

Tenth International Congress
on Sound and Vibration
7-10 July 2003 • Stockholm, Sweden

ORDER REDUCTION OF NONLINEAR TIME PERIODIC SYSTEMS

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Abstract

This work reports new approaches for order reduction of nonlinear systems with time periodic coefficients. First, the equations of motion are transformed using the Lyapunov-Floquet (LF) transformation, which makes the linear part of new set of equations time invariant. At this point, either linear or nonlinear order reduction methodologies can be applied. The linear order reduction technique is based on classical technique of aggregation and nonlinear technique is based on '*Time periodic invariant manifold theory*'. These methods do not assume the parametric excitation term to be small. The nonlinear order reduction technique yields superior results. An example of two degrees of freedom system representing a magnetic bearing is included to show the practical implementation of these methods. The conditions when order reduction is not possible are also discussed.

INTRODUCTION

In many engineering systems, like structures subjected to periodic loadings, asymmetric rotor bearing system, etc., we encounter a large set of nonlinear differential equations with periodic coefficients. These equations are generally difficult to solve and require special analytical tools. Though these differential equations involve large number of states, for the purpose of analysis, simulation and control application only a few dominating states are important. It is possible to retain these dominating ('master') modes and get rid of non-dominating ('slave') modes.

Hence, the original large order system of dimension n could be reduced to a relatively small order system of dimension m ($m \ll n$). This process of constructing equivalent small-scale system, which captures the dynamics of original system accurately, is called the ‘model order reduction’. The model reduction could be linear or nonlinear depending upon the methodology adopted.

In this work, a new order reduction technique for nonlinear time periodic system is proposed. It is based on the Lyapunov-Floquet transformation and ‘*Time periodic center manifold theory*’. These techniques were developed by Sinha and coworkers and successfully applied for stability and bifurcation studies of time periodic systems. ‘*Time periodic center manifold theory*’ states that there exists a time periodic relation between stable and critical states and it is possible to eliminate the stable states by expressing them in terms of critical states. Thus, the entire system dynamics is reduced to the dynamics on the center manifold. The same argument can be rephrased as- it is possible to find a time periodic relationship between the non-dominant (‘slave’) states and the dominant (‘master’) states. Using this relationship, entire system dynamics can be expressed in terms of the ‘master’ states by eliminating the ‘slave’ states. Thus, the number of states describing the system dynamics can be reduced. In this paper, we make use of the ‘reducibility’ condition derived by Sinha, et al. (2003) that determines whether (or not) a large-scale system could be reduced to a smaller system.

This paper is divided into four sections. Section 2 discusses linear and nonlinear order reduction procedures. In Section 3 an example is presented and finally, Section 4 contains the conclusions.

2. ORDER REDUCTION IN STATE SPACE USING INVARIANT MANIFOLD

2.1 Linear Order Reduction Procedure

Consider a linear time periodic system described by

$$\mathbf{M}(t)\ddot{\mathbf{y}} + \mathbf{C}(t)\dot{\mathbf{y}} + \mathbf{K}(t)\mathbf{y} = 0 \quad (1)$$

where \mathbf{y} is an m vector and $\mathbf{M}(t), \mathbf{C}(t), \mathbf{K}(t)$ are time periodic $m \times m$ matrices. It is assumed that the system is at least neutrally stable. The objective of order reduction is to replace this original linear system by a reduced order system described by

$$\mathbf{M}_q(t)\ddot{\mathbf{y}}_q + \mathbf{C}_q(t)\dot{\mathbf{y}}_q + \mathbf{K}_q(t)\mathbf{y}_q = 0, \quad q \ll m \quad (2)$$

where $\mathbf{y}_q(t)$ is a q vector corresponding to the ‘master’ (dominant) states $\mathbf{M}_q(t), \mathbf{C}_q(t), \mathbf{K}_q(t)$ are reduced order mass, damping and stiffness matrices of order $q \times q$, respectively. However, at this stage, there is no systematic way of selecting these q coordinates due to the presence of various resonances in the system.

The system described by equation (1) can be expressed as a set of $n (= 2m)$ first order equations given by

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t); \quad \mathbf{x} = (\mathbf{y}, \dot{\mathbf{y}})^T \quad (3)$$

$$\text{where } \mathbf{A}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}(t)\mathbf{K}(t) & -\mathbf{M}^{-1}(t)\mathbf{C}(t) \end{bmatrix}, \quad (4)$$

\mathbf{I} and $\mathbf{0}$ are $m \times m$ identity and null matrices, respectively.

An application of the transformation $\mathbf{x}(t) = \mathbf{Q}(t)\bar{\mathbf{y}}(t)$, where $\mathbf{Q}(t)$ is Lyapunov-Floquet transformation matrix (For details see Sinha and Pandiyan (1994)), to equation (3), produces

$$\dot{\bar{\mathbf{y}}} = \mathbf{R}\bar{\mathbf{y}}(t) \quad (5)$$

where $\bar{\mathbf{y}}$ is an n vector and \mathbf{R} is an $n \times n$ time invariant matrix. Further, the modal transformation $\bar{\mathbf{y}} = \mathbf{M}\mathbf{z}$, yields

$$\dot{\mathbf{z}} = \mathbf{J}\mathbf{z} \quad (6)$$

where \mathbf{J} is the Jordan form of \mathbf{R} matrix. Equation (6) may be partitioned as

$$\begin{Bmatrix} \dot{\mathbf{z}}_r \\ \dot{\mathbf{z}}_s \end{Bmatrix} = \begin{bmatrix} \mathbf{J}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_s \end{bmatrix} \begin{Bmatrix} \mathbf{z}_r \\ \mathbf{z}_s \end{Bmatrix}, \quad r + s = n \quad (7)$$

where $\mathbf{J}_r, \mathbf{J}_s$ are the Jordan blocks corresponding to ‘master’ (\mathbf{z}_r) and ‘slave’ (\mathbf{z}_s) states, respectively. \mathbf{J}_r is an $r \times r$ matrix while \mathbf{J}_s has the dimension $(n-r) \times (n-r)$. The dominant or ‘master’ states (\mathbf{z}_r) are the desired states to be retained for the linear order reduction and thus we approximate equation (7) by the ‘master’ states (\mathbf{z}_r) only. Then from equation (7) we have

$$\dot{\mathbf{z}}_r = \mathbf{J}_r \mathbf{z}_r \quad (8)$$

as the reduced order system. At this stage we would like to express the ‘slave’ states in the original coordinates (\mathbf{x}_s) as functions of ‘master’ states (\mathbf{x}_r). By using the L-F transformation $\mathbf{Q}(t)$ and the modal transformation \mathbf{M} , we obtain

$$\mathbf{x} = \mathbf{Q}(t)\mathbf{M}\mathbf{T}\mathbf{z}_r \equiv \mathbf{P}(t)\mathbf{z}_r \quad (9a)$$

where $\mathbf{T} = [\mathbf{I}_{r \times r} : \mathbf{0}_{r \times (n-r)}]^T$. \mathbf{I} and $\mathbf{0}$ are the identity and null matrices, respectively.

Equation (9a) may be partitioned as

$$\begin{Bmatrix} \mathbf{x}_r \\ \mathbf{x}_s \end{Bmatrix} = \begin{bmatrix} \mathbf{P}_{12}(t) \\ \mathbf{P}_{21}(t) \end{bmatrix} \mathbf{z}_r \quad (9b)$$

where \mathbf{x}_r are the ‘master’ states of \mathbf{x} that we would like to retain. From equation (9b), the ‘slave’ states \mathbf{x}_s can be expressed in terms of the ‘master’ states \mathbf{x}_r as

$$\mathbf{x}_s = \mathbf{P}_{21}(t)\mathbf{P}_{12}^{-1}(t)\mathbf{x}_r \quad (10)$$

Using this relationship in equation (3), r equations in \mathbf{x}_r may be obtained as

$$\dot{\mathbf{x}}_r = \bar{\mathbf{A}}_r(t)\mathbf{x}_r \quad (11)$$

Finally, these r first order equations in \mathbf{x}_r may be represented in an equivalent 2nd order form by using similarity transformations suggested by Sinha, et al. (1998).

$$\mathbf{M}_q(t)\ddot{\mathbf{y}}_q + \mathbf{C}_q(t)\dot{\mathbf{y}}_q + \mathbf{K}_q(t)\mathbf{y}_q = \mathbf{0}, \quad \mathbf{y}_q = \{\mathbf{x}_r, \dot{\mathbf{x}}_r\}^T, \quad r = 2q, q \ll m \quad (12)$$

2.2. Nonlinear Order Reduction Procedure

Consider a nonlinear time periodic system described by

$$\mathbf{M}(t)\ddot{\mathbf{y}} + \mathbf{C}(t)\dot{\mathbf{y}} + \mathbf{K}(t)\mathbf{y} + \mathbf{F}(\mathbf{y}, \dot{\mathbf{y}}, t) = \mathbf{0} \quad (13)$$

where \mathbf{y} , $\mathbf{M}(t)$, $\mathbf{C}(t)$, $\mathbf{K}(t)$ are defined as before and $\mathbf{F}(\mathbf{y}, \dot{\mathbf{y}}, t)$ is a nonlinear vector function such that $\mathbf{F}(\mathbf{0}, \mathbf{0}, t) = \mathbf{0}$. Once again, the objective is to replace this system by a reduced order system described by

$$\mathbf{M}_q(t)\ddot{\mathbf{y}}_q + \mathbf{C}_q(t)\dot{\mathbf{y}}_q + \mathbf{K}_q(t)\mathbf{y}_q + \mathbf{F}_q(\mathbf{y}_q, \dot{\mathbf{y}}_q, t) = \mathbf{0}, \quad q \ll m \quad (14)$$

As before, equation (13) can be rewritten as

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{f}(\mathbf{x}, t) \quad (15)$$

where \mathbf{x} and $\mathbf{A}(t)$ are defined in equation (3), and $\mathbf{f}(\mathbf{x}, t)$ is a nonlinear time periodic n vector. Applying the L-F Transformation $\mathbf{x}(t) = \mathbf{Q}(t)\bar{\mathbf{y}}(t)$ produces

$$\dot{\bar{\mathbf{y}}}(t) = \mathbf{R}\bar{\mathbf{y}}(t) + \mathbf{Q}^{-1}(t)\mathbf{f}(\bar{\mathbf{y}}, t) \quad (16)$$

After the modal transformation, $\bar{\mathbf{y}}(t) = \mathbf{M}\mathbf{z}(t)$, we have

$$\dot{\mathbf{z}}(t) = \mathbf{J}\mathbf{z}(t) + \mathbf{M}^{-1}\mathbf{Q}^{-1}(t)\mathbf{f}(\mathbf{z}, t) \equiv \mathbf{J}\mathbf{z}(t) + \mathbf{w}(\mathbf{z}, t) \quad (17)$$

where \mathbf{J} is a Jordan form of \mathbf{R} and $\mathbf{w}(\mathbf{z}, t)$ represents an appropriately defined nonlinear time varying vector consisting of monomials of z_j . In the absence of damping, the eigenvalues of \mathbf{J} are distinct and purely imaginary provided $\mathbf{K}(t)$ is positive definite.

Equation (17) may be partitioned as

$$\begin{Bmatrix} \dot{\mathbf{z}}_r \\ \dot{\mathbf{z}}_s \end{Bmatrix} = \begin{bmatrix} \mathbf{J}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_s \end{bmatrix} \begin{Bmatrix} \mathbf{z}_r \\ \mathbf{z}_s \end{Bmatrix} + \begin{Bmatrix} \mathbf{w}_r(\mathbf{z}_r, \mathbf{z}_s, t) \\ \mathbf{w}_s(\mathbf{z}_r, \mathbf{z}_s, t) \end{Bmatrix} \quad (18)$$

Here vectors $\mathbf{w}_r(\mathbf{z}_r, \mathbf{z}_s, t)$, $\mathbf{w}_s(\mathbf{z}_r, \mathbf{z}_s, t)$ are monomials of z_j (of order i) with periodic coefficients. In the spirit of work reported by Sinha and Pandiyan (1994) and Sinha, et al., (1998), we propose a nonlinear relationship between ‘master’ and ‘slave’ states as

$$\mathbf{z}_s = \sum_i \mathbf{h}_i(\mathbf{z}_r, t) \equiv \mathbf{H}(\mathbf{z}_r, t) \quad (19)$$

where $\mathbf{h}_i = \sum_{\bar{\mathbf{m}}} \bar{\mathbf{h}}_i(t) \mathbf{z}_1^{m_1} \dots \mathbf{z}_r^{m_r}$, $\bar{\mathbf{m}} = (m_1, \dots, m_r)^T$, $m_1 + \dots + m_r = i$, $i = 2, 3, \dots, k$. (20)

Here $\bar{\mathbf{h}}_i(t)$ are the unknown periodic vector coefficients with period $2T$. Substitution of equation (19) into equation (18), ordering terms and expanding $\bar{\mathbf{h}}_i(t)$ and $\mathbf{w}_s(\mathbf{z}_r, t)$ in Fourier series yields the following ‘reducibility’ condition. (For details, see Sinha, et al. (2003))

$$\frac{\bar{i} \nu \pi}{T} + \sum_{l=1}^r (m_l \lambda_l) - \bar{\lambda}_p \neq 0, \quad \forall \nu = 0, \pm 1, \pm 2, \dots; \quad p = 1, 2, \dots, s. \quad (21)$$

where $\lambda_1, \lambda_2, \dots, \lambda_r$ are the eigenvalues of the Jordan matrix \mathbf{J}_r and $\bar{\lambda}_p$; $p = 1, 2, \dots, s$ are the eigenvalues of \mathbf{J}_s .

It can be shown that if the ‘reducibility’ condition is satisfied then the vector $\mathbf{H}(\mathbf{z}_r, t)$ can be obtained and the ‘slave’ states can be expressed in terms of the ‘master’ states. However, when this reducibility condition is not satisfied, such a reduction is not possible and ‘slave’ coordinates cannot be expressed as functions of ‘master’ coordinates.

Once $\mathbf{H}(\mathbf{z}_r, t)$ has been determined, we obtain the equation for the ‘master’ states \mathbf{z}_r (c.f., equation (18)) as

$$\dot{\mathbf{z}}_r = \mathbf{J}_r \mathbf{z}_r + \mathbf{w}_r(\mathbf{z}_r, \mathbf{H}(\mathbf{z}_r, t), t) \quad (22)$$

This is the reduced order system in the \mathbf{z} domain. Now, again we make use of the L-F transformation matrix $\mathbf{Q}(t)$ and the modal matrix \mathbf{M} to map the results back to the original (\mathbf{x}) domain. Thus, the reduced order equation for the ‘master’ states \mathbf{x}_r can be obtained from equation (15) as

$$\dot{\mathbf{x}}_r(t) = \mathbf{A}_r(t) \mathbf{x}_r(t) + \tilde{\mathbf{f}}(\mathbf{x}_r, t) \quad (23a)$$

However, if one uses the linear relationship between \mathbf{x}_r and \mathbf{x}_s as indicated in equation (10), then the following reduced order model is obtained.

$$\dot{\mathbf{x}}_r(t) = \bar{\mathbf{A}}_r(t) \mathbf{x}_r(t) + \bar{\mathbf{f}}(\mathbf{x}_r, t) \quad (23b)$$

Once again, equation (23) can be transformed to the 2nd order form following Sinha, et al. (1998).

$$\mathbf{M}_q(t) \ddot{\mathbf{y}}_q + \mathbf{C}_q(t) \dot{\mathbf{y}}_q + \mathbf{K}_q(t) \mathbf{y}_q + \mathbf{F}_q(\mathbf{y}_q, \dot{\mathbf{y}}_q, t) = 0, \quad 2r = q, \quad q \ll m \quad (24)$$

3. EXAMPLE

The methodology proposed in Section 2 can be effectively applied to reduce the order of practical engineering structures modeled by nonlinear differential systems with time periodic coefficients. As an example, we reduce the order of a nonlinear 2 degrees of freedom system representing the dynamics of a magnetic bearing as presented by Schmidt and Tondl (1986). The equations, neglecting linear damping, are given by

$$\begin{aligned} \ddot{x} + \delta x^2 \dot{x} + (\omega_{n_1}^2 + \varepsilon \cos(\omega t))x + \gamma x^3 + axy &= 0 & (a) \\ \ddot{y} + \omega_{n_2}^2 y + bx^2 &= 0 & (b) \end{aligned} \quad (25)$$

where x and y represent displacements, δ is the nonlinear damping parameter, γ is the spring nonlinearity parameter, ε is the intensity of parametric excitation, ω is the frequency of parametric excitation, a , b are coupling parameters, and ω_{n_1} and ω_{n_2} are the linearized natural frequencies.

Equation (25) may be represented in state space form as

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(\omega_{n_1}^2 - \varepsilon \cos(\omega t)) & 0 & 0 & 0 \\ 0 & -\omega_{n_2}^2 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -(\gamma x_1^3 + ax_1 x_2) \\ -bx_1^2 \end{Bmatrix} \quad (26)$$

where $\{x_1 \ x_2 \ x_3 \ x_4\}^T = \{x \ y \ \dot{x} \ \dot{y}\}^T$

Applying L-F transformation $\mathbf{x}(t) = \mathbf{Q}(t)\bar{\mathbf{y}}(t)$ equation (26) can be written as

$$\dot{\bar{\mathbf{y}}} = \mathbf{R}\bar{\mathbf{y}} + \mathbf{Q}^{-1}(t)\mathbf{f}(\bar{\mathbf{y}}, t) \quad (27)$$

where $\bar{\mathbf{y}} = \{\bar{y}_1 \ \bar{y}_2 \ \bar{y}_3 \ \bar{y}_4\}^T$ and \mathbf{R} is a 4×4 time invariant matrix. By choosing the suitable set of values of parameters $(\gamma, a, b, \omega, \varepsilon)$, the system can be made to undergo ‘parametric resonance’, (3:1 and 2:1), ‘internal resonance’, ‘true internal resonance’ or ‘true combination resonance’. For a detailed discussion on these resonances we refer to Sinha, et al. (2003). These resonances are discussed in context of this example.

Case 1: No resonance of any kind

For this case, the numerical values of system parameters are given in table 1. It is to be observed that the intensity ε of the *excitation term is not small*. For this set of parameters, the Floquet exponents (eigenvalues of \mathbf{R} matrix) are $(\pm 0.90i, \pm 1.52i)$. It can be easily verified that the ratio of the angles of Floquet multipliers (or the ratio of Floquet exponents) for this set is 1.68, which implies they are not in ‘true internal resonance’ and the ‘reducibility’ condition in equation (21) is satisfied.

For the order reduction, we retain z_1, z_2 as ‘master’ coordinates (\mathbf{z}_r) and eliminate z_3, z_4 . Following the procedure outlined in Section 2.1, a linear order reduction was performed to obtain a relationship of the form of equation (11). Since $(x_1, x_3)^T = (x, \dot{x})^T$, $(x_2, x_4)^T = (y, \dot{y})^T$, we obtain relationships of the form

$$\begin{aligned} y &= a_1(t)x + a_2(t)\dot{x} \equiv q_1(x, \dot{x}, t), \\ \dot{y} &= b_1(t)x + b_2(t)\dot{x} \equiv q_2(x, \dot{x}, t) \end{aligned} \quad (28)$$

Now substituting equation (28) into equation (25a) the reduced order equation in x is obtained as

$$\ddot{x} + (\omega_n^2 + \varepsilon \cos 2\eta t)x + \gamma x^3 + axq_1(x, \dot{x}, t) = 0 \quad (29)$$

Equation (29) is a single degree of freedom approximation of the original 2 degrees of freedom system.

For nonlinear order reduction, we follow the procedure described in Section 2.2., and equation (19) takes the following explicit form.

$$\begin{aligned} z_3 &= z_1^2(0.27 \sin \pi t) + z_1 z_2(0.31 \cos \pi t + 0.19 \cos 3\pi t) + z_2^2(0.78 \sin \pi t + 0.14 \sin 3\pi t) \\ z_4 &= z_1^2(1.11 \cos \pi t - 0.04 \cos 3\pi t) + z_1 z_2(1.63 \sin \pi t + 0.15 \sin 3\pi t + 0.01 \sin 2\pi t) \\ &\quad + z_2^2(-0.36 \cos \pi t + 0.11 \cos 3\pi t) \end{aligned} \quad (30)$$

Using equation (30) and transforming back we arrive at a one DOF approximation of the original two DOF system as

$$\begin{aligned} \ddot{x} &+ (1 + 10 \cos 2\pi t)x + 3x^3 + 4x\dot{x}^2(-0.60 - 0.47 \cos 2\pi t + 0.05 \cos 4\pi t) \\ &+ 4x^3(-1.48 + 0.30 \cos 2\pi t + 0.19 \cos 6\pi t + 0.01 \cos 8\pi t) \\ &+ 4\dot{x}x^2(-2.45 \sin 2\pi t - 0.64 \sin 4\pi t - 0.05 \sin 6\pi t) = 0 \end{aligned} \quad (31)$$

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The results for this case are shown in figures 1 and 2. In figure 1, we compare the time traces for the original full-scale system, the reduced order systems obtained by linear as well as the nonlinear techniques. It can be seen that the nonlinear reduced model yields superior results and follows the response of actual system more accurately.

A comparison of time traces is not a very good measure as it represents the short-time system behavior. In order to compare the long-time behavior, we may use measures like the Poincaré map. It can be observed from figure 2 that the Poincaré map of the nonlinear reduced system (figure 2.c) closely resembles the Poincaré map of the actual system qualitatively as well as quantitatively (figure 2.a). The Poincaré map of the reduced order system by linear method (figure 2.b) matches the Poincaré map of actual system qualitatively but not quantitatively. Thus, it can be concluded that the nonlinear reduced order models portray the system dynamics more accurately than the linear reduced models.

Case 2: Parametric resonance

Parametric resonance takes place when the Floquet multipliers are real and repeated. For *small* ($\epsilon \ll 1$) parametric excitation, the frequency of periodic term ω and the natural frequency ω_{ni} satisfy certain integral relationship. When the system has only nonlinear coupling and the linear part of the system is decoupled, it (c.f., equation (25a)) resembles a Mathieu equation. The system states would comprise of resonant states and non-resonant states. In the z domain, these resonant states correspond to the zero entries in \mathbf{J} matrix and non-resonant states correspond to non-zero entries in \mathbf{J} matrix. For the present problem, if we choose the resonant states as the ‘master’ coordinates, we observe that the ‘reducibility’ condition is satisfied and order reduction is possible. However, if we choose non-resonant states as ‘master’ coordinates we observe that the ‘reducibility’ condition is not satisfied and hence the order reduction is not possible. However, one may apply linear order reduction technique but the results are not expected to be accurate.

4. DISCUSSION AND CONCLUSIONS

In conclusion, it can be stated that a rigorous technique for order reduction of general linear and nonlinear dynamical systems with time periodic coefficients has been presented. The parametric terms appearing in the linear parts of system equations are not assumed to be small. A mathematical condition is also included to determine whether a large-scale nonlinear parametrically excited system can be reduced to a lower order system. Application to a real engineering problem demonstrates the practical significance of the proposed techniques. Several extensions, generalizations and applications of these methods are under investigation.

ACKNOWLEDGMENT

Financial support provided by the Air Force Office of Scientific Research (AFOSR) and the University of Alaska Fairbanks under the contract No. F49620-01-1-0388 is gratefully acknowledged.

FIGURES

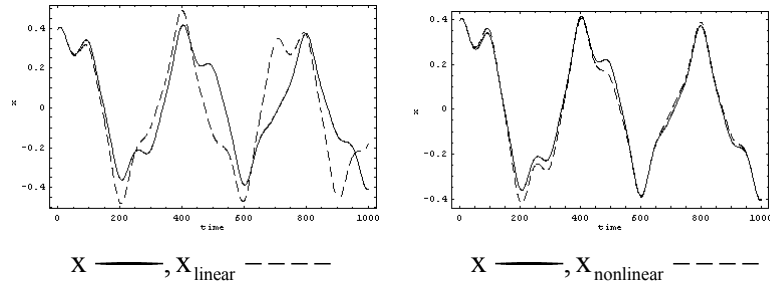


Figure 1: Time trace comparison of linear and nonlinear reduced models

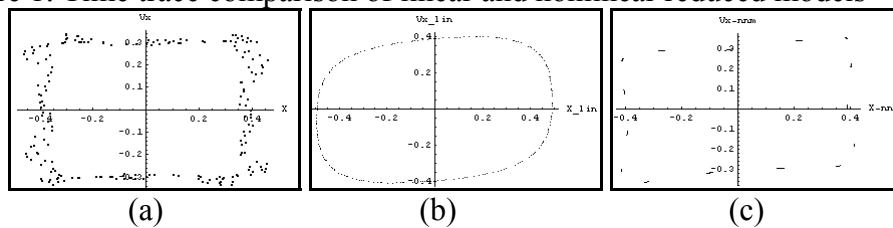


Figure 2: The Poincaré map of the a) original large-scale system b) reduced system using linear methodology c) reduced system using nonlinear methodology

TABLE

Table 1: Parameters for case 1

Parameter	ε	ω	γ	a	b	$\omega_{n_1}^2$	$\omega_{n_2}^2$
Value	10	2π	3	4	5	1	5

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