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Fuzzy Logic Control of a Parametrically Excited Rotating Beam

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ABSTRACT.
In this paper the control of a parametrically excited, rotating flexible beam is considered. Only the flap motion of the elastic beam is analyzed. Due to an axial harmonic excitation, the nonlinear equations of motion contain periodic coefficients. A fuzzy logic controller is designed such that the deflection of the beam tip can be limited to a desired value in a relatively short period of time. Such systems can be used as preliminary models for studying the flap dynamics and control of helicopter rotor blades and flexible mechanisms, among other systems.

Keywords: fuzzy logic control, nonlinear time-periodic systems

1 INTRODUCTION
The behavior of flexible bodies connected to moving supports has been studied for a long period of time. Beams attached to moving bases have received special attention in many technical papers dealing with elastic linkages, rotating machinery, robot manipulator arms.

To control the motion of the flexible links, both linear and nonlinear controllers were constructed and different kind of strategies were implemented. Adaptive control algorithms and on-line parameter identification techniques have been common features of the last generation controllers.

Cannon and Schmitz (1984) considered an elastic arm, modeled as a pinned-free beam, attached to a hub. The objective of their work was to carry out experiments designed to determine the necessary control torque applied at the base of the link using only the tip position measurement. A more complex system was analyzed by Berbyuk (1984). His work was related to the problem of controlling the plane rotational motions of two rigid bodies connected by an elastic rod. Asymptotic methods were used to obtain a solution of the control problem for some limiting cases.

Nathan and Singh (1991) treated the problem of controlling an elastic arm of two links based on variable structure system theory and pole assignment technique for stabilization. This design approach was motivated by a simple observation that the nonlinearity in the dynamics of an elastic robotic system is essentially due to the rigid modes (joint angles), and as the time derivatives of the rigid modes vanish, the remaining motion is only due to the elasticity. For the rigid modes a sliding controller was designed. The controller of the elastic
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modes was constructed using the pole assignment technique. A similar technique has been used by Singh et al. (1994) to control the motion of a flexible/rigid link robot.

Warren et al. (1995) designed a robust control system for a slewing shaft-beam system. The motivation for such a design was the control of flexible structures that need accurate pointing capabilities while rejecting structural vibrations.

Boghiu et al. (1996) studied the flap motion control of a rotating parametrically excited flexible beam. A linear controller, based on the Lyapunov-Floquet transformation, was constructed to suppress both the deflection angle and the elastic vibrations of the beam. The controller design was based on the idea suggested by Sinha and Joseph (1994).

In recent years, fuzzy logic controllers demonstrated their superiority in many applications over conventional control schemes. This is mainly due to their capacity to capture the approximate, inexact nature of a real world system. In fact, most of the dynamic processes have nonlinearities that are difficult to be exactly modeled in mathematical forms. Lim and Hiyama (1991) proposed proportional-integral and fuzzy logic controllers to control a two link rigid robot. The PI controller was used to ensure fast transient response and zero steady-state error. The fuzzy-logic controller was used to enhance damping characteristics of the system. However, they found that the gains adjustment of the PI controller requires a large effort and the control scheme does not compensate for the nonlinear effects of the robot system.

Liu and Lewis (1992) designed a feedback-linearization/fuzzy logic hybrid scheme for a robotic manipulator with link flexibility. The control scheme was composed of a feedback-linearization inner-loop control and a fuzzy linguistic outer-loop control. A reduced-order computed torque control was first used to linearize the whole system to a Newton's law-like system, then a linguistic fuzzy controller (of 33 if-then fuzzy rules) is used to command the rigid modes to track a desired trajectory while the residual vibrations are maintained as small as possible.

Kubica and Wang (1993) applied a fuzzy control strategy to control the rigid body and the first flexural mode of vibration in a single link robotic arm. Two fuzzy logic controllers were constructed. The first one was designed to govern the rigid motion of the beam as it was rotated from one position to another. The second controller was designed to attenuate the vibrations resulting from the rigid body motion. The results obtained showed an improvement over those obtained using conventional multivariable techniques.

Beale and Lee (1995) developed a fuzzy logic control scheme to investigate the vibration suppression of a flexible-rod slider mechanism. A three mode approximation of the beam was considered. From simulations it was found that the transverse deflection of the flexible link was significantly reduced.

In this paper, a mechanical system similar to the one presented in (Boghiu et al., 1996) is considered. However, the equations of motion are nonlinear and a fuzzy logic strategy is designed to control the nonlinear flap motion of the elastic beam.

2 THE SYSTEM MODEL

The diagram of the system is shown in Fig. 1. The system consists of a slender flexible beam AB cantilevered onto a rigid massless base. The beam has length \( L \), a constant flexural rigidity \( E I \) and a uniformly distributed mass per unit length \( \rho = m/L \), where \( m \) is the total mass. The base of length \( L_b = 2b \) has a
sinusoidal vibrating motion of amplitude $L_o$ and frequency $\omega$. The base can perform small rotational deflections $\phi(t)$ about the $x$ axis passing through the point $O$. A spring and a damper are connected to the base and the rigid link in order to avoid large rotation $\phi$. The whole system rotates in the horizontal plane with a constant angular velocity $\Omega$. Let $x$ be the position of a point $P$ on the beam with respect to the end $A$ of the base and $y$ be the elastic deflection. Only the flap motion of the elastic link is considered. Two reference frames are used: a “fixed” reference frame $(N)$, of unit vectors $i, j$ and $k$, whose origin is at $O$ and a rotating reference frame $(R)$, of unit vectors $m_1, m_2$ and $m_3$, with the origin at $O$ and attached to the rigid link $OO$. These are related by the transformation

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} \cos \Omega t & \sin \Omega t & 0 \\ -\sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix}.$$  

The position vector of point $P$ in the rotating reference frame $(R)$ is

$$r_P = [L_o \sin \omega t + (b + x) \cos \phi - y \sin \phi]m_1 + [y \cos \phi + (b + x) \sin \phi]m_3.$$  

The elastic deflection $y$ of the beam is computed as

$$y(x, t) = \sum_{i=1}^{\infty} \Psi_i(x)q_i(t),$$

where $\Psi_i(x)$ are the mode shapes of a cantilever beam. These are defined by the expression

$$\Psi_i(x) = \frac{\cosh(\lambda_i) - \cos(x)}{\sinh(\lambda_i) + \sin(\lambda_i)} \left(\sinh(x) - \sin(x)\right),$$

where

$$z = \frac{x \lambda_i}{L},$$

and $\lambda_i (i = 1, \ldots, \infty)$ are the consecutive roots of the transcendental equation

$$\cos(\lambda) \cosh(\lambda) = -1.$$  

The mode shape functions satisfy the orthogonality relations

$$\int_0^L \rho \Psi_i(x)\Psi_j(x) \, dx = m \delta_{ij}, \quad \int_0^L \Psi_i(x)\Psi_j''(x) \, dx = \frac{\lambda_i^4}{L^3} \delta_{ij},$$

where $\delta_{ij}$ is Kronecker delta function.

The velocity of the point $P$, in the fixed reference frame $(N)$, is computed using the expression

$$v_P = \frac{Rd}{dt} r_P + \Omega \times r_P,$$

where the first term of the right hand side represents the derivative with respect to time in the moving reference frame $(R)$. 

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The total kinetic energy of the system is
\[ K = \frac{1}{2} \int_0^L v_p \cdot v_p dx. \] (9)

The total potential energy of the system is computed as
\[ U = \frac{EI}{2} \int_0^L \frac{\partial^2 y}{\partial x^2} dx + \frac{EI}{2} \int_0^L P(x,t) \left( \frac{\partial y}{\partial x} \right)^2 dx + \frac{1}{2} k\phi^2 \] (10)

The second integral on the right hand side of Eq. (10) is the work performed by the axial force \( P(x,t) \) arising from centripetal effect. This force is computed by the expression
\[ P(x,t) = \int_0^L \tau_{p1} \Omega^2 dx, \] (11)
where \( \tau_{p1} \) is the first component of the position vector \( \tau_p \), i.e.,
\[ \tau_{p1} = L_0 \sin \omega t + (b + z) \cos \phi - y \sin \phi. \] (12)

The nonlinear equations of motion are of the form
\[ M\ddot{x} + f(x,t) = d u. \] (13)

For the same system, it was found (Boghiu et al., 1996) that the first mode is dominant, the influence of the other modes being negligible. Therefore the simulations presented here are based on one mode approximation. With this approximation \((n = 1)\), various quantities appearing in Eq. (13) are defined in the Appendix. It is to be observed that \( x \) and \( f(x,t) \) are \( 2 \times 1 \) vectors, \( M \) is a \( 2 \times 2 \) matrix, \( d \) is a column vector and \( u \) is a scalar.

Figure 1. The system model

3 FUZZY-LOGIC CONTROL (FLC)

In this section a fuzzy-logic controller is designed such that the total deflection of the beam tip \( y_t \) is limited to a desired value in a relatively short period of time. This deflection can be computed with the expression (Figure 1)
\[ y_t = L \sin \phi + y_L \cos \phi \] (14)
where \( y_L \) is the elastic deflection of the beam tip, computed as
The inputs to the fuzzy-logic controller are the angle $\phi$ and its time derivative $\phi'$. The output of the controller is the torque $u$ applied to the base of the elastic beam. For each input and output of the controller, a fuzzy set is associated. The vector representation of a fuzzy set has two components: an universe of discourse vector (also called the support vector) and a membership vector (also called the grade vector). The support vector represents the range of the input/output signal used to design the controller, while the grade vector can have any value between 0 and 1.

Figure 2. Basic configuration of a fuzzy-logic control system

3.1 Design procedure of a fuzzy logic controller
A basic FLC is shown in Figure 2 (Lin, 1994) and contains 4 principal components:

- a fuzzification interface (FI), similar to an A/D converter in digital control;
- a decision making logic (DML), similar to a digital controller;
- a defuzzification interface (DFI), similar to a D/A converter in digital control;
- a knowledge base (KB), which works like digital control theorems.

The fuzzification interface
The process of evaluating a fuzzy set at a given support value is called fuzzification. The evaluation of a fuzzy set consists of taking a support value and calculating its (fuzzy) grade $u$. Thus, the crisp information is converted into its fuzzy representation. The most common shapes of fuzzy sets are the triangles and trapezoids. As mentioned before, the input of the controller are the angle $\phi$ and its time derivative $\phi'$. The fuzzy sets associated with these two parameters are shown in Figure 3 for the angle $\phi(\mu_{\phi})$, the angular velocity $\phi'(\mu_{\phi'})$ and the control torque $u(\mu_{u})$. One can see that the support vector of the angle $\phi$ is between -1 rad and 1 rad. However, the angle $\phi$ is not restricted to this domain. The $\phi$ angle fuzzy set is a combination of trapezoids and triangles. It has 7 grades, that correspond to negative huge (NH), negative big (NB), negative small (NS), zero (ZE), positive small (PS), positive big (PB) and positive huge (PH) values of the angle $\phi$. For the angular velocity $\phi'$, the support vector is defined between -2 and 2 rad/s, but, of course, $\phi'$ can take any value. The fuzzy set associated is also a combination of trapezoids and triangles. It has only 3 grades, which correspond to negative (N), zero (Z) and positive (P) values of $\phi$. 

\[ y_L(t) = y(L, t) = \Psi_1(L) q_1(t). \]
The fuzzy set of the controller output $u$ (Figure 3) consists only of equidistant triangle grades. The control force has 5 components, that correspond negative big (NB), negative small (NS), zero (ZE), positive small (PS), positive big (PB) values of the controller output.

Intuitively, when $\phi$ and $\dot{\phi}$ are "zero", then the output of the controller should also be "zero".

**The decision-making logic**

The decision making logic is the module in which the controller output is generated. It uses the input fuzzy sets, and the decision is taken according to the values of the inputs (angle $\phi$ and $\dot{\phi}$). A consequence table is required and the controller output is generated using "If-Then" rules. The rule has two parts, an antecedent and a consequence. The antecedent is the "If" part and the consequence is the "Then" part.
The consequence table is a matrix of 7 columns (the number of grades of the \( \phi \) angle fuzzy-set) and 3 rows (the number of grades of \( \phi \) fuzzy-set). The consequence table is as follows:

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>NH</th>
<th>NB</th>
<th>NS</th>
<th>ZE</th>
<th>PS</th>
<th>PB</th>
<th>PH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{\phi} = N )</td>
<td>PB</td>
<td>PB</td>
<td>PS</td>
<td>PS</td>
<td>ZE</td>
<td>NS</td>
<td>NB</td>
</tr>
<tr>
<td>( \dot{\phi} = Z )</td>
<td>PB</td>
<td>PS</td>
<td>ZE</td>
<td>ZE</td>
<td>ZE</td>
<td>NS</td>
<td>NB</td>
</tr>
<tr>
<td>( \dot{\phi} = P )</td>
<td>PB</td>
<td>PS</td>
<td>ZE</td>
<td>NS</td>
<td>NS</td>
<td>NB</td>
<td>NB</td>
</tr>
</tbody>
</table>

As an illustration of the “If-Then” rules, let us suppose that \( \phi \) is negative by (NB) and \( \dot{\phi} \) is positive (P). For this case, one has:

IF \( \phi \) is NEGATIVE BIG and \( \dot{\phi} \) is POSITIVE,

THEN, \( u \) is POSITIVE SMALL.

Each time, according to the values of \( \phi \) and \( \dot{\phi} \), one or more “If-Then” rules are used to compute the control torque \( u \).

The defuzzification interface

Defuzzification is the operation of obtaining a crisp number from a fuzzy set based on the grades of the fuzzy set. Several defuzzification techniques are based on maximum grade, minimum grade and centroid. The maximum (minimum) grade method selects the support value associated with the maximum (minimum) grade. The results of these methods may be discontinuous in response to small changes in the original fuzzy sets’ grades. The centroid method returns the “center of mass” of a fuzzy set. The centroid of a fuzzy set changes in a continuous manner as the shape of the set changes. This method is used in this paper to generate the numerical value of the control torque \( u \) from its fuzzy set.

4 SIMULATIONS AND RESULTS

The following typical numerical values were used for simulations: lengths of elastic beam (aluminium) \( L = 1.0 \) m, mass/unit length \( \rho = 0.213 \) kg/m, flexural rigidity \( EI = 34.852 \) N, amplitude of the parametric excitation \( L_0 = 0.02 \) m, spring constant \( k = 10 \) Nm/rad, damper constant \( c = 2 \) Nms/rad.

The normalized natural frequencies of the linearized system were reported in Boghii et al. (1996), for three mode shapes. They correspond to \( \beta_1 = 4.74 \) rad/s, \( \beta_2 = 28.04 \) rad/s, \( \beta_3 = 169.33 \) rad/s, \( \beta_4 = 473.49 \) rad/s, for a nonrotating system \((\Omega = 0 \) rad/s). For the rotating system \((\Omega = 20 \) rad/s) one has \( \beta_1 = 6.76 \) rad/s, \( \beta_2 = 36.23 \) rad/s, \( \beta_3 = 177.8 \) rad/s, \( \beta_4 = 482.02 \) rad/s. In this work, simulations were performed for an excitation frequency of \( \omega = \beta_1 + \beta_2 \). In both cases, for this value of the excitation frequency, the uncontrolled system is neutrally stable.

Using the controller system procedure developed in the previous section, several simulations were performed. In all the cases, a typical initial condition of \( \phi(0) = 0.2 \) rad was used.
Figure 4. Damped-nonrotating system

Figure 4 shows the results of a damped-nonrotating system. The total beam tip deflection $y_t$ of the uncontrolled nonlinear system is shown in Figure 4.a. One can see that the system is neutrally stable. Figure 4.b shows the total beam tip deflection $y_t$ of the controlled system. It is observed that the zero deflection is reached in 0.4 s for the nonlinear controller. Figure 4.c shows the control torque $u$ generated by the nonlinear controller.

The results for the damped-rotating system are shown in Figure 5. Figure 5.a shows the behavior of the controlled system. The stabilization time is larger than in the previous case (the influence of the rotation), and the system is more oscillatory until the final position is reached. The control torque generated by the fuzzy-logic controller is shown in Figure 5.b.

5 CONCLUSIONS

The control problem associated with a rotating nonlinear elastic beam with a parametrically excited base is considered. A fuzzy logic controller has been designed to control the flap motion of the elastic beam by bringing the total deflection of the beam tip $y_t$ to zero. The controller inputs were the rigid angle $\phi$ and its time derivative and no information about the elastic coordinate $q_1$ was required for the design or implementation of the controller.

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Compared to other controllers, the fuzzy logic controller has the advantage that it does not require knowledge of the equations of motion and only a partial knowledge of the states of the system. The number of operations required to compute the control law is relatively small and hence a real time implementation is possible with a high sampling frequency. The memory storage is also small.

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REFERENCES


APPENDIX

The various quantities appearing in the nonlinear equation of motion (13) are defined as in the following.

- \( \mathbf{x} \) is the generalized coordinates vector defined by \( \mathbf{x} = [\phi, \theta, \dot{\phi}, \dot{\theta}]^T \);

- \( \mathbf{M} \) is the mass matrix defined as
  \[
  \mathbf{M} = \begin{bmatrix}
  mL^2/3 + qL^2 & F_1 \\
  F_1 & G_1
  \end{bmatrix},
  \tag{16}
  \]

The functions \( F_1 \) and \( G_1 \) are defined as
  \[
  F_1 = \int_0^L x \rho \dot{\Psi}_1(x) \, dx,
  \quad G_1 = \int_0^L \rho \dot{\Psi}_1^2(x) \, dx.
  \tag{17}
  \]

- \( \mathbf{f}(\mathbf{x}, t) \) is a nonlinear vector
  \[
  \mathbf{f}(\mathbf{x}, t) = [f_1, f_2]^T
  \tag{18}
  \]

where
  \[
  f_1 = \omega \left[ \frac{2}{3} \cos(2\phi) \sin \phi \right] + \left[ \frac{1}{3} \sin(2\phi) \cos \phi \right] + \left[ \frac{\omega}{2} \sin(2\phi) \cos \phi \right] + \left[ \frac{\omega}{2} \sin \phi \right] - \frac{\omega}{2} \sin(2\phi) \cos \phi
  \]

The following notations were used
  \[
  V_1 = \int_0^L \rho \dot{\Psi}_1(x) \, dx,
  \quad H_1 = \int_0^L \left( \dot{\Psi}_1^*(x) \right)^2 \, dx,
  \quad \Phi^0_{11} = \int_0^L \rho \left( \dot{\Psi}_1(x) \right)^2 \, dx,
  \quad \Phi^1_{11} = \int_0^L \rho \dot{\Psi}_1 \left( \dot{\Psi}_1(x) \right)^2 \, dx,
  \quad \Phi^2_{11} = \int_0^L \rho \left( \dot{\Psi}_1(x) \right)^2 \, dx,
  \quad \Phi^{11} = \int_0^L \left( \int_0^L \rho \dot{\Psi}_1(\xi) \, d\xi \right) \left( \dot{\Psi}_1(x) \right)^2 \, dx,
  \tag{19}
  \]

- \( \mathbf{d} = [1, 0]^T \) is the input vector;

- \( \mathbf{u} \) is the control torque applied to the moving base, as shown in Figure 1.