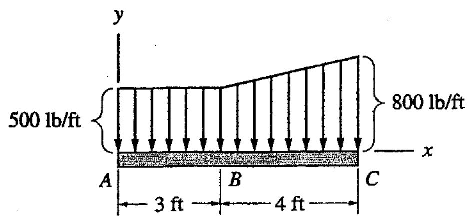
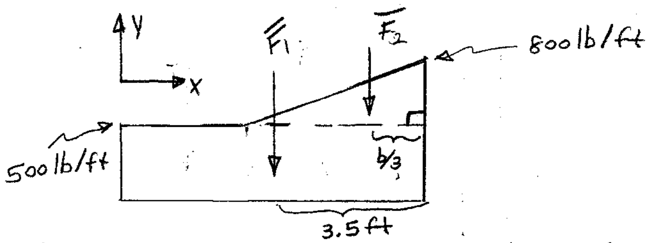
6A. For the distributed load shown below

a. Find the equivalent resultant force, $\mathbf{F_R}$. Write the equivalent resultant force as a Cartesian vector.

b. Find the moment resultant created by this distributed load about point C. Write the moment resultant as a Cartesian vector.





I have broken the distributed load up into a rectangular region and a triangular region.

$F_1 = 500\ \text{lb/ft}\ (7\ \text{ft}) = 3500\ \text{lb}$

$F_2 = \frac{1}{2} bh = \frac{1}{2}(4\ \text{ft})(800-500)\ \text{lb/ft}$

$F_2 = 600\ \text{lb}$

$\bar{F}_R = \bar{F}_1 + \bar{F}_2 = 3500\ \text{lb}(-\bar{j}) + 600\ \text{lb}(-\bar{j})$

$\bar{F}_R = 4100\ \text{lb}(-\bar{j})$

$\bar{M}_C^R = \bar{M}_C^{F_1} + \bar{M}_C^{F_2}$

$\bar{M}_C^{F_1} = 3500\ \text{lb}(3.5\ \text{ft})(\bar{k}) = 12{,}250\ \text{lbft}(\bar{k})$

$\bar{M}_C^{F_2} = 600\ \text{lb}(\ b/3)(\bar{k})$

$= 600\ \text{lb}(\ 4/3\ \text{ft})(\bar{k})$

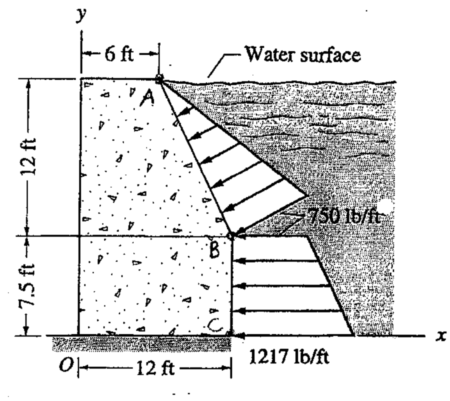
$\bar{M}_C^{F_2} = 800\ \text{lbft}(\bar{k})$

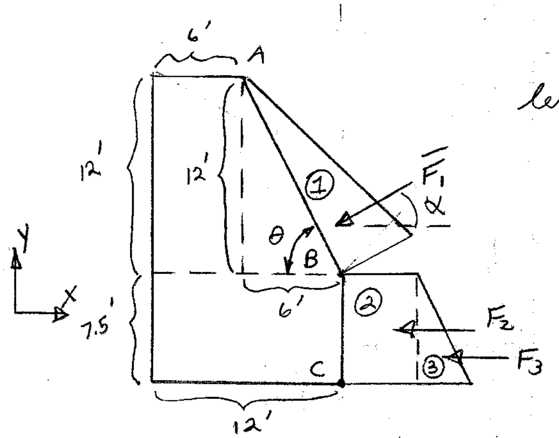
$\therefore \bar{M}_C^R = (12{,}250+800)\ \text{lbft}(\bar{k})$

$\bar{M}_C^R = 13050\ \text{lbft}(\bar{k})$

6B. Water pressure on a masonry dam produces the line-load distribution shown below.

a. Find the equivalent resultant force, $\mathbf{F_R}$, due to the two load distributions. Write the equivalent resultant force as a Cartesian vector.





length $AB = \sqrt{(6)^2 + (12)^2} = 13.4\,ft$

$\tan\theta = \frac{12}{6} \Rightarrow \theta = 63.4^\circ$

Triangular load distribution (1)

$F_1 = \frac{1}{2} bh = \frac{1}{2}(13.4\,ft)(750\,lb/ft)$

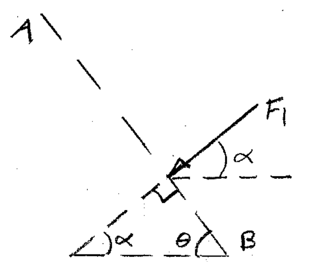
$F_1 = 5025\,lb$

Rectangular load distribution (2)

$F_2 = 750\,lb/ft\,(7.5\,ft)$

$F_2 = 5625\,lb$

Triangular distribution (3)



$\alpha = 90 - \theta = 90 - 63.4 = 26.6^\circ$

$F_3 = \frac{1}{2} bh = \frac{1}{2}(7.5\,ft)(1217 - 750)\,lb$

$F_3 = 1751\,lb$

Vector forms

$\bar{F}_1 = 5025\,lb \cos 26.6\,(-\bar{i}) + 5025\,lb \sin 26.6\,(-\bar{j})$

$\bar{F}_1 = 4493\,lb\,(-\bar{i}) + 2250\,lb\,(-\bar{j})$

$\bar{F}_2 = 5625\,lb\,(-\bar{i})$

$\bar{F}_3 = 1751\,lb\,(-\bar{i})$

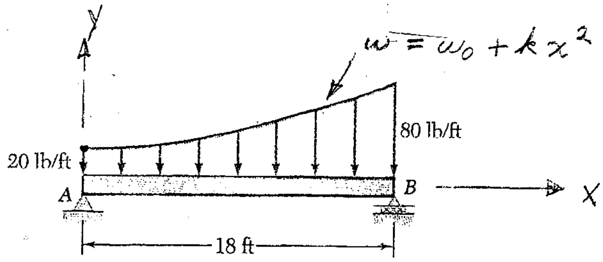
$\bar{F}_R = 11870\,lb\,(-\bar{i}) + 2250\,lb\,(-\bar{j})$

6C For the distributed load shown below

a. Determine the numerical values of ω_o and k in the equation for $\omega(x)$ by using the values of the line-load at points A and B.

b. Find the equivalent resultant force, $\mathbf{F_R}$. Write the equivalent resultant force as a Cartesian vector.

c. Find the moment resultant created by this distributed load about point **A**. Write the moment resultant as a Cartesian vector.



Part a

We have two constants in the equation that we need to find w_o and k. We have two points along the curve

$$w(x=0) = 20\,lb/ft \,, \quad w(x=18ft) = 80\,lb/ft$$

$$\therefore w(x=0) = w_o + k(0)^2 = w_o$$

$$\therefore w(x=0) = w_o = 20\,lb/ft$$

now, $w(x=18) = 80\,lb/ft = w_o + k(18)^2$

$$80\,lb/ft = 20\,lb/ft + k(18)^2$$

$$60\,lb/ft = k(18)^2$$

$$\frac{60\,lb/ft}{(18\,ft)^2} = k$$

$$.185 \frac{lb}{ft^3} = k$$

$$\therefore w(x) = 20 + .185x^2$$

Part b

$$F_R = \int w(x)\,dx$$

$$= \int_0^{18} (20 + .185x^2)\,dx$$

$$= \int_0^{18} 20\,dx + .185\int_0^{18} x^2\,dx$$

$$= 20x\Big|_0^{18} + .185\,\frac{x^3}{3}\Big|_0^{18}$$

$$= 360 + 360$$

$$F_R = 720\,lb \Rightarrow \bar{F}_R = 720\,lb(-\bar{j})$$

Part c

$$M_A = \int x\,w(x)\,dx$$

$$\int x\,w(x)\,dx = \int_0^{18} x(20 + .185x^2)\,dx$$

$$= 20\int_0^{18} x\,dx + .185\int_0^{18} x^3\,dx$$

$$= 20\,\frac{x^2}{2}\Big|_0^{18} + .185\,\frac{x^4}{4}\Big|_0^{18}$$

$$= 10(18)^2 + \frac{.185}{4}(18)^4$$

$$= 3240 + 4855$$

$$\int_0^{18} x\,w(x)\,dx = 8095\ lb\,ft$$

$$\bar{M}_A = 8095\,lb\,ft(-\bar{k})$$