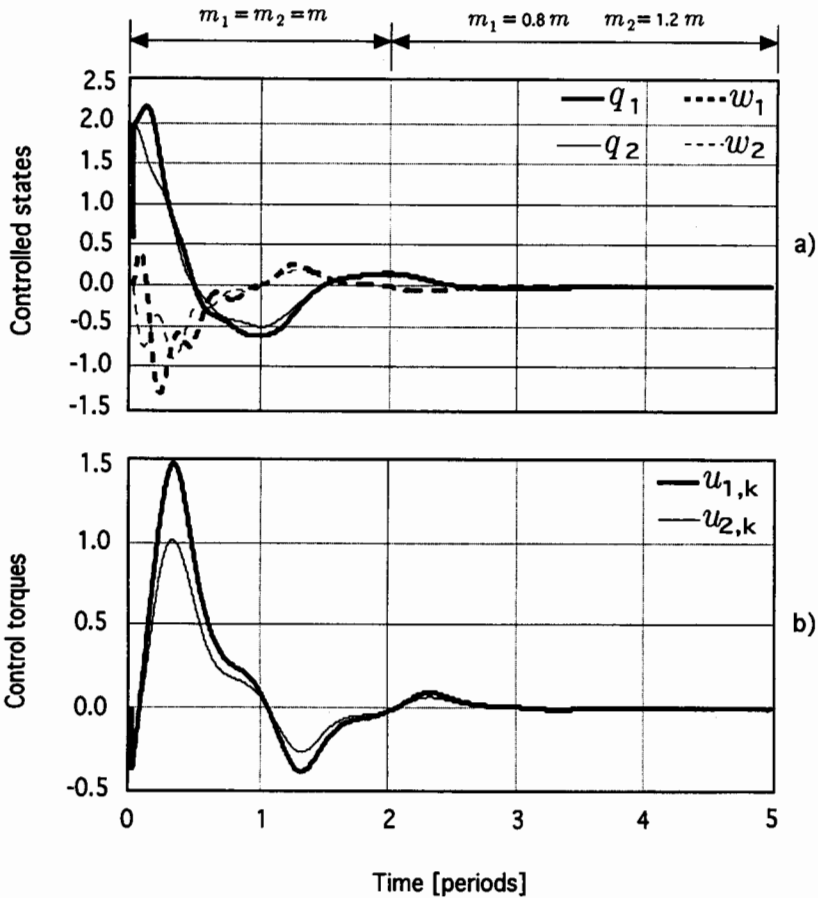


Figure 8. Damped-rotating system (robustness to mass variation) a) controlled states, b) control torques.

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