Order Reduction and Control of Parametrically Excited Dynamical Systems

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(Received 9 July 1999; accepted 6 October 1999)

Abstract: This paper provides methodology for designing reduced-order controllers for large-scale, linear systems represented by differential equations having time-periodic coefficients. The linear time-periodic system is first converted into a form in which the system stability matrix is time invariant. This is achieved by the application of Lyapunov-Floquet transformation. Then a completely time-invariant auxiliary system is constructed and order reduction algorithms are applied to this system to obtain a reduced-order system. The control laws are calculated for the reduced-order system by minimizing the least square error between the auxiliary and the transformed system. These control laws are then transformed to obtain the desired control action in the original domain. The schemes formulated are illustrated by designing full-state feedback and output feedback controllers for a five-mass inverted pendulum exhibiting parametric instability.

Key Words: Order reduction, control, parametric excitations, Lyapunov-Floquet transformation

NOMENCLATURE

$L-F$ = Lyapunov-Floquet transformation n, m, p, r = state, control vector, output vector, reduced-order state dimensions, respectively
$A(t) = n \times n$ system stability matrix
$B(t) = n \times m$ input gain matrix
$C(t) = p \times n$ output gain matrix
$x(t), \hat{x}(t) = n \times 1$ state/estimated state vector
$z(t), \hat{z}(t) = n \times 1 L-F$ transformed/estimated state vector
$\bar{z}, \bar{v} = n \times 1$ and $r \times 1$ auxiliary state/reduced state vectors
$u(t) = m \times 1$ control vector
$v_a(t)$ = the control vector for auxiliary system
$y(t) = p \times 1$ output vector
$Q(t) = n \times n$ $2T$ periodic $L-F$ transformation matrix
$\Phi(t) = n \times n$ state transition matrix (STM)
$F = n \times n$ Floquet transition matrix (FTM)

$\bar{B}$ = time-invariant input gain matrix for auxiliary system
$\bar{K}$ = time-invariant gain matrix for controller
$\bar{B}\bar{K}$ = has dimension $n \times r$
$\bar{T} = r \times n$ time-invariant order-reduction operator
$(.)^*$ = the conjugate transpose of the argument
$(.)^\#$ = the generalized inverse of the argument
$(.)^{-1}$ = the inverse of the argument
$\tilde{x}, \tilde{z}, \tilde{u}, \tilde{y}$ = dual quantities of x, z, u, y
$J_r = r \times r$ stability matrix of reduced-order system
$R = n \times n$ system stability matrix of the transformed system
$e = z - \bar{z} = n \times 1$ error variable representing the difference between the actual and the auxiliary state trajectory
$\hat{e} = x - \hat{x} = n \times 1$ estimation error variable representing the difference between the actual and the estimated state trajectory

1. INTRODUCTION

The mathematical description of many physical systems is often given by a set of a large number of ordinary differential equations with time-periodic coefficients. These equations, in general, are nonlinear, but under appropriate conditions, the equations of motion linearized about a stable periodic orbit or an equilibrium position determine the local stability of the system. The problem dealt with in this paper is that of control of instability in linear, dynamical systems with time-periodic coefficients.

Tremendous computational problems arise when one tries to apply the traditional control strategies to a system with a large number of equations with time-periodic coefficients. These traditional control strategies may include optimal control techniques, sliding mode techniques, and so on. The computational problems posed can be of convergence of a particular method or complexities in simulating a real system. The straightforward application of optimal control techniques (Kwakernaak and Sivan, 1972) results in a time-varying Riccati differential equation that obviously has a large dimension for a large-scale system. The application of sliding modes and related control techniques (Utkin, 1992) also results in a time-varying differential Riccati equation (for computation of sliding hyperplane) that has an order equal to the dimension of the state vector. Furthermore, the control laws resulting from sliding modes methodology are nonlinear even for a linear system. The control problem associated with large-scale, time-periodic systems has not been attempted owing to these complexities. Since the coefficients are time periodic, the time-varying eigenvalues do not characterize stability of such systems and one has to resort to Floquet theory that gives necessary and sufficient conditions for stability.

This kind of computational burden is almost absent when the system is time invariant. Hence, a better approach would be to transform a system with time-periodic coefficients into a system having a form that is suitable for the application of time-invariant control techniques. The time-periodic system is transformed by means of Lyapunov-Floquet transformation so that the system matrix of the transformed system is time invariant while the input gain matrix remains time varying. To apply time-invariant order reduction and control techniques, a completely time-invariant auxiliary system is constructed. The auxiliary system, though time invariant, is large scale if the original dynamic system is large scale. It is usually not necessary to control all the modes of a large-scale system as some modes may already be stable. Also, controlling all the modes requires a large amount of control effort. Hence, it is of great advantage to reduce the order of the constructed auxiliary system and then to obtain time-invariant control laws for a reduced-order auxiliary system. These time-invariant control laws when transformed back to the original domain yield the required control action. This methodology circumvents the tedious computations required in case of previously mentioned techniques. The problem of order reduction of large-scale, time-invariant dynamic systems has received a lot of attention by researchers. Order reduction algorithms in time and frequency domain can be found in various books and publications. A comprehensive survey of such techniques can be found in Mahmood and Singh (1981) and Mahmood, Hassan, and Darwish (1985).

The aim of this paper is to provide a methodology for designing reduced-order controllers for general dynamical systems of the type

$$\begin{aligned} \dot{x} &= A(t)x + B(t)u \\ y &= C(t)x, \end{aligned} \tag{1}$$

where $A(t)$, $B(t)$, and $C(t)$ are T periodic matrices.

2. MATHEMATICAL BACKGROUND

Consider a time-periodic system in state space given by

$$\dot{x} = A(t)x; \qquad A(t+T) = A(t). \tag{2}$$

If $\Phi(t)$ is the STM (state transition matrix) of (2), then it satisfies equation (2). Also, $\Phi(t+T) = F\Phi(t)$, where F is the FTM (Floquet transition matrix). The FTM is essentially the STM computed at the end of a principal period. The stability of time-varying system (2) is completely characterized by the eigenvalues of FTM called *Floquet multipliers*. The system is asymptotically stable iff all the Floquet multipliers lie within the unit circle in the complex plane. The STM of (2) can be factored as $\Phi(t) = Q(t)e^{Rt}$, where $Q(t) = Q(t+2T)$ is the Lyapunov-Floquet (L-F) transformation matrix with period $2T$ and R is a constant, real matrix. For details on L-F transformation, the reader is referred to Yakubovich and Starzhinskii (1975). The application of Lyapunov-Floquet transformation (L-F transformation), $x(t) = Q(t)z(t)$, transforms system (1) into a time-invariant system:

$$\dot{z} = Rz. \tag{3}$$

Systems (2) and (3) have the same stability characteristics as they are related by an invertible and bounded transformation matrix. Computation of the STM and L-F transformation matrix is a formidable task for time-periodic systems of large dimensions. A general approach would be to use a matrix differential equation $\dot{\Phi} = A(t)\Phi$ satisfied by Φ and numerically compute the STM using a Runge-Kutta type algorithm. Recently, Sinha and Wu (1991) developed an efficient computational scheme using Chebyshev polynomials, which is much superior to traditional numerical integration, to find the STM in case of time-periodic systems. Butcher and Sinha (1996) further refined the algorithm for the systems with equations of motion given by a set of ordinary differential equations having periodic coefficients and arbitrary order. This approach was called the "Hybrid Formulation" and is used in the present paper. The details of computing $Q(t)$ and $\Phi(t)$ using Chebyshev polynomials can be found in Sinha, Pandiyan, and Bibb (1996).

3. MATHEMATICAL FORMULATION

3.1. State Feedback

Consider a control problem associated with a periodic system

$$\dot{x} = A(t)\,x + B(t)\,u, \tag{4}$$

where $(A(t), B(t))$ forms a stabilizable pair. In this case, $C(t) = I_{n\times n}$, where $I_{n\times n}$ is an $n \times n$ identity matrix. Applying $L - F$ transformation $x = Q(t)z$ to (4), we get

$$\dot{z} = Rz + Q^{-1}(t)\,B(t)\,u. \tag{5}$$

The application of order reduction algorithms available in the literature requires the system to be completely time invariant. For this purpose, an auxiliary system

$$\dot{\bar{z}} = R\bar{z} + \bar{B}v_a \tag{6}$$

is constructed. The choice of $\bar{B}$ can be arbitrary since there is no constructive algorithm to arrive at a particular form. However, rank of $\bar{B}$ should be the same as that of $Q^{-1}(t)B(t)$. The order reduction of system (6) can be carried out using a suitable algorithm in the time domain. Following Mahmood and Singh (1981), two possible algorithms in the time domain are modal decomposition and aggregation. In this paper, we apply aggregation (Aoki, 1968) that comprises the linear transformation of auxiliary state $\bar{z}$ by a time-invariant transformation $\hat{T}$. Thus, transforming (6) using $\bar{v} = \hat{T}\bar{z}$, we obtain

$$\dot{\bar{v}} = J_r\bar{v} + \hat{T}\bar{B}v_a. \tag{7}$$

At this point, it is important to understand the structure of the order-reduction transformation matrix $\hat{T}$. It would be designed to incorporate the dominant modes of (6) into (7). If the time-invariant system is unstable, then the application of aggregation should retain all the unstable modes of the system in the reduced-order model. For such a system, $\hat{T}$ is given as a product $\bar{T}M^{-1}$, where M is a modal matrix of R and $\bar{T} = I_{r\times n}$ is a matrix of 0s and 1s. For all $j = 1, \ldots, n$, 1 in the jth column indicates that the jth mode has been carried over from system (6) to (7).

The state feedback law $v_a = -\bar{K}\bar{v}$ can be designed such that system (7) is asymptotically (exponentially) stable. The substitution of the control law in (7) and its subtraction from (6) gives the equation for error dynamics

$$\dot{e} = \left(R - \bar{B}\bar{K}\bar{T}M^{-1}\right)e + Q^{-1}Bu + \bar{B}\bar{K}\bar{T}M^{-1}z, \tag{8}$$

where $e = z - \bar{z}$ is an error variable. At this point, a suitable $\bar{K}$ is chosen from the reduced-order system (6) to satisfy two important conditions: (1) $(R - \bar{B}\bar{K}\bar{T}M^{-1})$ has all the eigenvalues with negative real parts. (2) The norm of the time-varying term $Q^{-1}Bu + \bar{B}\bar{K}\bar{T}M^{-1}z$ has to be as minimum as possible. The selection of $\bar{K}$ from the reduced-order system can be done by well-known algorithms like pole placement or quadratic cost minimization (optimal control). For the second condition to be fulfilled, the sum $Q^{-1}Bu + \bar{B}\bar{K}\bar{T}M^{-1}z$ has to be minimized because $B(t)$ appearing in the first term of the sum is a rectangular matrix. This idea has been suggested by Sinha and Joseph (1994) and used by Boghiu, Sinha, and Marghitu (1998).

We define an error vector $\xi(t) = B(t)u + Q(t)\bar{B}\bar{K}\bar{T}M^{-1}z$, and $u(t)$ is computed to minimize a performance index $\xi^T W\xi$, where W is a symmetric, positive-definite matrix of appropriate weights. The minimization of this performance index with respect to $u(t)$ yields the weighted least square solution for control action $u(t)$

$$\frac{\partial\left(\xi^T W\xi\right)}{\partial u} = B^T(t)\,W\left(B(t)\,u(t) + Q(t)\,\bar{B}\bar{K}\bar{T}M^{-1}z\right) = 0, \tag{9a}$$

or

$$u(t) = -\left(B^T(t)\,WB(t)\right)^{-1}B^T(t)\,WQ(t)\,\bar{B}\bar{K}\bar{T}M^{-1}z. \tag{9b}$$

When the weighting matrix is chosen to be the identity matrix,

$$u(t) = -B^{\#}Q\bar{B}\bar{K}\bar{T}M^{-1}z \tag{10a}$$

or

$$u(t) = -B^{\#}Q\bar{B}\bar{K}\bar{T}M^{-1}Q^{-1}x. \tag{10b}$$

The resulting closed-loop system is

$$\dot{x} = (A(t) - B(t)\,K(t))\,x, \tag{11}$$

where $K(t) = (B^{\#}Q\bar{B}\bar{K}\bar{T}M^{-1}Q^{-1}(t)$. The conditions for asymptotic stability of this system are discussed in Section 6.

3.2. Output Feedback

The control problem associated with equation (1) when $C(t)$ does not have rank equal to the dimension of the state vector is the output feedback problem. The pair $(A(t), B(t))$ is assumed stabilizable, and the pair $(A(t), C(t))$ is assumed to be detectable. The general theory of observer design is applicable here. The observer constructed here will be reduced order in the sense that it reconstructs only the states required by the reduced-order controller. The observer is a dynamical system given by

$$\begin{aligned} \dot{\hat{x}} &= A(t)\hat{x} + B(t)u + G(t)(y - \hat{y}) \\ \hat{y} &= C(t)\hat{x} \end{aligned} \tag{12}$$

such that the estimation error dynamics equation given by $\dot{\hat{e}} = (A(t) - G(t)C(t))\hat{e}$, where $\hat{e} = x - \hat{x}$ is exponentially stable. The observer gain matrix $G(t)$ is obtained using the *separation principle* and *duality*. The *dual system* (Kwakernaak and Sivan, 1972) of (1) is defined as

$$\begin{aligned} \dot{\tilde{x}} &= A^T(t)\tilde{x} + C^T(t)\tilde{u} \\ \tilde{y} &= B^T(t)\tilde{x}. \end{aligned} \tag{13}$$

The observer gain matrix $G(t)$ is given by the transpose of the controller gain matrix obtained for the first equation in (12). Assuming that there exists a $\tilde{u} = -\tilde{K}(t)\tilde{x}$ such that the resulting closed-loop system is exponentially stable, then $G(t) = \tilde{K}^T(t)$. The controller for the dual system is designed exactly in the same manner as described in the previous section. The controller is designed, following the same steps, and the state feedback is written in terms of the estimated state. It should be noted that the observer designed here is reduced order since it is actually the reduced-order controller for the dual system. Therefore, the modes that are retained in the reduced-order system of the observer are the same as those retained in the reduced-order system of the controller.

The resulting closed-loop system is called the *interconnected system* and is given by

$$\begin{aligned} \dot{x} &= A(t)x - B(t)K(t)\hat{x} \\ \dot{\hat{x}} &= G(t)C(t)x + (A(t) - B(t)K(t) - G(t)C(t))\hat{x}. \end{aligned} \tag{14}$$

The conditions for asymptotic stability of this system depend on asymptotic stability of the controller and the observer. These conditions are discussed in Section 6.

4. APPLICATIONS

The methodology devised in the previous section is applied to an inverted five-mass pendulum shown in Figure 1. The equations of motion can be written as

$$M_m\ddot{\eta} + K_k(t)\eta = \hat{B}(t)u(t). \tag{15}$$

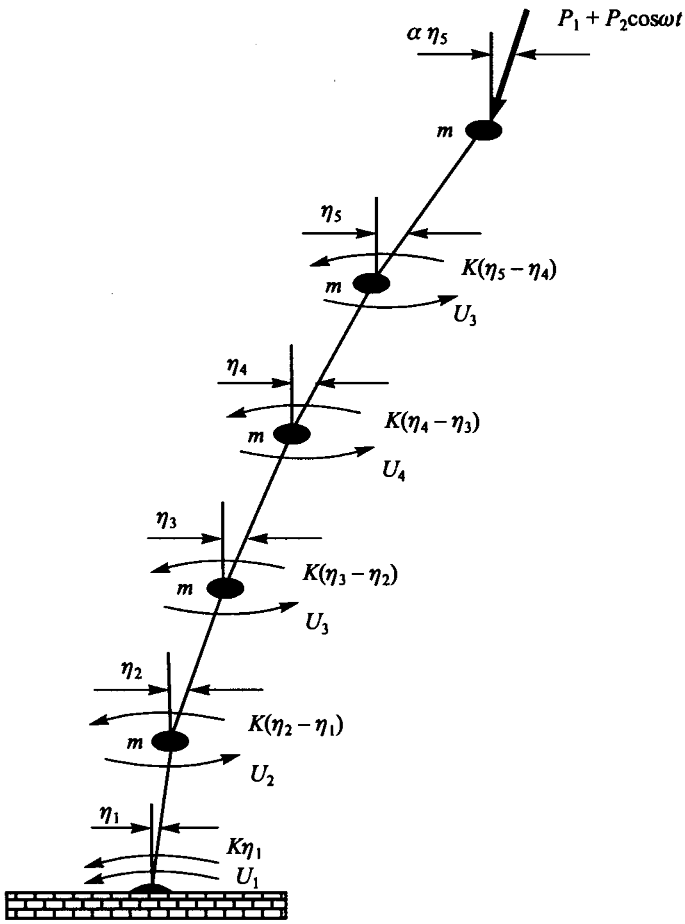

Figure 1. Structural diagram of five-mass inverted pendulum.

It has actuators for applying control torques at all the joints as shown. Hence, the input gain matrix $\hat{B}(t) = I_{5\times5}$, which may not be the case in general. M_m is a 5×5 mass matrix, and $K_k(t)$ is a 5×5 time-periodic stiffness matrix. The semi-follower, time-periodic load acting on the top of the pendulum contributes to the time-periodic terms of stiffness matrix. $\eta_{5\times1}$ is the vector of angular displacements of joints. M_m and $K_k(t)$ are given as

$$M_m = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 4 & 3 & 2 & 1 \\ 3 & 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$K_k(t) = \begin{bmatrix} \bar{k}(2-\bar{p}) & -\bar{k} & 0 & 0 & \bar{p}\bar{k}a \\ -\bar{k} & \bar{k}(2-\bar{p}) & -\bar{k} & 0 & \bar{p}\bar{k}a \\ 0 & -\bar{k} & \bar{k}(2-\bar{p}) & -\bar{k} & \bar{p}\bar{k}a \\ 0 & 0 & -\bar{k} & \bar{k}(2-\bar{p}) & \bar{k}(\bar{p}a-1) \\ 0 & 0 & 0 & -\bar{k} & \bar{k}(1-\bar{p}(1-a)) \end{bmatrix},$$

where $\bar{p} = (p_1 + p_2\cos(\omega t))\frac{l}{k}$ and $\bar{k} = k/ml^2$.

Equation (17) can be rewritten in the state-space form

$$\dot{x} = A(t)x + B(t)u$$

$$y(t) = C(t)x$$

$$A(t) = \begin{bmatrix} [0]_{5\times5} & [I]_{5\times5} \\ [M_m^{-1}K_k(t)]_{5\times5} & [0]_{5\times5} \end{bmatrix}, \quad B(t) = \begin{bmatrix} [0]_{5\times5} \\ [M_m^{-1}]_{5\times5} \end{bmatrix}.$$

$C(t) = [I_{10\times10}]$ for the full-state feedback problem, and $C(t) = [I_{5\times5}\,|0_{5\times5}|]$ for the output-feedback problem since all the states are not measured.

The values chosen for two typical parameter sets are as follows:

$$\text{set I} \quad \bar{k} = 2.0, \ \frac{p_1 l}{k} = 2.0, \ \frac{p_2 l}{k} = 1.2, \ a = 0.5, \ \omega = 2.0$$

$$\text{set II} \quad \bar{k} = 2.0, \ \frac{p_1 l}{k} = 2.0, \ \frac{p_2 l}{k} = 1.2, \ a = 1.0, \ \omega = 1.0.$$

For these sets, the existence of *Floquet exponents* (eigenvalues of matrix R, which is the system matrix for L-F transformed equations) with positive real parts indicates that the system is unstable because of parametric resonance.

- The Floquet exponents for parameter set I are ± 3.5756, $1.8747 \pm 0.3328i$, $-1.8747 \pm 0.3328i$, $\pm$ 1.8532, $-2.1028 \times 10^{-9} \pm 1.0625i$
- The Floquet exponents for parameter set II are $5.3268 \pm 0.7332i$, $-5.3268 \pm 0.7332i$, ± 3.3825, $1.5813 \times 10^{-8} \pm 0.5699i$, $-2.4898 \times 10^{-9} \pm 0.7175i$

Note that these have symplectic properties (Yakubovich and Starzhinskii, 1975) because of the fact that the system is Hamiltonian. The dominant modes of the system for each parameter set are taken as the modes corresponding to the Floquet exponents that have positive real parts as well as those with real parts very close to zero. In the case of set I, there are four exponents having positive real parts and two having real parts very close to zero. These numbers are 3 and 4 for set II. The Floquet exponents retained in the reduced-order model for parameter set I are 3.5756, $1.8487 \pm 0.3328i$, 1.8532, $-2.1028 \times 10^{-9} \pm 1.0625i$, and for parameter set II are $5.3268 \pm 0.7332i$, 3.3825, $1.5883 \times 10^{-8} \pm 0.5699i$, $-2.4898 \times 10^{-9} \pm 0.7175i$.

The algorithm used for order reduction is aggregation as given in Aoki (1968) (or Mahmood and Singh, 1981) and as described earlier in Section 3 of this paper. The order

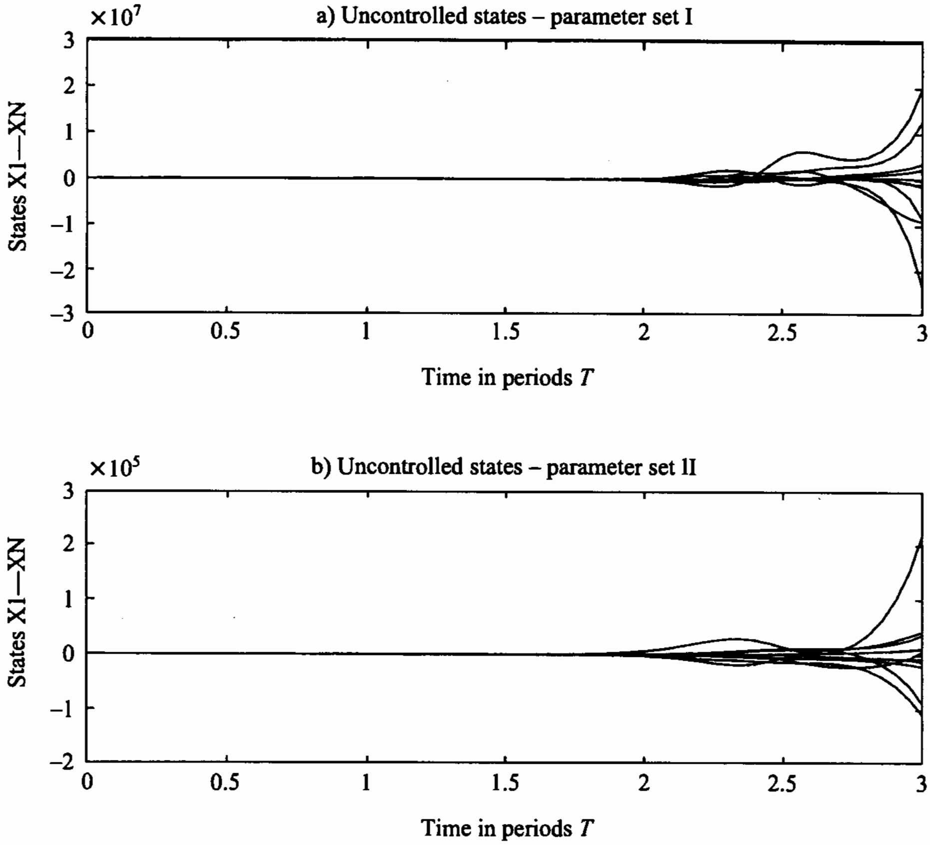

Figure 2. Uncontrolled trajectories: (a) parameter set I; (b) parameter set II.

of the reduced-order model for controller and observer in set I is 6 and that for set II is 7. This indicates that for parameter set I, a sixth-order model could be used to construct the control and state estimation gain matrices for the original tenth-order time-periodic system. For parameter set II, a seventh-order model is used for the same purpose. The time-invariant controller and observer gain matrices can be computed using the well-known algorithms such as pole placement or quadratic cost minimization.

The uncontrolled trajectories corresponding to parameter sets I and II are shown in Figures 2(a) and 2(b). The controlled trajectories for regulation to zero state in case of full-state feedback and output-feedback problems are shown in Figures 3(a) and 3(b) and 4(a) and 4(b) for sets I and II, respectively. The controlled trajectories of the unstable system are self-explanatory in establishing the effectiveness of the proposed methodology as the trajectories of the unstable system are controlled in four to five natural periods of the system. The time-invariant control and estimation gains for the results shown are $\bar{B}\bar{K} = M\bar{T}^{\#}[diag(15)]_{r\times r}$ for all the controllers designed and $\bar{B}\bar{K} = M\bar{T}^{\#}[diag(25)]_{r\times r}$ for all the observers designed, r being the order of the reduced-order model. Without loss of generality, we can choose matrix product $\bar{B}\bar{K}$ instead of just $\bar{K}$, owing to the flexibility in the choice of $\bar{B}$ as mentioned

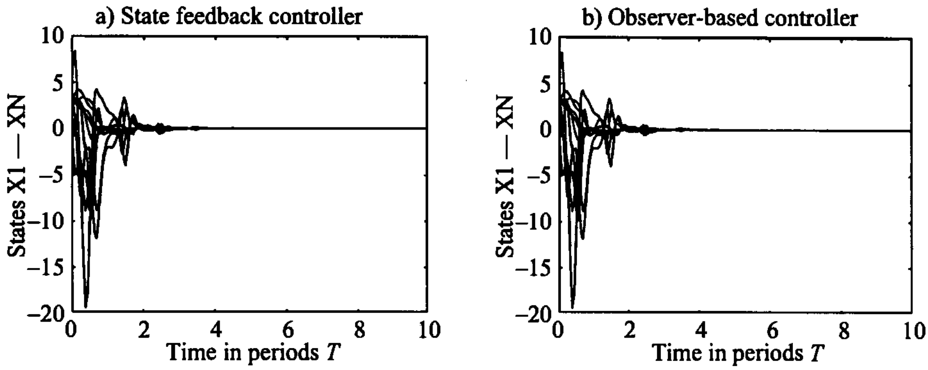

Figure 3. Controlled trajectories for parameter set I; (a) and (b) regulation.

in Section 3. In the present case, the time-invariant gain matrix $\bar{B}\bar{K}$ is designed using a standard pole-placement algorithm. The system is found to be Floquet stable for these values of controller and observer gains. For periodic systems, asymptotic stability is equivalent to Floquet stability, which means that the closed-loop systems represented by equations (11) and (14) should have all the Floquet exponents with negative real parts. This is exactly the type of stability achieved in the examples presented. Although this methodology appears to have worked well for general systems, some theoretical aspects of the stabilization problem deserve further clarification and remarks. These are presented in the next section.

5. REMARKS

It is observed that the closed-loop system given by equation (11) is asymptotically stable iff its $L-F$ transformed version is asymptotically stable. The same is true for equation (14). Applying L-F transformation $x(t) = Q(t)z(t)$ to equation (11) or equation (14), one obtains an equation of the form

$$\dot{z} = \bar{A}z + B'(t)\,z, \tag{16}$$

where $\bar{A}$ is the system matrix and $B'(t)$ is a bounded periodic matrix. System (16) should be considered as $L-F$ transformed closed-loop system (11) or (14). Since the two systems are related by a time-varying, bounded, invertible transformation (Lyapunov transformation), the stability characteristics of both the systems are identical. Therefore, it suffices to study the stability of equation (16). Equation (16) can be integrated to yield

$$z(t) = e^{\bar{A}t} z(t_0) + \int_{t_0}^{t} e^{\bar{A}(t-\tau)} B'(\tau)\, z(\tau)\, d\tau\,. \tag{17}$$

Now, if all the eigenvalues of $\bar{A}$ have negative real parts, the largest being $-a(a > 0)$, and $\| B'(t) \| < c_1$, and if $c_1 < a$, then according to a theorem by Bellman (1953), system (16) is asymptotically stable. This theorem is also known as small gain theorem (Vidyasagar and

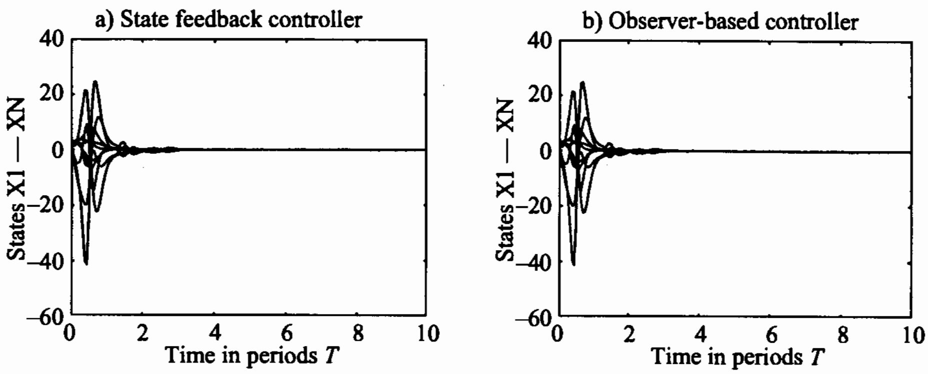

Figure 4. Controlled trajectories for parameter set II; (a) and (b) regulation.

Desoer, 1975). The application of this theorem to closed-loop systems obtained earlier is straightforward. Going back to closed-loop system (11), we have

$$\dot{z} = Rz - Q^{-1}BB^{\#}Q\bar{B}\bar{K}\bar{T}M^{-1}z. \tag{18}$$

Following the approach by Lee and Balas (1998), equation (18) becomes

$$\dot{z} = \left(R - \bar{B}\bar{K}\bar{T}M^{-1}\right)z + \Delta\left(t\right)z, \tag{19}$$

where $BB^{\#} = I_{n\times n} + \Delta_B\left(t\right)$ and $\Delta(t) = Q^{-1}\Delta_B\, Q\bar{B}\bar{K}\bar{T}M^{-1}$. Equation (21) has the same form as equation (18). Now, if $(R - \bar{B}\bar{K}\bar{T}M^{-1})$ and $\Delta(t)$ satisfy the conditions stated in the theorem, then system (20), which is the closed-loop system, is asymptotically stable. It has been mentioned before that the choice of time-invariant gain matrix $\bar{K}$ is critical. The eigenvalue with the largest real part of $(R - \bar{B}\bar{K}\bar{T}M^{-1})$ and the norm of $\Delta(t)$ compete against each other in establishing the stability conditions. Following Keel, Bhattacharyya, and Howze (1988), a search algorithm for an appropriate $\bar{K}$ can be devised. The search algorithm is based on the optimization of a cost function that arises from the process of guaranteeing stability by Lyapunov's second method. The cost function depends on $\bar{K}$, and a minimal value of the cost functional gives optimal $\bar{K}$, which is required to perform its twofold role. This algorithm is ad hoc but nevertheless useful. Implementation of these algorithms has not been pursued in this study, partly owing to the fact that these provide sufficient conditions only, and therefore the results are bound to be very conservative. However, necessary and sufficient conditions for systems with time-periodic coefficients are provided by Floquet theory. Therefore, the approach adopted to achieve stable closed-loop systems has been one of trial and error. Starting from a guessed $\bar{K}$, its value is updated iteratively until a closed-loop system, asymptotically stable in the Floquet sense, is obtained. It is planned to devise an optimal constructive algorithm to guarantee exponentially stable closed-loop performance.

6. CONCLUSIONS

The problem of order reduction of large-scale systems with time-periodic coefficients is posed and solved in a systematic fashion for the first time in the literature. The solution to the problem presents a practical method of controller designs for large-scale time-periodic systems. Control of time-periodic systems via order reduction is an elegant straightforward scheme for designing full-state feedback controllers or observer-based controllers as it incorporates the advantages of both L-F transformation and order reduction. The approach uses the idea of transforming a time-periodic system into a time-invariant one using Lyapunov-Floquet transformation followed by order reduction. The simplicity in computing the control laws is the direct consequence of this idea. The generalizations of this technique are conceptually obvious and simple. This methodology can be generalized for nonlinear large-scale systems that are not critical using normal form theory, provided we have efficient algorithms to perform near identity transformations. The computation of control laws is just a matter of multiplying matrices, which makes the scheme really suitable for large-scale applications in real time.

Acknowledgments. Financial support provided by the National Science Foundation (Grant no. 9713971) is gratefully acknowledged. The computer time on CRAY C90 provided by the Alabama Supercomputer Authority is also acknowledged.

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