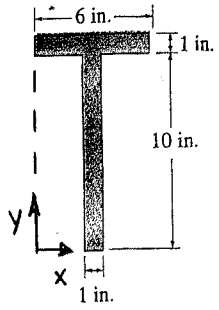
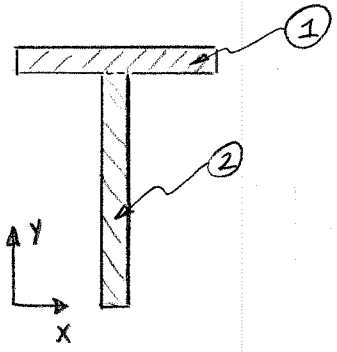
13A For the area shown below, find the location $(\overline{x}, \overline{y})$ of the centroid using the given coordinate system.



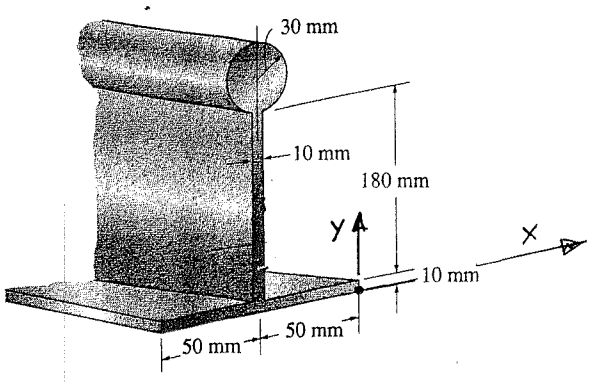


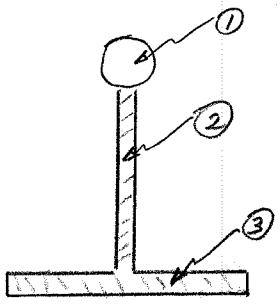
section	A	$\tilde{x}$	$\tilde{y}$	$\tilde{x}A$	$\tilde{y}A$
①	$6in^2$	3	10.5	18	63
②	$10in^2$	3	5	30	50
	$16in^2$			$48in^3$	$113in^3$

$$\overline{x} = \frac{48in^3}{16in^2} = 3in$$

$$\overline{y} = \frac{113in^3}{16in^2} = 7.06in$$

13B For the area shown below, find the location $(\bar{x}, \bar{y})$ of the centroid using the given coordinate system



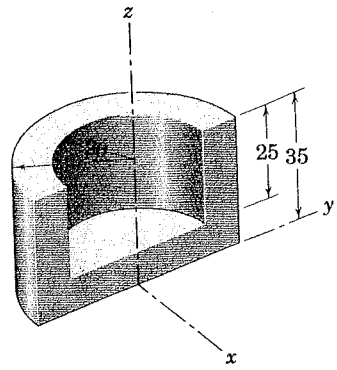


section	A	$\tilde{x}$	$\tilde{y}$	$\tilde{x}A$	$\tilde{y}A$
1	$\pi(.03)^2$ $(.00283)$	$-.05$	$.220$	$-.000142$	$.000623$
2	$(.010)(.18)$ $(.0018)$	$-.05$	$.100$	$-.00009$	$.00018$
3	$(.10)(.01)$ $(.001)$	$-.05$	$.005$	$-.00005$	$.000005$
	$\Sigma A = .00563$			$\Sigma \tilde{x}A = -.000282$	$.000808$

$$\bar{x} = \frac{-.000282}{.00563} = -.05\,m = -50\,mm$$

$$\bar{y} = \frac{.000808}{.00563} = .144\,m = 144\,mm$$

13C For the volume shown below, find the location $(\bar{x}, \bar{y}, \bar{z})$ of the center of gravity using the given coordinate system.



section	V	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	$\tilde{x}V$	$\tilde{y}V$	$\tilde{z}V$
large semicircular	$\frac{1}{2}\pi(30)^2(35)$ (49480)	$\frac{4(30)}{3\pi}$ (-12.7)	0	17.5	-628000	0	866000
small semicircular	$-\frac{1}{2}\pi(20)^2(25)$ (-15707)	$\frac{4(20)}{3\pi}$ (-8.49)	0	(10+12.5) (22.5)	+133000	0	-353000
	$\Sigma A = 33,773$				$\Sigma\tilde{x}V = -495,000$	0	513000

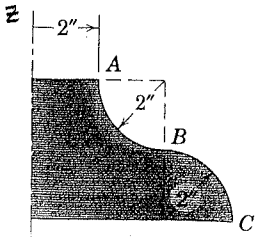
$$\bar{X} = \frac{-495,000}{33773} = -14.7$$

$$\bar{Y} = 0$$

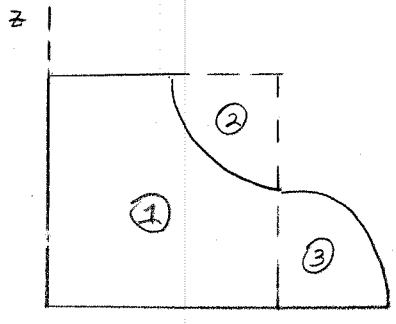
$$\bar{Z} = \frac{513000}{33,773} = 15.2$$

130 The area shown below is rotated 180 degrees about the z axis.

a. Determine the volume generated by this area as it is rotated 180 degrees.
b. Determine the surface area generated by curved line ABC as it is rotated 180 degrees.



a) Volume calculation



large rectangle (1)

$Vol^{(1)} = \theta \bar{r} A$

$\theta = \pi$
$\bar{r} = 2in$
$A = 16in^2$

$Vol^{(1)} = \pi(2)(16) = 100in^3$

¼ circle (2)

$Vol^{(2)} = \theta \bar{r} A$

$\theta = \pi$
$\bar{r} = (4 - \frac{4r}{3\pi}) = (4 - 8/3\pi) = 3.15in$
$A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2in)^2 = \pi$

$Vol^{(2)} = \pi(3.15)(\pi) = 31.1in^3$

¼ circle (3)

$Vol^{(3)} = \theta \bar{r} A$

$\theta = \pi$
$\bar{r} = 4 + \frac{4r}{3\pi} = 4 + 8/3\pi = 4.85in$
$A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2in)^2 = \pi in^2$

$Vol^{(3)} = \pi(4.85)\pi = 47.9in^3$

Total volume

$Vol = Vol^{(1)} - Vol^{(2)} + Vol^{(3)}$

$Vol = 100 - 31.1 + 47.9$

$Vol = 117in^3$

b) Surface area calculation

Curve AB

$SA = \theta \bar{r} L$

$\theta = \pi$

$\bar{r} = 4 - \frac{2r}{\pi} = 4 - \frac{2(2)}{\pi} = 2.73\,in$

$L = \frac{1}{4}\pi d = \frac{1}{4}\,2\pi r = \frac{1}{2}\pi r = \frac{1}{2}\pi(2in) = \pi\,in$

$SA^{AB} = \pi(2.73in)(\pi in) = 26.9\,in^2$

Curve BC

$SA = \theta \bar{r} L$

$\theta = \pi$

$\bar{r} = 4 + \frac{2r}{\pi} = 4 + \frac{2(2)}{\pi} = 5.27\,in$

$L = \frac{1}{4}\pi d = \pi\,in$

$SA = \pi(5.27in)(\pi in)$

$SA^{BC} = 52.0\,in^2$

Total surface area $= SA^{AB} + SA^{BC} = 26.9\,in^2 + 52in^2$

$SA^{TOT} = 78.9\,in^2$