Non-conservative oscillations of a tool for deep hole honing

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Abstract

The dynamics of a rotating tool, commonly employed in deep hole honing, is considered. A mathematical model of the process including a dynamic representation of tool, workpiece surface and honing stones interaction is suggested and analyzed. It is shown that interaction forces are non-conservative. The honing tool is modeled as a rotating continuous slender beam with a mandrel attached at an intermediate cross-section. The transverse oscillations of tool shaft cause variation in expansion pressure and change the interaction forces on the surface of the workpiece. The interaction forces are nonlinear and non-conservative in the general case including delayed functions. The expansion pressure of stones turns out to be a critical parameter in a honing process. Since the removal of stock increases linearly with pressure, the productivity can be improved by increasing expansion pressure, but in certain conditions may cause dynamic instability of the tool shaft. On the other hand, the lower expansion pressure and feed rates increase accuracy. Another source of shaft instability is due to the asymmetry of the tool that leads to discrepancy of the system stiffness in the transverse directions. As a result, under certain conditions unstable parametric vibrations may occur. Needless to say, the dynamic behavior of the tool can considerably influence the machined surface formation. In this study a model of honing surface formation is introduced and integrated into the model of process simulation. The system response is studied for different parameters in a non-dimensional form.

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Thus it is possible to analyze a set of real processes by applying the similarity conditions. The variation in the machined surface having various initial discrepancies from ideal cylinder is also analyzed. © 2004 Elsevier B.V. All rights reserved.

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1. Introduction

The machining of deep holes is one of the most complicated manufacturing operations. In particular, this process is used for manufacturing high depth pumps, high strength tubes of reactor batteries, and devices for polyethylene extrusion under high pressure. The central geometric features of such products are long cylinders with precise high quality deep holes with length-to-diameter ratios of 200–250. The most widely used operation for high precision manufacturing of long cylindrical details is honing on a horizontal-honing machine tool. The traditional setup has a rotating tool shaft engaged in reciprocal translation and loaded transversely, and in tension or compression also. In the last decade, honing has become a process better described as bore finishing, because stock removal rates have increased substantially. It is now practical to remove 0.4 mm (0.016") or more during rough honing from hard steel bore [1]. This fact results in considerable increase of cutting forces acting on the tool.

The main feature of honing is that the machined surface is mutually based on honing stones of the tool. It is ensured by a special tool design (with additional degrees of freedom) that provides its self-adjustment. Common honing tool designs include a single stone (Fig. 1) or multiple stones (Fig. 2), or consist of Krossgrinding tool designs [2]. Experimental research shows a direct correlation between surface quality and the number of tool stones [3,4]. Multi-stone tools are used for bores of 15–150 mm diameter and have multiple contact points in order to provide improved force distributions, better geometry, longer tool life, and (possibly) faster stock removal. Cone-fed expansion mechanisms or coolant-fed tools are commonly used, providing constant contact of guide shoes and stones with the workpiece surface. Contact forces of the tool stones and workpiece are non-conservative and their resultant acts upon the rotating hone shaft. The magnitude of this load may be large enough to cause dynamic instability. Stabilization of the dynamic system behavior is one of the main conditions of high precision attainment.
Another problem is that, if the honing stones are arranged asymmetrically, then the cutting forces acting on a shaft are time-varying due to the rotation of the tool. In this case dynamic instability may occur due to parametric resonance. Moreover, if the tool is non-eccentric, the shaft may precess if the rotation speed is near the eigenfrequency of the shaft. Therefore, the frequencies of tool vibration become important in analyzing the dynamic behavior.

The experimental research of honing forces variation and their influence on surface geometry are presented in [3,4]. Many researchers have studied the honing process, but deep hole honing dynamics have not yet been analyzed. Kim and Choi [5] carried out a frequency analysis of a honing system, but a continuous tool was modeled as a discrete system, combining the mandrel, single honing stone and two guide shoes. The authors did not consider the non-conservative contact forces. The influence of axial loads on the eigenvalues of a rotating long shaft with an intermediate disk was analyzed in [6]. The modeling of continuous multi-stone honing tool dynamics under non-conservative interaction forces is presented in [7]. The frequency analysis of continuous single stone honing tool is given in [8].

In this paper the honing tool is considered as a continuous rotating system. Honing forces are calculated as a result of the non-conservative interaction of the stones and the machined surface. The model of the honing tool and shaft vibrations are reduced to a system of linear differential equations with time-periodic coefficients. The model of new surface formation is introduced in order to calculate the surface geometry variation. This model include time-delayed displacement and, in general, it is nonlinear when cutting is discontinuous. The full model of tool dynamics is combined of the tool and shaft vibrations model; model of cutting forces and model of new surface formation. The system is nonlinear, with variable time-delayed functions and time periodic coefficients. It can be analyzed numerically, only.

2. Model development

Honing operations are such that the tool can be simulated as a shaft with intermediate rotating disk (mandrel) of finite width and diameter with mass, rotational and spin inertia. The schematic diagram of the model is shown in Fig. 3. The tool for deep hole honing is the system of abrasive stones and guiding shoes distributed along the cylindrical surface with radius, $R$, and length, $l_0$. 
These stones are pressed to the machined surface in the radial direction by a special cone expansion mechanism that is modeled as spring of stiffness $k$ and equivalent damping $c$ (Fig. 4). The interacting forces at the tool/workpiece interface are also shown in Fig. 4.

We assume that the tool can be modeled as a flexible rotating shaft with a point disk (hone) of mass $M_D$ with rotation and spin inertia $J_1$ and $J_0$, respectively. The disk and shaft rotate with angular velocity $\omega$ and move in the axial direction with constant feed rate $V_0$. The shaft is considered absolutely rigid axially, and therefore its axis has only displacements in the transverse direction. This transverse displacement changes the interaction forces between the stones and the disk. As we assume that the stones do not vibrate significantly, the interaction forces between the stone and the machined surface are the same as between the stone and the disk. These forces are distributed on the external honing stone surface and depend upon the shaft displacement and bending angle. We assume that cutting loads can be presented as the normal pressure $p$ and tangential distributed force $\tau_\Sigma$. The tangential force $\tau_\Sigma$ acts like dry friction in the direction opposite to the relative velocity $V_\Sigma$ of the tool/workpiece contact point. The magnitude of this load is considered to be proportional to the normal pressure, $p$, at this point as follows:

$$\tau_i = \tau_\Sigma \cos \beta = f_i p \cos \beta$$

$$\tau_z = -\tau_\Sigma \sin \beta = -f_z p \sin \beta$$  \hspace{1cm} (1)
where $\tau_r, \tau_z$ are projections of the distributed interacting forces on the circumferential direction and $z$ axis as shown in Fig. 4. $\beta$ is angle between the direction of contact point velocity and the tangent direction ($\sin \beta = V_0 / V_z$) and $f_t$ is cutting force coefficient.

The cutting force coefficient $f_t$ is assumed to be approximately equal to the dry friction factor. In turn, the normal pressure is proportional to the displacement of the shaft axis given by

$$p = p_{01} + k(u + \partial_x z) \cos \varphi + k(v - \partial_y z) \sin \varphi$$  \hspace{1cm} (2)

where $p_{01}$ is initial normal expansion pressure of hone for axial position of shaft; $k$ is the equivalent stiffness of the honing stone; $u, v$ are the displacements of the shaft in the transverse directions referred to the coordinate system rotating together with hone; $\partial_x, \partial_y$ are the angles of shaft axis rotation with respect to the $x$ and $y$ axes, respectively.

We consider that the stone length $l_0$ is significantly less than the shaft length, $L$. Then the distributed forces interacting between the tool and the workpiece surface are changed by the resultant force and moment applied at the centroid of the disk. In the case of a multi-stone tool, we derive the equation for the resultant by integrating the distributed forces along the cylindrical surface of the tool. If the tool has three or four sticks the integration can be replaced by simple summation. It follows from the conditions of force equilibrium acting in the $x, y$ and $z$ directions that

$$P_x = \int_{-l_0/2}^{l_0/2} \int_{0}^{2\pi} (p_x + \tau_x) R d\varphi dz = f_t Rk\pi l_0 v \cos \beta - Rk\pi l_0 u$$

$$P_y = \int_{-l_0/2}^{l_0/2} \int_{0}^{2\pi} (p_y + \tau_y) R d\varphi dz = -Rk\pi l_0 v - Rk\pi l_0 f_t u \cos \beta$$  \hspace{1cm} (3)

$$P_z = \int_{-l_0/2}^{l_0/2} \int_{0}^{2\pi} \tau_z d\varphi dz = -2f_t p_{01} R l_0 \pi \sin \beta$$

and it follows from the moment analysis that

$$T_x = \int_{-l_0/2}^{l_0/2} \int_{0}^{2\pi} (p_x z - \tau_z y + \tau_z v) R d\varphi dz = -\frac{Rk\pi l_0^3}{12} \partial_x + \frac{f_t Rk\pi l_0^3 \cos \beta}{12} \partial_y - R^2 k\pi l_0 f_t v \sin \beta$$

$$T_y = \int_{-l_0/2}^{l_0/2} \int_{0}^{2\pi} (p_y z - \tau_x y + \tau_x u) R d\varphi dz = -\frac{Rk\pi l_0^3}{12} \partial_y - \frac{f_t Rk\pi l_0^3 \cos \beta}{12} \partial_x + R^2 k\pi l_0 f_t u \sin \beta$$

$$T_z = \int_{-l_0/2}^{l_0/2} \int_{0}^{2\pi} (-p_y y + p_x x - \tau_x y + \tau_x u) R d\varphi dz = -2f_t \pi R^2 l_0 p_{01} \cos \beta$$  \hspace{1cm} (4)

In the case of a single stone tool (Fig. 5), the projections of the resultant radial interacting force are derived as

$$P_x = -2c \sin^2 \theta \frac{\partial u}{\partial t} - 2k \sin^2 \theta + k \sin \theta (L_2 - L_3)$$

$$P_y = -c (1 + 2 \cos^2 \theta) \frac{\partial v}{\partial t} - k (1 + 2 \cos^2 \theta) v - k \cos \theta (L_2 + L_3) + k L_1$$  \hspace{1cm} (5)
where $P_x$, $P_y$ are projections of the resultant radial interacting force; $c$ is equivalent radial damping factor of honing stone; $u$, $v$ are the displacements of the shaft in the transverse direction referred to the rotating axes; $L_j$ is discrepancy between the surface radius under $j$th stone and radius of ideal cylinder; $\theta$ is angle of tool guiding shoes relative disposition.

In the relations (5) the additional terms due to the machined surface discrepancy from ideal cylindrical surface are taken into account. The derivation of the discrepancy of surface radius in detail later is considered.

It follows from the conditions of equilibrium that

$$T_z = -2p_0 f_k R \cos \beta (\cos \theta + 1)$$
$$P_z = \mp p_0 f_k \sin \beta (\cos \theta + 1)$$

where $A_k$ is the stone contact surface area.

It can be seen from Eqs. 3, 4, 6 that the axial component of the resultant force $P_z$ and moment $T_z$ are independent of the transverse displacement of the shaft. Therefore, the transverse and torsional vibrations of the tool shaft can be analyzed separately. The dynamic behavior of the tool is fully defined by the transverse vibrations of the shaft under constant axial force and moment, and the non-conservative transverse loads are linearly dependent on the displacement and rotation of the shaft axis.

Consider the flexural vibration of a shaft of circular cross-section, rotating about its own axis with angular velocity $\omega$. The shaft is subjected to a resultant force, $P$, and a moment, $T$, at the center of the disk mass. The equations of shaft vibration subjected to concentrated loads have been derived in [9, 10]. Complex eigenvalues have been analyzed by Lee and Yun [11] in the case of axial force and moment with constant magnitude. The equations of motion of a rotating shaft with a rigid, intermediate disk subjected to compressive load and torque have been derived by Voronov and Fatalchuck [6] and analyzed by the Galerkin method. This analysis considers the equations of transverse vibrations of a rotating beam with intermediate disk loaded by a resultant force and moment in a moving basis $\{e_1, e_2, e_3\}$ attached to the beam axial line relative to the stationary rotating straight axis (see Fig. 6). The equations as recorded in reference [10] are
where \( s \) is coordinate along shaft axis; \( s_1 \) is coordinate of disk; \( m_0 \) is shaft mass per unit length; \( u_2 \), \( u_3 \) are deflections of axial line in attached basis \( \{e_1,e_2,e_3\} \); \( \vartheta_2 \), \( \vartheta_3 \)—slope with respect to \( e_2 \), \( e_3 \) axes, respectively; \( \Delta Q_2 \), \( \Delta Q_3 \), \( \Delta M_2 \), \( \Delta M_3 \) are internal transverse forces and bending moments; \( EJ_{22} \), \( EJ_{33} \) bending rigidities of shaft; \( GJ_{11} \) is torsional rigidity of shaft; \( P_z, T_z \) are axial components of resultant interacting force and moment; \( P_2, P_3 \) are transverse components of resultant interacting force and moment in basis \( \{e_1,e_2,e_3\} \); \( H(s - s_1) \) is Dirac Delta function; \( H(s - s_1) \) is Heavyside function.

The rotating shaft implies that the interaction forces and moments are also rotating such that the components of force \( P \) and moment \( T \) derived in the basis \( \{x,y,z\} \) should be reduced to the basis \( \{e_1,e_2,e_3\} \). Relations between the coordinate systems are as follows:

\[
\begin{align*}
\vartheta_2 &= -\frac{\partial u_1}{\partial s} \quad \vartheta_3 = \frac{\partial u_2}{\partial s} \\
\vartheta_2 &= \cos \varphi \quad \vartheta_3 = \sin \varphi
\end{align*}
\]

where \( \varphi = \omega t \). Therefore, after substituting these relations equations (7) will include time periodic coefficients.

However, in the case of axisymmetrical arrangement of stones, the time-varying coefficients vanish.

Considering that the bending rigidities \( EJ_{22} \) and \( EJ_{33} \) are equal for a circular shaft and introducing dimensionless complex variables, we can simplify the equation of shaft lateral vibration to
\[
\begin{align*}
\frac{\partial^2 U}{\partial \tau^2} - \gamma \frac{\partial^4 U}{\partial \tau^2 \partial \xi^2} + 2iv_1 \frac{\partial^3 U}{\partial \tau \partial \xi^2} + \alpha \frac{\partial U}{\partial \tau} + \frac{\partial^4 U}{\partial \xi^4} - iM_{10} \left[ 1 - H(\xi - \xi_1) \right] \frac{\partial^3 U}{\partial \xi^3} \\
- \tilde{Q}_{10} \left[ 1 - H(\xi - \xi_1) \right] \frac{\partial^2 U}{\partial \xi^2} = P(\tau) \delta(\xi - \xi_1)
\end{align*}
\]

where

\[
\begin{align*}
\xi &= \frac{s}{L}; \quad U = u_2 + iu_3; \quad \vartheta = \vartheta_3 - i\vartheta_2; \quad \tau = p_0 t; \quad p_0 = \sqrt{\frac{EJ_{33}}{m_0 L^4}}; \quad v = \frac{\omega}{p_0}; \\
\gamma &= \left[ \frac{J_{33}}{AL^2} + \frac{J}{m_0 L^2} \delta(\xi - \xi_1) \right] \\
\tilde{M}_{10} &= \frac{T_z L}{EJ_{33}}; \quad \tilde{Q}_{10} = \frac{P_z L^2}{EJ_{33}}; \quad \tilde{P}(\tau) = (P_2 + iP_3)L^2/EJ
\end{align*}
\]

\(A\) is shaft cross-sectional area and \(\alpha\) is the generalized external damping factor.

In the case of initial ideal cylindrical surface machining Eq. (8) is homogeneous in spite of the presence of terms on the right-hand side. The relations \(P(\tau)\) include terms that are proportional to the displacement of the tool and have time-periodic coefficients. Applying the Galerkin method, we will find two-term approximate solution of Eq. (6) as

\[
U = \sum [f_2(\tau) + if_3(\tau)] \psi_j(\xi)
\]

where \(f_2(\tau), f_3(\tau)\) are unknown time varying functions and \(\psi_j(\xi)\) are coordinate functions satisfying all the boundary conditions. The frequencies and modes of axisymmetric tool were numerically calculated and are given in [7]. Here the eigenmodes of a non-rotated unloaded shaft are used.

For the simply supported beam \(\psi_j(\xi) = \sin(j\pi\xi)\), and for clamped–hinged beam

\[
\psi_j(\xi) = K_2(\lambda_j)K_3(\lambda_j\xi) - K_1(\lambda_j)K_4(\lambda_j\xi),
\]

where \(K_i\) are Krylov’s functions [10], \(\lambda_1 = 3.926, \lambda_2 = 7.068, \lambda_n = (4n + 1)\pi/4\).

After applying the Galerkin method we arrive at the set of ordinary differential equations

\[
M \ddot{f} + B(\tau) \dot{f} + C(\tau) f = D(\tau - \tau_f)
\]

where \(f = (f_{21}, f_{22}, f_{31}, f_{32})^t\) is a transposed vector of unknown functions, \(M\) is a constant matrix and \(B(\tau), C(\tau)\) are \(T\)-periodic matrices of numeric coefficients which depend on boundary conditions and magnitudes of system parameters.

Further

\[
B(\tau + T) = B(\tau); \quad C(\tau + T) = C(\tau)
\]

\(D(\tau - \tau_f)\) includes the delayed functions appearing due to dependence of the surface radius discrepancy from the ideal cylindrical surface on the position of stones as at present time and at time when the surface was formed at previous pass as well. The guiding stones are not cutting but supporting on the surface that was machined at a previous pass of cutting stone.
3. A model of new surface formation

Let us assume that the thickness of the cutting layer during one pass of the honing stone is insignificant in comparison with the shaft's transverse displacement. Therefore, it does not change the normal pressure of the interacting forces in the relation (2). In turn, the coordinate of the machined surface is derived on the basis of the Preston hypothesis as follows: the velocity of the machined cutting layer removal is proportional to the work done by the friction forces acting on an element of surface at that point [4] (Fig. 7), i.e.,

\[
\frac{dY}{dt} = p \frac{V_S}{k_y \sigma_0}
\]

where \( Y = Y(t) \) is the coordinate of the machined surface under the honing stone, \( p \) is the normal pressure at the honing stone, \( V_S \) is the resultant speed of movement of the honing stone, \( k_y \) is the dimensionless coefficient of the cutting layer removal which depends on the tool and workpiece material and geometry and is derived empirically, \( \sigma_0 \) is the characteristic strength of the machined material.

We assume that at each time instant we have point contact of the honing stone and workpiece surface and therefore we derive the coordinate of the machined surface as follows [12]:

\[
Y(t) = Y(t - T_J) + p \frac{V_S \Delta t}{k_y \sigma_0}
\]

where \( \Delta t \) is time of stone contact with the certain point of surface and \( t - T_J \) is time when this point of surface was cut at previous pass.

The normal pressure \( p \) must be always positive or zero but not negative in no circumstances. Thus the model of new surface formation is nonlinear. If initial compression \( p_{01} \) is greater than the increment due to shaft transverse vibration the honing stone cuts metal in accordance with (13). If stone loses contact with the surface the normal pressure vanishes and the surface coordinate remains unchanged. In order to begin calculations of \( Y(t) \) we need a prescribed initial surface coordinate \( Y_0 \) that can be specified as zero or surface with distortions or as random. In general, as the tool rotates and the workpiece has reciprocating axial motion, the coordinate

![Fig. 7. Schematic of machining by honing tool sticks.](image-url)
$Y(t)$ is a function of time. On the other hand, it is a function of the contact point of the workpiece surface $Y(Z, \phi)$ in the cylindrical coordinate system. While machining proceeds, the axial coordinate $Z$ and $\phi$ are varying in time: $Z = Z_0 + V_0 t$, $\phi = \phi_0 + \omega t$. Therefore, by integrating Eq. (11) in time we determine the position of shaft. Applying relation (2) we calculate normal pressure at honing stone and then recalculate by (13) the new generated surface coordinate under cutting stone at each step of integration. The discrepancies $L_j$ in (5) are to be determined as follows $L_j(t) = Y(Z, \phi)$. As surface was formed at previous pass of honing stone we obtain equations with time delay, that is in general variable. If the axial speed is equal to zero, we can analyze the process of honing of the same surface $Z = Z_0$ for short cylinders without axial reciprocation and variation of the surface while processing. In this case $T_j$ is constant and is equal to the period of shaft rotation if cutting stone does not loose the contact with machining surface. This algorithm was implemented in MATLAB for numerical simulation of surface formation while machining.

4. Numerical simulation

4.1. Frequency analysis

The analysis of system eigenvalues is carried out by setting the right hand side of Eq. (11) equal to zero, which means that we assume that initial surface is ideal cylinder $Y_0 = 0$ and cutting is absent. We should put also $\phi = 0$ in Eq. (11) to exclude time periodic coefficients in case of non-axisymmetric tool. This yields constant matrices $B$ and $C$ and Eq. (11) reduces to

$$M \ddot{\phi} + B \dot{\phi} + C \phi = 0$$  

(14)

The eigenfrequencies of Eq. (14) are complex under the arbitrary case of load behavior and fixture conditions. Therefore, it is difficult to calculate the explicit values of the complex frequencies. In this study, the lower frequencies are of greatest interest, as they define the loss of stability. The algorithm of frequencies derivation is presented in [6,7] where they were determined numerically or applying approximate Galerkin method for different fixture conditions. This algorithm was implemented in MATLAB and the results are presented in the figures where the real and imaginary parts of the complex eigenvalues are plotted.

The numerical analysis shows the influence of initial expansion pressure $p_{01}$, equivalent stiffness of tool $k$, and rotational speed $\omega$ on the system eigenvalues. For numerical simulation of the system, the following typical set of data was used:

$$l_0 = 0.02 \text{ m}; \quad L = 1.1 \text{ m}; \quad d = 0.018 \text{ m}; \quad R = 0.024 \text{ m}; \quad \omega = 0 - 1000 \text{ rpm};$$

$$p_{01} = 0.5 - 10 \text{ MPa}$$

$$k = 0 - 1 \times 10^6 \text{ N/m}; \quad \alpha = 0 - 0.5; \quad f_c = 0 - 0.5; \quad s_1/l = 0.5 \text{ and } 0.7$$

By assuming $p_{01}$ and damping $\alpha$, $c$ and $f_c$ equal to zero, we can obtain eigenfrequencies of the non-loaded rotating shaft transverse vibrations. The two lower frequencies' dependence on rotational speed $\omega$ is presented in Fig. 8. In this case the system is conservative and eigenvalues have a zero
imaginary parts of eigenvalues; \( fr=0, c=0, s_1 = 0 \)

Fig. 8. Two first frequencies versus speed of rotation \((s_1 = 0.5—\text{solid line}, s_1 = 0.75—\text{dashed line})\).

real component and their imaginary components are equal to the frequencies of shaft transverse vibrations. The behavior at higher frequencies is shown in Fig. 9. As seen, two pairs of frequencies diverge near the corresponding frequency of the shaft without rotation and damping.

The influence of \( p_{01} \) and damping become more complex as the system becomes non-conservative. By increasing \( p_{01} \) we see the axial components of force and torque rise simultaneously, and due to the presence of torque, the system becomes non-conservative and the eigenvalues become complex (Figs. 10 and 11). It can be seen from the figures that the eigenvalues’ real parts can be negative or positive. In the case of a negative real component, we have stable motion, and if even one eigenvalue has a positive real part the motion is unstable with increasing amplitude. The system has a critical value of \( p_{01} = p^* \) at the border of stability. As for the eigenvalues’ imaginary components, they are decreasing with increasing \( p_{01} \). The critical value of \( p^* \) depends considerably on the system damping factor \( \alpha \), cutting force coefficient \( f_t \), and method of tool fixturing. The influence of the equivalent tool stiffness is shown in Figs. 12 and 13. In this case, the eigenvalues’ imaginary components (i.e. frequencies of vibrations) are increasing, but the real part may be positive or negative. By increasing \( k \) we also can achieve the critical value of tool stiffness \( k^* \), where shaft vibrations become unstable.

It should be noted that in case of an axially symmetric tool, the system becomes non-conservative for any value of initial stiffness due to the rise of the torque moment [9]. The situation is not the same in the case of single-stone tool. The tool has one honing stone and two guiding shoes disposed by an angle \( \theta \) from the vertical line. If the angle \( \theta \) is equal \( \pi/3 \), the tool is axially symmetric. For any other angle it is non-symmetric as it changes the relation of the tool rigidities in the transverse direction of axes \( x \) and \( y \). If \( \theta \) is varied from \( 0 \) to \( \pi/3 \), pairs of eigenfrequencies converge on each other and then diverge as \( \theta \) is increased from \( \pi/3 \) to \( \pi/2 \) (Fig. 14). At the value
Imaginary parts of eigenvalues; \( \omega = \omega_0 = p_{01} = 0 \)

Fig. 9. Third and forth frequencies versus speed of rotation \((s_1 = 0.5-\text{solid line}, s_1 = 0.75-\text{dashed line})\).

Eigen values; \( \omega = 500 \text{ rpm}, c = 0.1 \)

Fig. 10. Variation of system eigenvalues versus initial stiffness of tool \((x = 0.2, f_i = 0.2-\text{solid line}; x = 0.1, f_i = 0.2-\text{dashed line}; x = 0.2, f_i = 0.15-\text{asterisks})\).

\( \theta = \pi/3 \) the tool becomes symmetric with respect to the vertical axis and the eigenfrequencies become multiple. In the case of \( \theta \) close to \( \pi/3 \) and tool damping factor \( c = 0 \), the system becomes
Fig. 11. Variation of system eigenvalues versus initial stiffness of tool ($\alpha = 0.2, f_i = 0.2$—solid line; $\alpha = 0.1, f_i = 0.2$—dashed line; $\alpha = 0.2, f_i = 0.15$—asterisks).

Fig. 12. Variation of system eigenvalues versus hone stiffness factor $k$ ($\alpha = 0.2, f_i = 0.2$—solid line; $\alpha = 0.1, f_i = 0.2$—dashed line; $\alpha = 0.2, f_i = 0.15$—asterisks).
non-conservative as a non-zero positive part of the complex eigenvalues arise. If damping $c = 0.2$ the eigenvalues are complex but their real part is negative (Fig. 15), i.e., oscillations are damped.
The influence of other parameters on eigenvalues remains the same as those for the symmetrical tool. The frequencies and modes thus obtained are used in the following to perform a stability analysis of the system.

4.2. Dynamic stability analysis ($\phi = \omega t; D = 0$)

Let us consider now the case of non-axisymmetric tool vibrations. In this case we have to analyze Eq. (11), which contains time-periodic coefficients. Rayleigh–Hill's method [11], the perturbation technique [12], averaging and asymptotic methods [15], and the Lyapunov–Floquet method [16] are some of the commonly used mathematical methods in the analysis of linear periodic systems. It is well known that the Rayleigh–Hill approach is only suitable for determining the stability boundaries for such systems. The perturbation and averaging methods have their own limitations due to the fact that they can only be applied to relatively smaller systems where the periodic coefficients can be expressed in terms of small parameter. Therefore, Floquet analysis coupled with numerical integration of state transition matrix (monodromy matrix) or its analytical derivation by applying the solution expansion in Chebyshev polynomials [17] is employed.

In order to determine monodromy matrix we transform Eq. (11) to normal Cauchy form given by

$$\dot{X} = A(\tau) \cdot X$$

where $X = \{f_{21}, f_{22}, f_{31}, f_{32}, f_{31}, f_{32}, f_{33}\}^T$, $A(\tau)$ is a matrix composed of coefficients of Eq. (11) and is time periodic, i.e., $A(\tau + T) = A(\tau)$, where $T$ is the period of tool rotation $T = 2\pi/\omega$.

In order to analyze the dynamic stability we apply the Floquet method [14]. The fundamental solution matrix of Eq. (15) at time $T$ is determined. Its eigenvalues $\mu$ are called the Floquet
multipliers and in general they are complex. If the absolute value of the maximal multiplier is less than one the vibrations are stable. If at least one multiplier has absolute value $|\mu| > 1$ the system vibrations are unstable. The border of stability corresponds to the case when $|\mu| = 1$. We can also calculate the Lyapunov characteristic exponents as

$$\chi = \frac{1}{T} (\ln |\mu| + i \text{Arg} \mu)$$

(16)

If the characteristic exponents have negative real parts the system is stable. This algorithm was implemented in MATLAB®. The program plots the diagram of stability in the plane of two prescribed parameters. It is possible to follow how the multipliers and characteristic numbers change along the arbitrary line drawn on the plane of varying parameters.

The results of numerical analysis for some values of system parameters are presented in Figs. 16-21. The influence of tool stiffness, tool internal damping, and disposition of tool guiding shoes on the stability domain as a function of the rotation speed was observed. The instability regions are plotted for several varying parameters versus the frequency of rotation. Asterisks fill the unstable regions. In the Fig. 16 the instability regions are presented for the case when the internal damping of tool and external damping are absent. In this case the system is conservative and eigenvalues are completely imaginary: $\lambda_1 = 15.19 \cdot i; \lambda_2 = -15.19 \cdot i; \lambda_3 = 48.10 \cdot i; \lambda_4 = -48.10 \cdot i$.

It is seen from Fig. 16 that there exist a few instability regions. The domain of principal resonance is near the natural frequencies $p_1$ and $p_2$ ($p_1 = \lambda_1 = |\lambda_2|; p_2 = |\lambda_3| = |\lambda_4|$) and near the doubled frequency of excitation $2p_1$ and $2p_2$. In the figures asterisks fill the unstable region and it expands if the magnitude of the tool stiffness $k$ increases.

The variation of the multipliers while crossing the unstable domain near the first natural frequency $p_1$ is presented in Fig. 17. It is seen that multiplier with maximal absolute value on the

![Stability Diagram](image-url)

Fig. 16. Influence of tool stiffness on stability domain ($s_1 = 0.5$, $\theta = \pi/6$, $a = 0$, $c = 0$, $p_{01} = 1$ Mpa).
boundary of stability is $\mu = 1$, which corresponds to $T$-periodic motion of the shaft. The variation of Lyapunov exponent in Fig. 17 can be observed. The next region near twice the natural frequency $2p_1$ corresponds to $2T$-periodic motion as a multiplier takes the value $\mu = 1$ on the boundary of the instability domain. The same regions of $T$-periodic and $2T$-periodic stability appear near the second frequency and higher ones but they are more narrow.
The influence of internal tool damping on stability domain in Fig. 18 is presented. This diagram is plotted in the plane of internal damping and speed of rotation. It is seen from the plot that we have the same regions of principal resonance near $p_1$ and $2p_1$ while in addition there is a region of combination resonance covering the region higher than line $c_{cr}$ corresponding to the critical value of internal damping factor. The variation of multipliers and exponents at the intersection of this line in Fig. 19 is presented.
The influence of tool shoes disposition characterizing the asymmetry of the tool is more complex and is shown in Figs. 20 and 21. The region presented in Fig. 20 is in the vicinity of the first free vibration frequency and corresponds to the $T$-periodic principal resonance, while the instability region in Fig. 21 is near twice the first frequency and corresponds to the $2T$-periodic motion.

4.3. Simulation of surface formation

Numerical simulation of the machined surface formation was calculated for several types of the initial surface profile. It was assumed that initial surface has a radial distortion that is constant along the workpiece axis. An elliptical profile, a sinusoidal profile of varying amplitude, and a random initial surface profile (Fig. 22) were considered. The parameter $e$ characterizing the machined surface quality was defined as

$$e = \frac{R_{\text{max}} - R_{\text{min}}}{R_{\text{max}} + R_{\text{min}}}$$

(17)

where $e$ is eccentricity of machined surface and $R_{\text{max}}$, $R_{\text{min}}$ are the surface maximum and minimum radii, respectively.

As an illustration of numerical simulation, the variation of the surface radius versus the angle of tool rotation $\phi$ and number of tool turns is presented in Fig. 23 for the case of honing without axial feed when the honing stone moves on the same surface ($\beta = 0$).
The influence of tool stones angular orientation on the variation of surface quality parameter $e$ is presented in Fig. 24. It is seen that in the case $\theta = \pi/4$ and $\theta = 75^\circ$, the parameter $e$ increases due to system vibrations and at $\theta = \pi/3$ the parameter $e$ decreases, i.e. the surface is corrected. The influence of tool stones stiffness $k$ on variation of surface quality parameter $e$ while machining is presented in Fig. 25. It is seen that an increase of the stiffness $k$ results in an increasing removal velocity of the machined material. However, the surface error increases in this case ($k = 500$ kN/m and $k = 100$ kN/m) due to vibration excitation. Therefore the system damping should be increased to prevent the vibration excitation. Tool rotation speed affects the process in a different manner. The plot of surface eccentricity variation in time for different speeds of tool rotation (600 rpm and 1800 rpm) is shown in Fig. 26. It is seen that the correction of error at the same number of turns is greater in the case of machining at lower speeds of rotation. This can be explained due to an increase in the dynamic forces. The numerical analysis shows if the
tool radial stiffness is rather small the transverse vibrations can be unstable with growing amplitude. The results of the system phase parameters variation while processing for case of honing the surface with initial elliptical distortion and radial tool stiffness $k = 3 \text{kN/m}$ are shown in Fig. 27. The numerical analysis for this case is incorrect because the model of new surface formation (13) can be used only for magnitudes of cut chip commensurable with the size of abrasive tool
Fig. 26. Variation of eccentricity vs tool turn number for different speed of rotation ($\theta = \pi/6$, $k = 500$ kN/m, $p_{01} = 1$ MPa, $c = 1$ kN/s/m).

Fig. 27. Plot of system phase parameters variation for unstable case, ($\theta = \pi/6$, $k = 3\times10^3$ N/m $p_{01} = 1$ MPa, $c = 1$ kN/s/m).

grains. But these results can be used in process design for choosing rational tool parameters and cutting conditions.
5. Conclusions

The developed model allows the simulation of the deep hole honing process in a wide variety of industrial conditions. The results are presented in a non-dimensional form, and can be interpreted for wide set of real system parameters. The model allows calculation of system frequencies and conditions when vibrations are stable. An analysis of the simulation results yields an understanding of the influence of the system parameters on the machined surface formation. These may be summarized as

- An increase of the initial pressure and the radial tool stiffness may cause the shaft to become unstable with transverse vibrations at critical values of $p_0 > p^*$ and $k > k^*$.
- In the case of non-symmetric tool stone arrangement the system dynamic instability appears due to a parametric resonance mechanism.
- The maximal speed of material machining and a reduction in surface errors are observed at $\theta = \pi/3$.
- The speed of material machining increases with the growth of tool stiffness.
- The proper selection of tool parameters and operating conditions is necessary to avoid the loss of system stability.

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References


