SOLUTION (15.1)

Known: A pinion with 32 teeth and 8 diametral pitch meshes with a gear having 65 teeth.

Find: Calculate the standard center distance.

Schematic and Given Data:

Assumptions:
1. The gears are spur gears.
2. The gears have teeth of standard involute profile.
3. The gears mesh along their pitch circles.

Analysis:
1. From Eq. (15.3): \( P = \frac{N}{d} = \frac{N_p}{d_p} = \frac{N_g}{d_g} \)
2. \( P = 8, N_p = 32 \). Hence, \( d_p = 4.0 \) in.
3. \( N_g = 65 \). Hence, \( d_g = 8.125 \) in.
4. The center distance, \( c = \frac{d_g + d_p}{2} = 6.0625 \) in.

Comments:
1. If the gears did not mesh at the theoretical pitch circles the measured (actual) center distance would not be equal to the sum of the theoretical pitch circle radii of the gears.
2. It should be evident that meshing gears must have the same diametral pitch.
SOLUTION (15.2)

**Known:** A spur gear has a size 8 diametral pitch.

**Find:** Calculate the thickness of the spur gear tooth measured along the pitch circle.

**Schematic and Given Data:**

![Schematic and Given Data](image)

**Assumption:** The gear has teeth of standard involute profile.

**Analysis:**

1. From Eq. (15.5); \( pP = \pi \)
   \[ P = 8. \text{ Hence, } p = 0.3927 \text{ in.} \]
2. Tooth thickness, \( t = \frac{p}{2} \): \( t = 0.1963 \text{ in.} \)
**SOLUTION (15.3)**

**Known:** A pair of gears with a known gear ratio and at a specified center distance have a diametral pitch of 6.

**Find:** Determine the number of teeth in each gear.

**Schematic and Given Data:**

![Diagram of two gears with given data](image)

- \( P = 6 \)
- \( \frac{N_g}{N_p} = 2 \)
- Speed ratio 2:1
- \( c = 5 \text{ in.} \)
- \( d_p \)
- \( d_g \)

**Assumptions:**

1. The gears are spur gears.
2. The gears have teeth of standard involute profile.
3. The gears mesh along their pitch circles.

**Analysis:**

1. A gear ratio of 2:1 indicates a 2:1 ratio in the gear diameters. Hence \( d_g = 2d_p \).
2. Center distance, \( c = (d_g + d_p)/2 \). Hence,
   \[ d_g + d_p = 10 \text{ in.} \]
   Hence, \( d_p = 10/3 \text{ in.} \)
3. Substituting \( d_g = 2d_p \) in part 2, we obtain
   \[ 3d_p = 10 \text{ in.} \]
   Hence, \( d_p = 10/3 \text{ in.} \)
4. Diametral pitch, \( P = N_p/d_p \), and \( N_p = (10/3)(6) = 20 \).
5. Since the gear ratio is 2:1, the number of teeth on the gear, \( N_g = 2N_p \).
   Therefore, \( N_g = 40 \).

**Comments:**

1. If the gear teeth were not of involute profile it would still be possible to have a constant speed ratio provided it is ensured that the pitch point is stationary.
2. It should be evident that meshing gears must have the same diametral pitch.
3. If the diametral pitch were chosen to be higher, then the number of teeth would be greater on both the pinion and gear (other parameters being kept the same).
SOLUTION (15.4)

**Known:** A pinion with 20 teeth and 6 diametral pitch meshes with a gear having 55 teeth.

**Find:** Calculate the standard center distance.

**Schematic and Given Data:**

![Diagram of gears](image)

- \( N_p = 20 \)
- \( P = 6 \)
- \( N_g = 55 \)
- \( d_p \)
- \( d_g \)
- \( c \)

**Assumptions:**
1. The gears are spur gears.
2. The gears have teeth of standard involute profile.
3. The gears mesh along their pitch circles.

**Analysis:**
1. From Eq. (15.3): \( P = \frac{N}{d} = \frac{N_p}{d_p} = \frac{N_g}{d_g} \)
2. \( P = 6, N_p = 20 \). Hence, \( d_p = 3.33 \) in.
3. \( N_g = 55 \). Hence, \( d_g = 9.17 \) in.
4. The center distance, \( c = \frac{d_g + d_p}{2} = 6.25 \) in.

**Comments:**
1. If the gears did not mesh at the theoretical pitch circles the measured (actual) center distance would not be equal to the sum of the theoretical pitch circle radii of the gears.
2. It should be evident that meshing gears must have the same diametral pitch.
SOLUTION (15.5)

Known: A spur gear has a 6 diametral pitch.

Find: Calculate the thickness of the spur gear tooth measured along the pitch circle.

Schematic and Given Data:

Assumption: The gear has teeth of standard involute profile.

Analysis:
1. From Eq. (15.5); \( pP = \pi \)
   \( P = 6 \). Hence, \( p = 0.5236 \) in.
2. Tooth thickness, \( t = p/2 : t = 0.262 \) in.
SOLUTION (15.6)

**Known:** A pair of gears with a known gear ratio and at a specified center distance have a diametral pitch of 4.

**Find:** Determine the number of teeth in each gear.

**Schematic and Given Data:**

![Diagram](image)

**Assumptions:**
1. The gears are spur gears.
2. The gears have teeth of standard involute profile.
3. The gears mesh along their pitch circles.

**Analysis:**
1. A gear ratio of 3:1 indicates a 3:1 ratio in the gear diameters. Hence $d_g = 3d_p$.
2. Center distance, $c = (d_g + d_p)/2$. Hence, $d_g + d_p = 12$ in.
3. Substituting $d_g = 3d_p$ in part 2, we obtain $4d_p = 12$ in. Hence, $d_p = 12/4 = 3.0$ in.
   □
5. Since the gear ratio is 2:1, the number of teeth on the gear, $N_g = 3N_p$.  
   Therefore, $N_g = 36$. □

**Comments:**
1. If the gear teeth were not of involute profile it would still be possible to have a constant speed ratio provided it is ensured that the pitch point is stationary.
2. It should be evident that meshing gears must have the same diametral pitch.
3. If the diametral pitch were chosen to be a larger number, then the number of teeth would be greater on both the pinion and gear (other parameters being kept the same).
SOLUTION (15.7D)

Known: A web site address is given as http://www.grainger.com.

Find: Search the web site and select a spur gear with 32 pitch, 14.5° pressure angle, and 20 teeth. List the manufacturer, description, and price of the gear.

Analysis: The web site provides the following information:

- Mfg. Name: Boston Gear
- Description: Spur Gear-32 Pitch, Steel-14 1/2 Deg. Pressure Angle-20 Teeth
- Price: $11.28

SOLUTION (15.8)

Known: A pinion of known pitch and number of teeth rotates at 2000 rpm and drives a gear at 1000 rpm.

Find: Determine the number of teeth on the gear, theoretical center distance and circular pitch.

Schematic and Given Data:

![Schematic diagram](image)

Assumptions:
1. The gears are spur gears.
2. The gears have teeth of standard involute profile.
3. The gears mesh along their pitch circles.

Analysis:
1. A 2:1 speed ratio requires a 1:2 ratio in number of teeth. Hence, \( N_g = 40 \)

2. \( d_g = \frac{40}{8} = 5 \text{ in.}, \; d_p = \frac{20}{8} = 2.5 \text{ in.} \)

   Theoretical center distance, \( c = \frac{d_g + d_p}{2} \):

   \[
   c = \frac{5 + 2.5}{2} = 3.75 \text{ in.}
   \]
3. Circular pitch, \( p = \frac{\pi}{P} \): \( p = \frac{\pi}{8} \) in.

**Comments:**

1. If the gear teeth were not of involute profile it would still be possible to have a constant speed ratio provided it is ensured that the pitch point is stationary.
2. If the gears did not mesh along the pitch circles the speed ratio would not be constant and also the center distance would not be equal to the sum of the pitch circle radii of the gears.
3. It should be evident that meshing gears must have the same diametral pitch.

---

**SOLUTION (15.9)**

**Known:** A pair of spur gears provide a given speed ratio at a specified center distance and have a diametral pitch of 8.

**Find:** Determine the numbers of teeth and the pitch diameters of the gears.

**Schematic and Given Data:**

![Diagram of spur gears](image)

**Assumption:** The spur gears mesh at the pitch circles.

**Analysis:**

1. For \( c = 7.5 \) in., \( d_p + d_g = 15 \) in.
2. For 4:1 speed ratio, \( d_g = 4d_p \); Hence \( 5d_p = 15 \) in., \( d_p = 3 \) in.; \( d_g = 12 \) in.
3. For \( P = 8 \), \( N_p = 24 \); \( N_g = 96 \)

**Comments:**

1. The assumption to have spur gears meshing at the pitch circles ensures that the given center distance is the theoretical center distance and is equal to the sum of the pitch radii of the gears.
2. If the diametral pitch were chosen to be higher then the number of teeth would be more on both the pinion and gear (other parameters being kept the same).
SOLUTION (15.10)

**Known:** A pinion with known module and number of teeth rotates at 2400 rpm and drives a gear at 800 rpm.

**Find:** Determine the number of teeth on the gear, circular pitch and theoretical center distance.

**Schematic and Given Data:**

![Schematic](image)

- \( N_p = 24 \)
- \( m = 2 \text{ mm} \)
- \( 2400 \text{ rpm} \)
- \( N_g = ? \)
- \( 800 \text{ rpm} \)

**Assumption:** The spur gears mesh along their pitch circles.

**Analysis:**
1. For the 3:1 velocity ratio, \( N_g = 24 \times 3 = 72 \)  
2. \( p = \pi m = 2\pi \text{ mm} \)  
3. \( d = Nm \); Hence \( d_p = 48 \text{ mm}, d_g = 144 \text{ mm} \)
4. \( c = \frac{48 + 144}{2} = 96 \text{ mm} \)

**Comments:**
1. Similar to the diametral pitch, the module must be the same for a pair of meshing gears.
2. If the module were a higher value the pitch diameters of the gears and the theoretical center distance would have been higher (other parameter values remaining the same).
SOLUTION (15.11)

Known: A pair of spur gears of known velocity ratio, center distance, diametral pitch, and pressure angle are given.

Find: Draw a full-size layout of the spur gears and label the following: (a) pitch circle, (b) base circle, (c) pressure angle, (d) addendum (for both the pinion and the gear), (e) dedendum (for the pinion only)

Assumption: The gears mesh along their pitch circles.

Schematic and Given Data and Analysis:

For velocity ratio = 4, \( d_p = 4d_g \)
For \( c = 10 \) in., \( d_p + d_g = 20 \) in.
Therefore, \( d_p + 4d_p = 20 \) in.
and \( d_g = 4 \) in., \( d_g = 16 \) in.
Addendum = \( \frac{1}{5} = 0.2 \) in.
Dedendum = \( \frac{1.25}{5} = 0.25 \) in.

Note: This drawing is not drawn to scale.
SOLUTION (15.12)
**Known:** For a pair of standard 20° full-depth spur gears the diametral pitch, velocity ratio, number of teeth on the pinion and its direction of rotation are given.

**Find:** Draw a full-size layout of the spur gears in the region of tooth contact and show the following: (a) pitch circle, addendum circle, dedendum circle, and base circle of the gear, (b) interference, (c) path of contact, (d) angle of recess for the pinion and the gear.

**Assumption:** The gears mesh along their pitch circles.

**Schematic and Given Data and Analysis:**

![Diagram of gear layout](image)

- $d_p = \frac{N_p}{P} = \frac{24 \text{ teeth}}{4 \text{ teeth/in.}} = 6 \text{ in.} = d_p$
- $d_g = (d_p)(\text{velocity ratio}) = 6 \text{ in.}(2) = 12 \text{ in.} = d_g$
- Addendum = $1/P = 1/4 \text{ in.}$
- Dedendum = $1.157/P = 0.289 \text{ in.}$

**Note:** This drawing is not drawn to scale.
SOLUTION (15.13)

**Known:** A pair of involute gears of known base circle diameters with (a) center distance = 120 mm and (b) center distance = 100 mm is given.

**Find:** Determine the pressure angles of the gears for cases (a) and (b), and the ratio of pitch diameters.

**Schematic and Given Data:**

![Diagram showing involute gears with base circle diameters and pressure angle.]

**Assumption:** The spur gears mesh along their pitch circles.

**Analysis:**
1. Let $r_{bp}$ and $r_{bg}$ represent pinion and gear base circle radii, respectively.

   $$r_p = \frac{r_{bp}}{\cos \phi} \quad \text{and} \quad r_g = \frac{r_{bg}}{\cos \phi}$$

   In case (a), $c = r_p + r_g = 120 \text{ mm} = \frac{30}{\cos \phi} + \frac{60}{\cos \phi}$;

   $\cos \phi = 0.75, \phi = 0.7227 \text{ rad} = 41.4^\circ$

2. Similarly, in case (b), $\cos \phi = \frac{90}{100} = 0.9, \phi = 0.4510 \text{ rad} = 25.8^\circ$

3. $d_g/d_p = d_{bg}/d_{bp} = 120/60 = 2 \text{ (for any center dist.)}$

**Comments:**
1. With fixed base radii, reduction in center distance resulted in a reduction in pressure angle.
2. Changes in the pressure angle for the gear pair did not affect the ratio of pitch diameters since the pressure angle must be the same for meshing gears and the ratio of base diameters is fixed in this case.
SOLUTION (15.14)

**Known:** A gear has a known outside diameter, diametral pitch and pressure angle.

**Find:** Determine the pitch diameter, the circular pitch, the addendum, the dedendum of the gear, and the number of gear teeth.

**Schematic and Given Data:**

![Diagram of a gear showing addendum, dedendum, and pitch circle.]

**Assumptions:**
1. The gear is a spur gear.
2. The gear has teeth of standard involute profile.

**Analysis:**
1. Addendum, \( a = 1/P = 0.05 \) in.
   Pitch diameter, \( d_p = [(\text{outside diameter}) - 2a] \)
   \[ d_p = 3.0 - (2)(0.05) = 2.9 \text{ in.} \]
2. Module, \( m = 1/P = 0.05 \) in. = 1.27 mm. From Eq (15.5), circular pitch, \( p = \pi m \).
   Therefore, \( p = 3.99 \text{ mm.} \)
3. Dedendum, \( d = 1.25 a = 1.25(0.05) = 0.0625 \) in.
   Hence, \( d = 0.0625 \) in.
   From Eq (15.4), the number of gear teeth, \( N = d_p/m \)
   Therefore, \( N = 58. \)
SOLUTION (15.15)
Known: A pair of mating gears of known pressure angle, numbers of teeth, and center distance is given. The pinion has stub teeth and the gear has full involute teeth.

Find: Calculate the contact ratio and the diametral pitch.

Schematic and Given Data:

![Diagram of gears with given dimensions](image)

Assumptions:
1. The gears are spur gears.
2. The gears mesh along their pitch circles.

Analysis:
1. The ratio of the number of teeth, \( N_g / N_p = 36 / 18 = 2/1 \). Hence, \( d_p / d_g = 2/1 \).
   Center distance, \( c = (d_p + d_g) / 2 \)
   Therefore, \( d_p = 20/3 \), \( d_g = 40/3 \)
2. Diametral pitch, \( P = N / d = 2.7 \)
3. For full deep involute teeth, the gear addendum, \( a = 1 / P = 0.37 \). The addendum circle radius of the gear \( r_{ag} = r_g + a = 7.037 \) in. From the textbook, the addendum for a 20° stub system is 0.8/P. The addendum of the pinion, \( a_p = 0.296 \) in. The addendum circle radius of the pinion, \( r_{ap} = r_g + a_p = 3.63 \) in.
4. The base circle radius of the gear, \( r_{bg} = r_{pg} \cos \phi = (20/3) \cos 20° = 6.26 \) in. The base circle radius of the pinion, \( r_{bp} = (10/3) \cos 20° = 3.33 \)\( \cos 20° = 3.13 \)
5. From Eq. (15.10), the base pitch, \( p_b = (\pi d_b / N) \), \( p_b = 1.09 \) in.
6. From Eq. (15.9), contact ratio,

\[
CR = \frac{\sqrt{r_{ag}^2 - r_{bp}^2} + \sqrt{r_{bg}^2 - r_{bp}^2} - c \sin \phi}{p_b}
\]

\[
= \frac{\sqrt{3.63^2 - 3.13^2} + \sqrt{7.037^2 - 6.26^2} - 10 \sin 20°}{1.09} = 1.498
\]

Hence, \( CR = 1.5 \)
SOLUTION (15.16)

**Known:** For a given pair of mating gears, the pressure angle, diametral pitch and the numbers of teeth are known.

**Find:**
(a) Determine the arc of approach, arc of recess, arc of action, base pitch and the contact ratio.
(b) Estimate the addendum, dedendum, circular pitch, tooth thickness and the base diameter for the pinion and gear.

**Schematic and Given Data:**

![Diagram of gear system]

**Assumptions:**
1. The gears are spur gears.
2. The gears mesh along their pitch circles.
3. The gears have teeth of standard involute profiles.
4. Interference (contact below the base circle) does not occur.

**Analysis:**
1. Diametral pitch, \( P = N_g/d_g \). Hence, \( d_g = 2.625 \text{ in.} \). Since \( N_g/N_p = d_g/d_p = 84/17 \), \( d_p = 0.53125 \text{ in.} \). Center distance, \( c = (d_g + d_p)/2 = 1.57813 \text{ in.} \).
2. Addendum, \( a = 1/P = 0.03125 \text{ in.} \).
   Dedendum, \( d = 1.25a = 0.039 \text{ in.} \).
   From Eq. (15.5), circular pitch, \( p = 0.098 \text{ in.} = 2.5 \text{ mm.} \).

15-15
3. Tooth thickness, \( t = p/2 = 1.25 \text{ mm.} \)

4. Base circle diameter of the gear, \( d_{bg} = d_g \cos \phi \)
   \( d_{bg} = 2.625 \cos 20^\circ = 2.47 \text{ in.} \)

   Base circle diameter of the pinion, \( d_{bp} = d_p \cos \phi \)
   \( d_{bp} = 0.53125 \cos 20^\circ = 0.49921 \text{ in.} \)

5. From Eq. (15.10), base pitch, \( p_b = (\pi d_b)/N \). Therefore, \( p_b = 0.092 \text{ in.} \)

6. Addendum circle radius of the gear, \( r_{ag} = 1.34 \text{ in.} \)
   Addendum circle radius of the pinion, \( r_{ap} = 0.296 \text{ in.} \)
   From Eq. (15.9); contact ratio,
   \[ CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - \pi \sin \phi}{p_b} \]
   \[ = \frac{\sqrt{0.296^2 - 0.249^2} + \sqrt{1.34^2 - 1.235^2} - 1.5781 \sin 20^\circ}{0.092} = 1.525 \]

7. Arc of action \( = (CR)p = (1.525)(0.098) = 0.15 \text{ in.} \)

**Comment:** Although an assumption was made that interference (contact below the base circle) would not take place, a calculation needs to be performed to determine whether there will be interference when standard full-depth teeth are used. Following SAMPLE PROBLEM 15.1, we first determine the base circle radii of pinion and gear. From Eq. 15.11, \( r_{bp} = (0.53125/2) \cos 20^\circ = 0.2496 \text{ in. and } r_{bg} = (2.625/2) \cos 20^\circ = 1.2333 \text{ in.} \) Substitution in Eq. 15.8 gives \( r_{ap(max)} = 0.59467 \) for the pinion and \( r_{ag(max)} = 1.34624 \) for the gear. The limiting outer gear radius is equivalent to an addendum of \( r_{ag(max)} - r_g = 0.03374 \text{ in.} \), whereas a standard full-depth tooth has an addendum of \( 1/P = .03125 \text{ in.} \). So, the use of standard teeth should not cause interference.

**SOLUTION (15.17)**

**Known:** For a pair of mating gears, the diametral pitch, center distance and the number of teeth are given. The center distance is increased by 0.125 in.

**Find:** Determine the contact ratio and the pressure angle.

**Schematic and Given Data:**

![Diagram of gear system with dimensions and labels]
Assumptions:
1. The gears are physically the same spur gears as in Problem 15.16.
2. The base and outside diameters of the gear and pinion remain the same as in Problem (15.16).
3. The gears mesh along their actual (not theoretical) pitch circles.
4. The gears have teeth of standard involute profile.

Analysis:
1. If the center distance is increase by 0.125 in., the pinion and gear will no longer mesh -- see the following diagram.
2. If the center distance is increased by 0.0125 in., the gears will mesh along their actual (not theoretical) pitch circles.

3. From the analysis of Problem (15.16) and the given data, center distance, \( c = 1.5781 + 0.0125 = 1.5906 \) in.
4. \( N_p/N_g = d_p/d_g = 17/84 \). Hence, \( d_g = 4.9412d_p \).
   \( c = (d_p + d_g)/2 = 1.5906 \). Solving for \( d_g \) and \( d_p \), \( d_g = 2.64575 \) in., \( d_p = 0.53545 \) in.
5. \( d_{bg} = 2.4667 = d_g \cos \phi = 2.64575 \cos \phi \). Solving for \( \phi \), \( \phi = 21.20^\circ \). Therefore, pressure angle, \( \phi = 21.20^\circ \).
6. Addendum circle radius of the gear, \( r_{ag} = 1.34 \) in.
Addendum circle radius of the pinion, \( r_{ap} = 0.249 \) in.
From Eq. (15.9); contact ratio,

\[
CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c \sin \phi}{P_b}
\]

\[
= \frac{\sqrt{0.296^2 - 0.249^2} + \sqrt{1.34^2 - 1.235^2} - 1.5906 \sin 21.20^\circ}{0.092} = 1.1394
\]

SOLUTION (15.18)

**Known:** A pair of mating spur gears of known module, pressure angle and numbers of teeth is given.

**Find:**
(a) Sketch drawings showing geometric details.
(b) Determine the lengths of path of contact, angles of approach and recess, and contact ratio.

**Schematic and Given Data:**

Note: This drawing is not drawn to scale.
Assumption: The spur gears mesh along their pitch circles.

Analysis:
1. From Eq. (15.4); \( d_p = m N_p = 8(30) = 240 \text{ mm} \)
   \( d_g = m N_g = 6(60) = 360 \text{ mm} \)
   \( \text{addendum} = a = m = 8 \text{ mm} \)
   \( \text{dedendum} = 1.25m = 10 \text{ mm} \)

2. From Eq. (15.10); \( p_b = \frac{\pi d_{bp}}{N_p} = \frac{\pi (240 \cos(0.35))}{30} \)
   \( p_b = 23.6 \text{ mm} \)
   contact ratio = length of line of action/p_b
   from the drawing, CR = 38/23.6 = 1.610.

Comments:
1. Increasing the contact ratio of the gear pair for the same numbers of teeth can be achieved by increasing the diametral pitch for the gears.
2. If the gears have a 6 mm module, then the contact ratio CR can be calculated as follows.
   From Eq. (15.4); \( d_p = m N_p = 6(30) = 180 \text{ mm} \)
   \( d_g = m N_g = 6(60) = 360 \text{ mm} \)
   \( \text{addendum} = a = m = 6 \text{ mm} \)
   \( \text{dedendum} = 1.25m = 7.5 \text{ mm} \)

   From Eq. (15.10); \( p_b = \frac{\pi d_{bp}}{N_p} = \frac{\pi (180 \cos(0.35))}{30} \)
   \( p_b = 17.7 \text{ mm} \)
   contact ratio = length of line of action/p_b
   from the drawing, CR = 38/17.7 = 2.15.

SOLUTION (15.19)
Known: A pair of standard spur gears of known pressure angle, center distance and velocity ratio is given. Number of teeth on pinion is specified.

Find:
(a) Determine diametral, circular and base pitches.
(b) Sketch drawings showing geometric details.
(c) Determine if interference results from choice of standard tooth proportions.
(d) Determine length of path of contact from drawing and compute contact ratio.
Schematic and Given Data:

Assumption: The spur gears mesh along their pitch circles.

Analysis:
1. \( c = 10 = r_p + r_g \) and \( r_g = 4r_p \);
   hence, \( r_p = 2 \) in. and \( r_g = 8 \) in.
   \( P = N_p/d_p = 20/4 \); hence, \( P = 5 \)
   \( p = \pi/5 \approx 0.6280 \) in.
   \( p_b = p \cos \phi = 0.628 \cos 20^\circ = 0.590 \) in.

2. Addendum = \( \frac{1}{5} \) in. = 0.2 in.
   Dedendum = \( 1.25 \times \frac{1}{5} \) = 0.25 in.

3. \( r_{ap(max)} = 4.0 \) in., \( r_{ag(max)} = 8.3 \) in.
   Hence, no interference.

4. Path of contact \( \approx 1.0 \) in.
CR = \frac{\text{Path of contact}}{P_b} = \frac{1.0}{.590} = 1.695

Hence, CR = \text{roughly 1.7}

\text{SOLUTION (15.20)}

\text{Known: A pair of standard spur gears of known pressure angle, center distance and velocity ratio is given. Number of teeth on pinion is specified.}

\text{Find: Compute the contact ratio using equations in Section 15.3 and compare with graphical results of Problem 15.19.}

\text{Schematic and Given Data:}

\text{Assumption: The gears mesh along their pitch circles.}

\text{Analysis:}

1. \text{From Eq. (15.11), } r_{bp} = r_p \cos \phi = 2.0 \cos 20^\circ = 1.879 \text{ in.}

r_{pg} = r_g \cos \phi = 8 \cos 20^\circ = 7.517 \text{ in.}
From Eq. (15.8),

\[ r_{ap}(\text{max}) = \sqrt{1.879^2 + 10.0^2 \sin^2 20^\circ} = 3.90 \text{ in.} \]

\[ r_{ag}(\text{max}) = \sqrt{7.517^2 + 10.0^2 \sin^2 20^\circ} = 8.26 \text{ in.} \]

(this agrees with graphical solution)

2. From Eq. (15.9), (with \( r_{ap} = 2.2 \) and \( r_{ag} = 8.2 \) from Problem 15.19)

\[ CR = \frac{\sqrt{2.2^2 - 1.879^2} + \sqrt{8.2^2 - 7.517^2} - 10 \sin 20^\circ}{0.590} \]

\[ CR = 1.69 \] (which is more accurate than the graphical solution.)

**Comment:** The contact ratio can be increased by choosing a greater number of teeth on the gears and/or increasing the diametral pitch.

---

**SOLUTION (15.21)**

**Known:** A two stage spur gear speed reducer is given which uses a countershaft and identical gear pairs in each stage. Gear and shaft geometry is specified such that input and output shafts are collinear.

**Find:**

(a) Determine the rotation speeds of countershaft and output shaft, pitch diameters and circular pitch of the pinion and gear.

(b) Determine the torques carried by each shaft assuming (i) 100% gear efficiency and (ii) 95% efficiency of each gear pair.

(c) Determine the radial loads applied to bearings on countershaft.

**Schematic and Given Data:**

![Schematic diagram of a spur gear speed reducer](image_url)
Assumptions:
1. The gears mesh along their pitch circles.
2. The shafts are all parallel.
3. Friction losses in the bearings can be neglected (given).
4. All the gear radial and tangential load is transferred at the pitch point.
5. Bending deflection of the countershaft is negligible.
6. The location of the bearing loads can be idealized to points due to small bearing widths.
7. The location of the tooth loads can be idealized to points due to small tooth face width.
8. The gears are rigidly connected to their shafts.

Analysis:
1. The pitch diameters: \( d_p = \frac{15}{5} = 3 \text{ in.}, \quad d_g = \frac{45}{5} = 9 \text{ in.} \)
   - The circular pitch: \( p = \frac{\pi}{5} = 0.63 \text{ in.} \)
   - rpm of (b) = \( 1200 \times \frac{15}{45} = 400 \text{ rpm} \)
   - rpm of (c) = \( 400 \times \frac{15}{45} = 133.3 \text{ rpm} \)
2. In case (i):
   - From Eq. (1.2),
     \[
     \text{the torque in shaft (a)} = \frac{9549(1 \text{ kW})}{1200 \text{ rpm}} = 7.96 \text{ N\cdot m}
     \]
   - the torque in (b) = \( T_b = 7.96 \times \frac{45}{15} = 23.88 \text{ N\cdot m} \)
   - the torque in shaft (c) = \( T_c = 23.88 \times \frac{45}{15} = 71.64 \text{ N\cdot m} \)
   - In case (ii):
     - \( T_b = (23.88)(0.95) = 22.69 \text{ N\cdot m} \)
     - \( T_c = (22.69)\frac{45}{15}(0.95) = 64.65 \text{ N\cdot m} \)
3. \( r_p = 1.5 \text{ in.} = 38.1 \text{ mm}, \quad r_g = 4.5 \text{ in.} = 114.3 \text{ mm} \)
   - tangential force on the motor pinion = \( \frac{7.96 \text{ N\cdot m}}{0.0381 \text{ m}} = 209 \text{ N} \)
   - radial force on the motor pinion = \( 209 \text{ N} \cdot \tan 25^\circ = 97.5 \text{ N} \)
4. Forces on the countershaft pinion are 3 times as large i.e., = 627 N and 292.5 N
5. For the horizontal plane:
\[ \Sigma M_A = 0 : 209(25) + 627(125) - B_H(100) = 0 \]
hence, \( B_H = 836.0 \text{ N} \)
\[ \Sigma F = 0 : 836.0 - 627 + 209 - A_H = 0 \]
hence, \( A_H = 418.0 \text{ N} \)
6. For the vertical plane:
\[ \Sigma M_A = 0 : 97.5(25) - 292.5(125) + B_V(100) = 0 \]
hence, \( B_V = 341.25 \text{ N} \)
\[ \Sigma F = 0 : 97.5 + 292.5 - 341.25 - A_V = 0 \]
hence, \( A_V = 48.75 \text{ N} \)
7. The radial loads are:
\[ A_r = \sqrt{418^2 + 48.75^2} = 420.83 \text{ N} \]  
\[ B_r = \sqrt{836^2 + 341.25^2} = 902.96 \text{ N} \]

Comments:
1. The effect of power losses in each stage (efficiency 95%) was to decrease the torque transmitted to the output while keeping the speeds ratios the same. Thus, each stage reduces the torque transmitted by a factor.
2. The effect of considering friction losses in the bearings will also be to reduce the torque transmitted to the output shaft.
3. Large bending deflections of the countershaft will render our calculation of force and moment equilibrium inaccurate due to changes in the force directions and moment arms.
4. If the bearing widths or gear face widths were significant relative to the length of the countershaft, location of the forces and reactions as point loads would lead to inaccurate estimates of bearing loads.
5. The pinion of the countershaft has higher tooth loads than the gear of the countershaft leading to higher radial loads on the bearing closer to the pinion.
SOLUTION (15.22)

Known: A two stage spur gear speed reducer is given which uses a countershaft and identical gear pairs in each stage. Gear and shaft geometry is specified such that input and output shafts are collinear.

Find:
(a) Determine the rotation speeds of countershaft and output shaft, pitch diameters and circular pitch of the pinion and gear.
(b) Determine the torques carried by each shaft assuming (i) 100% gear efficiency and (ii) 95% efficiency of each gear pair.
(c) Determine the radial loads applied to bearings on countershaft.

Schematic and Given Data:

Assumptions:
1. The gears mesh along their pitch circles.
2. The shafts are all parallel.
3. Friction losses in the bearings can be neglected (given).
4. All the gear radial and tangential load is transferred at the pitch point.
5. Bending deflection of the countershaft is negligible.
6. The location of the bearing loads can be idealized to points due to small bearing widths.
7. The location of the tooth loads can be idealized to points due to small tooth face width.
8. The gears are rigidly connected to their shafts.
Analysis:
1. The pitch diameters: \( d_p = 21/5 = 4.2 \text{ in.} \), \( d_g = 62/5 = 12.4 \text{ in.} \)

   The circular pitch: \( p = \pi/5 = 0.63 \text{ in.} \)

   rpm of (b) = 1200 \times \frac{21}{62} = 406.5 \text{ rpm}

   rpm of (c) = 400 \times \frac{21}{62} = 135.5 \text{ rpm}

2. In case (i):

   From Eq. (1.2),

   \[
   \text{the torque in shaft (a)} = \frac{9549(1 \text{kW})}{1200 \text{ rpm}} = 7.96 \text{ N} \cdot \text{m}
   \]

   \[
   \text{the torque in (b)} = T_b = 7.96 \times \frac{62}{21} = 23.5 \text{ N} \cdot \text{m}
   \]

   \[
   \text{the torque in shaft (c)} = T_c = 23.5 \times \frac{62}{21} = 69.4 \text{ N} \cdot \text{m}
   \]

   In case (ii):

   \[
   T_b = (23.5)(0.95) = 22.325 \text{ N} \cdot \text{m}
   \]

   \[
   T_c = (22.325)\frac{62}{21}(0.95) = 62.6 \text{ N} \cdot \text{m}
   \]

3. \( r_p = 2.1 \text{ in.} = 53.34 \text{ mm} \), \( r_g = 6.2 \text{ in.} = 157.5 \text{ mm} \) tangential force on the motor pinion = \( \frac{7.96 \text{ N} \cdot \text{m}}{0.05334 \text{ m}} = 149 \text{ N} \)

   radial force on the motor pinion = \( (149 \text{ N})\tan 25^\circ = 69.5 \text{ N} \)

4. Forces on the countershaft pinion are 3 times as large; i.e., \( = 447 \text{ N} \) and \( 208.5 \text{ N} \)

5. For the horizontal plane:

   \[
   \Sigma M_A = 0 : 149(25) + 447(125) - B_H(100) = 0
   \]

   hence, \( B_H = 596.0 \text{ N} \)

   \[
   \Sigma F = 0 : 596.0 - 447 + 149 - A_H = 0
   \]

   hence, \( A_H = 298.0 \text{ N} \)

6. For the vertical plane:

   \[
   \Sigma M_A = 0 : 69.5(25) - 208.5(125) + B_V(100) = 0
   \]

   hence, \( B_V = 243.25 \text{ N} \)
\[ \Sigma F = 0 : \ 69.5 + 208.5 - 243.25 - A_v = 0 \]

\[ \text{hence, } A_v = 34.75 \text{ N} \]

7. The radial loads are:

\[ A_r = \sqrt{298^2 + 34.75^2} = 300: A_r = 300 \text{ N} \]
\[ B_r = \sqrt{596^2 + 243.25^2} = 643.73: B_r = 644 \text{ N} \]

Comments:

1. The effect of power losses in each stage (efficiency 95%) was to decrease the torque transmitted to the output while keeping the speeds ratios the same. Thus, each stage reduces the torque transmitted by a factor.

2. The effect of considering friction losses in the bearings will also be to reduce the torque transmitted to the output shaft.

3. Large bending deflections of the countershaft will render our calculation of force and moment equilibrium inaccurate due to changes in the force directions and moment arms.

4. If the bearing widths or gear face widths were significant relative to the length of the countershaft, location of the forces and reactions as point loads would lead to inaccurate estimates of bearing loads.

5. The pinion of the countershaft has higher tooth loads than the gear of the countershaft leading to higher radial loads on the bearing closer to the pinion.

---

**SOLUTION (15.23)**

**Known:** A two stage spur gear speed reducer of specified geometry is given.

**Find:** Determine the radial loads applied to the countershaft bearings.

**Schematic and Given Data:**

![Schematic Diagram]
Assumptions:
1. The gears mesh along their pitch circles.
2. Friction losses in gears and bearings can be neglected (given).
3. The shafts are all parallel.
4. All the gear radial and tangential load is transferred at the pitch point.
5. Bending deflection of the countershaft is negligible.
6. Gear face width and bearing widths are negligible relative to countershaft length.
7. The gears are rigidly connected to their shafts.

Analysis:

1. Pitch diameter of input pinion $= \frac{N}{P} = \frac{18}{9} = 2$ in.
2. For the horizontal plane:
   $\Sigma M_A = 0$:
   $20 \text{ lb (2 in.)} + 35.55 \text{ lb (10 in.)} - B_H(12 \text{ in.}) = 0$
   hence, $B_H = 32.96$ lb
   $\Sigma F = 0 : 20 \text{ lb} + 35.55 \text{ lb} - 32.96 \text{ lb} - A_H = 0$
   hence, $A_H = 22.59$ lb
3. For the vertical plane:
   $\Sigma M_A = 0$:
   $9.33 \text{ lb (2 in.)} - 16.58 \text{ lb (10 in.)} + B_V(12 \text{ in.}) = 0$
   hence, $B_V = 12.26$ lb
   $\Sigma F = 0 : -9.33 \text{ lb} + 16.58 \text{ lb} - 12.26 \text{ lb} + A_V = 0$
   hence, $A_V = 5.01$ lb
4. The bearing radial loads are:
   $A_{rad} = \sqrt{22.59^2 + 5.01^2} = 23.14$ lb, hence, $A_{rad} = 23.14$ lb
   $B_{rad} = \sqrt{32.96^2 + 12.26^2} = 35.17$ lb, hence, $B_{rad} = 35.17$ lb
Comments:
1. The effect of considering friction losses in the gears and bearings is to reduce the torque transmitted to the output shaft while keeping the speed ratios the same. A reduction in torque transmitted will result in lower gear tooth loads and hence lower radial loads on the countershaft bearings.
2. The pinion on the countershaft has higher tooth loads than the gear on the countershaft because the pinion has a smaller radius while transmitting the same torque. The higher tooth load on the pinion leads to a higher radial load on the bearing closer to the pinion.
3. The effect of choosing a smaller diametral pitch for the gears in the second reduction stage is to provide larger teeth to withstand the higher torques and tooth loads of the second stage.
4. If load sharing between teeth is considered, the transfer of gear tooth forces is not strictly at the pitch point and will lead to different radial forces in the bearings.

SOLUTION (15.24)
Known: A two stage spur gear speed reducer of specified geometry is given.

Find: Determine the radial loads applied to the countershaft bearings.

Schematic and Given Data:

Assumptions:
1. The gears mesh along their pitch circles.
2. Friction losses in gears and bearings can be neglected (given).
3. The shafts are all parallel.

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4. All the gear radial and tangential load is transferred at the pitch point.
5. Bending deflection of the countershaft is negligible.
6. Gear face width and bearing widths are negligible relative to countershaft length.
7. The gears are rigidly connected to their shafts.

Analysis:

1. Pitch diameter of input pinion $= \frac{N}{P} = 20/9 = 2.22$ in.

2. For the horizontal plane:
   \[ \Sigma M_A = 0 : \]
   \[ 18.02 \text{ lb (2.0 in.)} + 32.04 \text{ lb (10 in.)} - B_H(12 \text{ in.}) = 0 \]
   Hence, $B_H = 29.7$ lb
   \[ \Sigma F = 0 : 18.02 \text{ lb} + 32.04 \text{ lb} - 29.7 \text{ lb} - A_H = 0 \]
   Hence, $A_H = 20.36$ lb

3. For the vertical plane:
   \[ \Sigma M_A = 0 : \]
   \[ 8.4 \text{ lb (2.0 in.)} - 14.94 \text{ lb (10 in.)} + B_V(12 \text{ in.}) = 0 \]
   Hence, $B_V = 11.05$ lb
   \[ \Sigma F = 0 : -8.4 \text{ lb} + 14.94 \text{ lb} - 11.05 \text{ lb} + A_V = 0 \]
   Hence, $A_V = 4.51$ lb

4. The bearing radial loads are:
   \[ A_{rad} = \sqrt{20.36^2 + 4.51^2} = 20.85 \text{ lb} \]
   \[ B_{rad} = \sqrt{29.7^2 + 11.05^2} = 31.69 \text{ lb} \]

Comments:
1. The effect of considering friction losses in the gears and bearings is to reduce the torque transmitted to the output shaft while keeping the speed ratios the same. A reduction in torque transmitted will result in lower gear tooth loads and hence lower radial loads on the countershaft bearings.
2. The pinion on the countershaft has higher tooth loads than the gear on the countershaft because the pinion has a smaller radius while transmitting the same torque. The higher tooth load on the pinion leads to a higher radial load on the bearing closer to the pinion.

3. The effect of choosing a smaller diametral pitch for the gears in the second reduction stage is to provide larger teeth to withstand the higher torques and tooth loads of the second stage.

4. If load sharing between teeth is considered, the transfer of gear tooth forces is not strictly at the pitch point and will lead to different radial forces in the bearings.

SOLUTION (15.25)

Known: For a single stage speed reducer, gear geometry, overhang and the horsepower transmitted are specified.

Find: Estimate the forces on the pinion, gear and shafts.

Schematic and Given Data:
Assumptions:
1. The gears are spur gears.
2. The gears mesh along their pitch circles.
3. Friction losses in the gears and bearings can be neglected.
4. The shafts are parallel.
5. Bending deflection of each shaft is negligible.
6. The gears are rigidly connected to their shafts.
7. The weight of the gear on its shaft can be neglected.

Analysis:
1. Pitch diameter of the input pinion, \( d_p = N/p = 18/6 = 3 \) in.
   Pitch line velocity, \( V = \pi dn/12 = \pi (3)1800/12 = 1413.7 \) ft/min.
2. From Eq. (15.14), the power transmitted, \( \dot{W} = (F_t V/33000) \)
   \[
   F_t = \frac{(0.5)(33000)}{1413.7} = 11.67 \text{ lb}
   \]
   From Eq. (15.12), \( F_T = F_t \tan \phi = (11.67) \tan 20^\circ \). Hence, \( F_t = 4.25 \text{ lb} \).
   Therefore, the force on the pinion, \( F = \sqrt{F_t^2 + F_T^2} \)
   Hence, \( F = \sqrt{11.67^2 + 4.25^2} = 12.42 \) lb.
3. From Newton's third law, the force on the gear tooth equals the force on the pinion tooth.

SOLUTION (15.26)
Known: Three identical spur gears are used to transmit power from a motor to a machine through an idler with all three gears simply supported between identical bearings.

Find: Determine the relative loadings on the six bearings.

Schematic and Given Data:
**Assumptions:**
1. The gears mesh along their pitch circles.
2. Friction losses in the gears and bearings are negligible.
3. All the tooth loads are transferred at the pitch point.
4. The shafts are all parallel.

**Analysis:**
1. The tangential gear force = \( \frac{\text{motor torque}}{\text{gear radius}} \)
   
   let this be equal to 2 (in arbitrary units)
2. For the motor shaft:

   ![Diagram of motor shaft](image)

   bearing radial loads are \( \sqrt{1 + \tan^2 \phi} \)

   for \( \phi = 25^\circ \), load = 1.10
3. For the idler shaft:

   ![Diagram of idler shaft](image)
bearing radial loads = 2.0
4. For the output shaft it is the same as the motor shaft.
5. Conclusion: Idler shaft bearing loads are nearly double those applied to the other shafts.

Comments:
1. The equal and opposite radial tooth loads on the idler cancel each other and hence do not contribute to the bearing loads for the idler in this case. However, if the radii of the input and output gears were not equal the radial tooth loads would be unequal and would then effect the bearing loads for the idler.
2. Friction losses in the gears and bearings reduce the torque transferred to the driven gears. A reduction in torque and consequently tooth loads would result in reduction in bearing loads.
3. A reduced pressure angle of 20° for the gears would reduce the radial loads on the motor and machine shaft bearings to 1.06 units but retain each idler bearing load at 2.0 units.
SOLUTION (15.27)

**Known:** Three spur gears transmit power from a motor shaft to a machine shaft in a given geometric arrangement. The middle gear acts as an idler and is supported by two bearings.

**Find:**
(a) Determine the radial load on idler shaft bearings for a given direction of motor shaft rotation.
(b) Determine the radial load on the bearings for the motor shaft rotation opposite to (a).
(c) Give an explanation as to why answers to (a) and (b) are different.

**Schematic and Given Data:**

![Diagram of gear system]

**Assumptions:**
1. The gears mesh along their pitch circles.
2. All the gear tooth loads are static and are transmitted at the pitch point.
3. Friction losses in the gears and bearings are negligible.
4. Shaft bending deflections can be neglected.

**Analysis:**
(a) Pitch diameter of idler $= \frac{N}{P} = \frac{32}{8} = 4.0$ in.
Bearing radial loads: \( R_A = 289 \text{ lb}, R_B = 96.1 \text{ lb} \)

(b) Bearing loads are reduced by factor of 90/192.9 to give:
\( R_A = 135 \text{ lb}, R_B = 45 \text{ lb} \)

(c) Reversing direction of rotation reversed tangential forces, causing tangential and radial components to subtract, rather than add.
Comments:
1. This problem illustrates the use of gear trains for purposes other than strictly speed or torque changing. Idlers are frequently used to convey rotary motion short distances from driver to driven shafts or to drive multiple shafts.
2. The location of bearings for the idler shaft in this problem caused a large radial load on the bearing closer to the idler and a smaller radial load on the other bearing. Although location of bearings on either side of the idler could have equalized the radial loads, such an arrangement may or may not be allowed by space constraints or accessibility requirements in actual applications.
3. For cases in which the rotation of the shaft is reversed from that shown in the figure, the explanation in part (c) reveals that the arrangement of the gears can be changed, putting the driven gear to the left of the idler in the figure to obtain lower bearing loads.

SOLUTION (15.28)
Known: Three identical spur gears are used to transmit power from a motor to a machine through an idler. Motor rpm is specified.

Find:
(a) Determine the gear most vulnerable to tooth bending fatigue failure.
(b) Determine the values for V, P, p, K_v, J

Schematic and Given Data:

Assumption:
1. The gears mesh along their pitch circles and transmit all the load at the pitch point.
2. Friction losses can be neglected and load sharing is absent.

Analysis:
(a) The gear most vulnerable to tooth bending fatigue failure is the idler because it is subjected to 2-way bending; others are bent only 1-way, thus:
\( V = \frac{\pi (8)}{12} \cdot 1000 \quad \Rightarrow \quad V = 2094 \text{ ft/min} \)

\( P = 80/8 \quad \Rightarrow \quad P = 10 \text{ teeth/in.} \)

\( p = \pi/10 \quad \Rightarrow \quad p = 0.314 \text{ in.} \)

\( K_v = \frac{50 + \sqrt{2094}}{50} \quad \Rightarrow \quad K_v = 1.92 \)

\( J = 0.375 \quad \text{ (Fig. 15.23b; no load sharing)} \)

**Comments:**
1. Larger diameters and higher rpm for gears produce larger values for \( K_v \).
2. A pressure angle of 20° instead of 25° would reduce the value of the geometry factor from 0.375 to 0.31.

**SOLUTION (15.29)**

**Known:** A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

**Find:** Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based only on bending fatigue.

**Schematic and Given Data:**

- **Pinion:** \( N_p = 20 \) rpm
- **Gear:** \( N_g = 40 \) rpm
- **Material:** Steel, heat treated to 350 Bhn
- **Standard full depth teeth**
- **Accurate mounting**
- **P = 8**
- **\( \phi = 20^\circ \)**
- **b = 1 in.**
- **Design life:** 5 yrs, 60 hr/wk, 50 wk/yr operation
- **Top quality hobbing operation for manufacturing**

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Assumptions:
1. The gears mesh along their pitch circles.
2. All the gear tooth loads are transmitted at the pitch point.
3. There is no loadsharing between the teeth.
4. The electric motor and blower constitute uniform load driver and driven equipment.
5. Top quality hobbing operation for manufacturing corresponds to curve C in Fig. 15.24 (to estimate velocity factor \(K_v\)).

Analysis:
1. From Eq. (15.17): \(\sigma = \frac{F_i P}{b J} K_v K_o K_m\)
   \(K_v\) requires finding the pitch line velocity as,
   \(V = \frac{\pi d n}{12} = \frac{\pi(20/8)1100}{12} = 720 \text{ fpm}\)
   from Curve C of Fig. 15.24,
   \(K_v = \frac{50 + \sqrt{720}}{50} = 1.54\)
   from Fig. 15.23(a), \(J = 0.24\) (for the pinion, as it is weaker - and with no load sharing)
   Also, \(K_m = 1.6\) (from Table 15.2 - probably best judgement)
   and \(K_o = 1.0\) (from Table 15.1- uniform driving and driven torque)
   Therefore,
   \(\sigma = \frac{F_i(8)}{(1.0)(0.24)} (1.54)(1.0)(1.6) = 82.1 F_i\)
2. From Eq. (15.18):
   \(S_n = S_n' C_L C_G C_s k_r k_t k_{ms}\)
   \(= (250 \times 350)(1)(1)(0.66)(0.814)(1)(1.4)\)
   \(= 65,812 \text{ psi}\)
   where
   \(S_n' = 250 \text{ (Bhn)} = 250 \times 350 \text{ psi for infinite life,}\)
   since design life = 5 yr \(\times\) (50 wk/yr) \(\times\) (60 hr/wk) \(\times\) (60 min/hr) \(\times\) 1100 rpm
   \(= 9.9 \times 10^8 > 10^6 \text{ cycles}\)
   \(C_L = 1.0,\)
   \(C_G = 1.0 \text{ since } P > 5,\)
   \(C_s = 0.66 \text{ from Fig. 8.13}\)
   \(k_r = 0.814 \text{ from Table 15.3}\)
   \(k_t = 1 \text{ and } k_{ms} = 1.4 \text{ since the pinion is not an idler}\)
3. For SF = 1.5 : 82.1(1.5 F_i) = 65,812
   hence, \(F_i = 534.4 \text{ lb}\)
   \(\dot{W} = \frac{F_i V}{33,000} = \frac{(534.4)(720)}{33,000} = 11.66 \text{ hp}\)
   Answer : approximately 11.7 hp
Comments:
1. The bending stresses can be reduced for the specified rpm by decreasing P or increasing b. But these parameters as well as the factors $K_v$ and $J$ are closely interrelated. Decreasing P for the same number of teeth increases pitch diameter which leads to larger pitch line velocity and hence to larger values of $K_v$ and $\sigma$. Decreasing P for the same pitch diameter decreases the number of teeth resulting in a smaller value of $J$ and a larger value of $\sigma$. Increasing the value of b requires accurate mounting and manufacturing to utilize the entire face width and ultimately tends to increase the value of $K_m$. Thus choice of suitable values for gear geometry parameters for specific applications requires balancing the parameter values with other side effects.
2. In this problem the design life of the gear pair did not enter into the solution except to determine whether the gears were to be rated for finite or infinite life.

SOLUTION (15.30)
Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: Determine the Bhn of gear so that gear and pinion teeth have the same factor of safety with respect to bending fatigue.

Schematic and Given Data:

Assumptions:
1. The gears mesh along their pitch circles.
2. All the gear tooth loads are transmitted at the pitch point.
3. There is no loadsharing between the teeth.
4. The electric motor and blower constitute uniform load driver and driven equipment.
5. Top quality hobbing operation for manufacturing corresponds to curve C in Fig. 15.24 (to estimate velocity factor $K_v$).
Analysis:
1. \( J \) for gear = 0.285 from Fig. 15.23a

2. From the analysis of Problem 15.29, gear tooth stress is only \( \frac{0.24}{0.285} \) times pinion stress

3. Since all parameters and factors are identical for the pinion and gear except \( Bhn \) and \( C_s \), \( (\text{gear } Bhn) \cdot C_s \) could theoretically be

\[
(350 \cdot 0.66) \left( \frac{0.24}{0.285} \right) = 194.5
\]

From Fig. 8.13 by trial and error:

\[ \text{gear } Bhn = 274 \text{ (and } C_s = 0.71) \]

Comment: The gear material can have a lower strength than the pinion material because the stress concentrations at the root of the gear teeth are lower than at the root of the pinion teeth as a result of the gear having a larger diameter.

SOLUTION (15.31)
Known: A spur gear speed reducer is driven by an electric motor and drives a load involving "moderate shock". The gear teeth are standard full depth and of specified geometry and material. Required life is \( 10^6 \) pinion revolutions for a specified transmitted load.

Find: Determine an estimate of the reliability of the speed reducer with respect to bending fatigue failure.

Schematic and Given Data:

Assumptions:
1. The spur gears mesh at the pitch circles.
2. Load sharing is not expected since the cutting process is of average quality.
3. The effects corrected by the velocity factor, \( K_v \), correspond to the middle of the range in Fig. 15.24 with manufacture by form cutters.
4. The pinion is driven by a uniform power motor while the gear drives a load involving "moderate shock" (given).
5. The tooth fillet radius is approximately equal to 0.35/P (to enable the use of Fig. 15.23 to estimate geometry factor J).

**Analysis:**
1. From Fig. 15.23(a), with no load sharing, \( J = 0.24 \).
   From Eq. (15.13a),
   
   \[
   V = \frac{\pi d n}{12} = \frac{\pi N_{p}n_{p}}{12P} = \frac{\pi(18)(1500)}{(12)(10)} = 706.8 \text{ ft/min}
   \]
   From Fig. 15.24, with \( V = 706.8 \text{ ft/min} \), \( K_V = 2.0 \)
   From Table 15.1, \( K_0 = 1.25 \)
2. From Eq. (15.17) applied to the pinion:
   \[
   \sigma = \frac{F_t P}{b J} K_V K_0 K_m = \frac{100(10)}{1.0(0.24)} (2.0)(1.25)(1.8)
   \]
   \( \sigma = 18750 \text{ psi} = 18.75 \text{ ksi} \)
3. From Eq. (15.18) applied to the pinion:
   \[
   S_n = S_n' C_L C_G C_s k_r k_t k_{ms}
   \]
   \[
   S_n = (65)(1)(1)(0.72)k_r(1.4) = 65.52 k_r \text{ ksi}
   \]
   since, \( S_u \equiv 500(\text{BHN}) = 500(260) \text{ psi} = 130 \text{ ksi} \).
   \[
   S_n' = S_u/2 = 65 \text{ ksi} \text{ and for bending loads, } C_L = 1.0, \text{ for } P > 5, C_G = 1.0,
   \text{ from Fig. 8.13, } C_s = 0.72.
   \]
   Therefore \( 18.75 = 65.52 k_r \); hence, \( k_r = 0.29 \)
4. Similarly for the gear, \( J = 0.27 \), \( S_n' = 58.75 \text{ ksi} \),
   \( C_s = 0.75 \); hence, \( k_r = 0.30 \)
5. From Table 15.3, reliability is \( >> 99.999\% \) \( \blacksquare \)

**Comments:**
1. The reliability estimated in this problem is based on considering failure only by bending fatigue. A more accurate estimate of reliability must consider failure by surface fatigue also.
2. Increasing the hardness of the gears will result in new choices in transmitting a higher load and higher rpm or choosing a smaller face width or a larger diametral pitch (i.e., with finer teeth).
3. The choice of a harder material for the pinion gives approximately the same reliability for both the pinion and gear in this case. Thus choice of a harder material for the pinion reflects consistency in the strength design of the gears.
SOLUTION (15.32)

**Known:** An identical pair of standard full depth spur gears of given geometry and material rotate at a given rpm.

**Find:** Determine an estimate of the horsepower that can be transmitted with 99% reliability based on tooth bending fatigue.

**Schematic and Given Data:**

![Schematic](image)

- **Identical spur gears:**
  - \( N = 60 \)
  - \( \phi = 20^\circ \)
  - \( P = 12 \)
  - \( b = 1.0 \text{ in.} \)
- Alloy steel material, case hardness 680 Bhn, core hardness 500 Bhn.
- Tooth profiles finished and ground to requirement of curve A in Fig. 15.24.
- Overload factor, \( K_o = 1.1 \)

**Assumptions:**
1. The gears are mounted on accurate mountings to mesh along their pitch circles.
2. Loading of the gears involves only mild shock (given).
3. High precision gears with fine ground tooth profiles allow the use of curve A in Fig. 15.24 to estimate velocity factor \( K_v \) (given).
4. Load sharing between teeth can be assumed to estimate geometry factor \( J \) (given).
5. The core hardness will be used to estimate the strength of the tooth with respect to bending fatigue (given).
6. The tooth fillet radius is approximately equal to \( 0.35/P \) (to enable use of Fig. 15.23 to estimate geometry factor \( J \)).
7. Neither of the spur gears act as idler gears in the described application.
8. The operating temperature for the gears is less than 160 °F.

**Analysis:**

1. Pitch line velocity,
   \[
   V = \frac{\pi d n}{12} = \frac{\pi (60/12)5000}{12} = 6545 \text{ fpm}
   \]

2. Velocity factor, \( K_v = \sqrt{\frac{78 + \sqrt{6545}}{78}} = 1.43 \)
3. From Fig. 15.23(a), geometry factor, \( J = 0.451 \)
4. From Table 15.2, the mounting factor, \( K_m = 1.3 \) and \( K_o = 1.1 \) (given)

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5. From Eq. (15.17):
\[
\sigma = \frac{F_t (12)}{(1.0)(0.451)} (1.43)(1.1)(1.3) = 54.4 \text{ F_t}
\]

6. From Eq. (15.18):
\[
S_n = (125 \text{ ksi})(1)(1)(.73)(.814)(1)(1.4) = 104 \text{ ksi}
\]
Since,
\[
S_n' = \left(\frac{500}{4}\right) = 125 \text{ ksi},
\]
\[C_L = 1.0,\]
\[C_G = 1 \text{ for } P > 5,\]
\[C_s = 0.73 \text{ from Fig. 8.13,}\]
\[k_t = 0.814 \text{ from Table 15.3,}\]
\[k_t = 1 \text{ and } k_{ms} = 1.4 \text{ (for one-way bending)}.\]

7. Equating stress \(\sigma\) and strength \(S_n\),
\[54.4 \text{ F_t} = 104,000 \text{ psi} \; \text{ hence F_t} = 1911.76 \text{ lb}\]
Horsepower that can be transmitted,
\[
\dot{W} = \frac{F_t V}{33,000} = \frac{1911.76(6545 \text{ fpm})}{33,000} = 379 \text{ hp}.
\]

Comments:
1. The horsepower that can be transmitted was estimated here based only on tooth bending fatigue, a more accurate estimate of the horsepower rating must consider the possibility of failure by surface fatigue.
2. Use of the core hardness to estimate the bending strength resulted in a smaller horsepower rating. This is a conservative assumption since the highest bending stress occurs on the tooth surface which has a higher hardness.
3. If either gear were acting as an idler the teeth would have been loaded in two way bending for that gear and the effective bending strength would have reduced by 40% resulting in a lower horsepower rating.
SOLUTION (15.33)

**Known:** Three identical standard full depth spur gears of given geometry and material rotate at a given rpm.

**Find:** Determine an estimate of the horsepower that can be transmitted with 99% reliability based on tooth bending fatigue.

**Schematic and Given Data:**

![Diagram of spur gears with specifications]

- 5000 rpm
- Identical spur gears:
  - \(N = 60\)
  - \(\phi = 20^\circ\)
  - \(P = 12\)
  - \(b = 1.0\) in.
- Alloy steel material, case hardness 680 Bhn,
  core hardness 500 Bhn.
- Tooth profiles finished and ground to requirement of curve A in Fig. 15.24.
- Overload factor, \(K_0 = 1.1\)

**Assumptions:**

1. The gears are mounted on accurate mountings to mesh along their pitch circles.
2. Loading of the gears involves only mild shock (given).
3. High precision gears with fine ground tooth profiles allow the use of curve A in Fig. 15.24 to estimate velocity factor \(K_v\) (given).
4. Load sharing between teeth can be assumed to estimate geometry factor \(J\) (given).
5. The core hardness will be used to estimate the strength of the tooth with respect to bending fatigue (given).
6. The tooth fillet radius is approximately equal to 0.35/P (to enable use of Fig. 15.23 to estimate geometry factor \(J\)).
7. One of the spur gears acts as an idler gear in the described application.
8. The operating temperature for the gears is less than 160 °F.

**Analysis:**

1. Pitch line velocity,
   \[ V = \frac{\pi d n}{12} = \frac{\pi (60/12)5000}{12} = 6545 \text{ fpm} \]

2. Velocity factor, \(K_v = \sqrt{\frac{78 + \sqrt{6545}}{78}} = 1.43\)

3. From Fig. 15.23(a), geometry factor, \(J = 0.451\)
4. From Table 15.2, the mounting factor, 
\( K_m = 1.3 \) and \( K_o = 1.1 \) (given) 

5. From Eq. (15.17):
\[
\sigma = \frac{F_t (12)}{(1.0)(0.451)} (1.43)(1.1)(1.3) = 54.4 \ F_t
\]

6. The gear most vulnerable to tooth bending fatigue is the idler because it is subjected to 2-way bending; others are bent only 1-way, thus \( k_{ms} = 1 \). From Eq. (15.18):
\[
S_n = (125 \ ksi)(1)(1)(.73)(.814)(1)(1.0) = 74.3 \ ksi
\]
Since,
\[
S_n' = \sqrt{\frac{500}{4}} = 125 \ ksi, \\
C_L = 1.0, \\
C_G = 1 \text{ for } P > 5, \\
C_s = 0.73 \text{ from Fig. 8.13,} \\
k_r = 0.814 \text{ from Table 15.3,} \\
k_l = 1 \\
and \ k_{ms} = 1 \text{ for the idler (two-way bending).}
\]

7. Equating stress \( \sigma \) and strength \( S_n \),
\[
54.4 \ F_t = 74,300 \ psi \ ; \text{ hence } F_t = 1365 \ lb
\]
Horsepower that can be transmitted,
\[
\dot{W} = \frac{F_t V}{33,000} = \frac{1365(6545 \ fpm)}{33,000} = 270.7 \ hp.
\]

Comments:
1. The horsepower that can be transmitted was estimated here based only on idler tooth bending fatigue, a more accurate estimate of the horsepower rating must consider the possibility of failure of the idler gear tooth by surface fatigue.
2. Use of the core hardness to estimate the bending strength resulted in a smaller horsepower rating. This is a conservative assumption since the highest bending stress occurs on the tooth surface which has a higher hardness.
3. The input and the output gears would be loaded only in one way bending and the effective bending strength for each of these gears would be 1.4 times larger resulting in a higher horsepower capacity for these two gears.
SOLUTION (15.34)

Known: For a pair of spur gears the pressure angle, modulus, number of teeth, and the speed of the pinion are given.

Find: Determine graphically the sliding velocity between the teeth (a) at the start of contact, (b) at the pitch point, and (c) at the end of contact.

Schematic and Given Data:

Assumption: The spur gears mesh along their pitch circles.

Analysis:
1. module, \( m = \frac{d}{N} \)
   Therefore, \( r_p = 90 \text{ mm}, r_g = 180 \text{ mm} \)
2. Addendum, \( a = m \)
   Therefore, \( r_{ap} = 96 \text{ mm}, r_{ag} = 186 \text{ mm} \)
**Case (a):** Start of contact

3.

![Diagram of gear and pinion with labeled vectors](image)

**Figure (a)**

4. From Fig. (a): \( r_{PA} = 82 \) mm, \( r_{GA} = r_{ag} = 186 \) mm

   Hence, \( V_{PA} = \omega_{PA} r_{PA} = \frac{210(2\pi)}{60} (82) = 1803 \) mm/s

   \( V_{GA} = \omega_{GA} r_{GA} = \frac{105(2\pi)}{60} (186) = 2045 \) mm/s

5. From Fig. (a), the sliding velocity is 760 mm/s.

**Case (b):** Pitch point

6. Sliding velocity = 0 [See Fig. 15.26(b)]
Case (c): End of contact

8. From Fig. (c): \( r_{gB} = 169 \) mm, \( r_{pB} = r_{ap} = 96 \) mm

Hence, \( V_{pB} = \omega_p r_{pB} = \frac{210(2\pi)}{60} (96) = 2111 \) mm/s

\( V_{gB} = \omega_g r_{gB} = \frac{105(2\pi)}{60} (169) = 1858 \) mm/s

9. From Fig. (c), the sliding velocity is 560 mm/s.

SOLUTION (15.35)

Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based on surface durability.
Schematic and Given Data:

Material: Steel, heat treated to 350 Bhn
Standard full depth teeth
Accurate mounting
P = 8
ϕ = 20°
b = 1 in.
Design life: 5 yrs, 60 hr/wk,
50 wk/yr operation
Top quality hobbing operation for manufacturing

Assumptions:
1. The gears mesh along their pitch circles.
2. The gear tooth loads are transmitted at the pitch point.
3. Tooth contact surfaces are approximated by cylinders.
4. Surface stresses are unaffected by lubricant and sliding friction.

Analysis:

1. From Eq. (15.24): \( \sigma_H = C_P \sqrt{\frac{F_t}{b d_p I}} K_v K_o K_m \)

   with \( I = \frac{\sin \phi \cos \phi R}{2} \frac{R + 1}{2} = \frac{\sin 20^\circ \cos 20^\circ}{2} \frac{2}{2 + 1} = 0.107 \)
   and \( b = 1 \text{ in.}, K_v = 1.54, K_o = 1.0, K_m = 1.6, \)
   \( d_p = N_p/P = (20/8) \text{ in.} \) (from the analysis of Problem 15.25),

   Therefore: \( \sigma_H = 2300 \sqrt{\frac{F_t (1.54)(1)(1.6)}{(1.0)(20/8)(0.107)}} = 6980.5 \sqrt{F_t} \)

2. From Eq. (15.25): \( S_H = S_{fe} C_{Li} C_R \)

   \( S_{fe} = 0.4 \text{ (Bhn)} - 10 \text{ ksi} = (0.4)(350) - 10 = 130 \text{ ksi} \)
   design life = 1100 cyl/min \( \times 60 \text{ min/hr} \times 60 \text{ hr/wk} \)
   \( \times 50 \text{ wk/yr} \times 5 \text{ yr} = 9.9 \times 10^8 \text{ cycles} \)

   hence, \( C_{Li} = 0.8 \)

   \( S_H = 130(0.8)(1) = 104 \text{ ksi} \)

3. For SF = 1.5:

   \( 104,000 = 6980.5 \sqrt{1.5} F_t \); \( F_t = 148 \text{ lb} \)

   \( \hat{W} = \frac{F_t V}{33,000} = \frac{148(720)}{33,000} = 3.23 \text{ hp} \)

Therefore, the horsepower rating with respect to surface durability is approximately 3.2 hp.
**Comment:** The horsepower rating of the gear pair is much lower when analyzed with respect to surface durability than with respect to bending fatigue (Problem 15.29). With other choices of material and geometry the opposite result can also occur. This problem illustrates the need for considering both bending fatigue and surface durability in the design and analysis of gears.

**SOLUTION (15.36)**

**Known:** A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

**Find:** Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based on surface durability and bending fatigue.

**Schematic and Given Data:**

![Schematic diagram of gears with specifications]

**Assumptions:**
1. The gears mesh along their pitch circles.
2. The gear tooth loads are transmitted at the pitch point.
3. Tooth contact surfaces are approximated by cylinders.
4. Surface stresses are unaffected by lubricant and sliding friction.

**Analysis:**

*For surface durability:*

*For the pinion:*

1. From Eq. (15.24): \( \sigma_H = C_p \sqrt{ \frac{F_t}{b d_p I} K_v K_o K_m } \)

   with \( I = \frac{\sin \phi \cos \phi}{2} \frac{R}{R + 1} = \frac{\sin 20^\circ \cos 20^\circ}{2} \frac{2}{2 + 1} = 0.107 \)

   and \( b = 1 \text{ in.}, K_v = 1.54, K_o = 1.0, K_m = 1.6, \)

   \( d_p = N_p/P = (20/8) \text{ in.} \) (from the analysis of Problem 15.29),
Therefore, \( \sigma_H = 2300 \sqrt[2]{\frac{F_i (1.54)(1)(1.6)}{(1.0)(20/8)(0.107)}} = 6980.5 \sqrt{F_i} \)

2. From Eq. (15.25): \( S_H = S_{fe} C_{Li} C_R \)
   \( S_{fe} = 0.4 \) (Bhn) - 10 ksi = (0.4)(400) - 10 = 150 ksi
   design life = 1100 cyl/min \( \times \) 60 min/hr \( \times \) 60 hr/wk
   \( \times \) 50 wk/yr \( \times \) 5 yr = 9.9 \( \times \) 10^8 cycles
   hence, \( C_{Li} = 0.8 \)
   \( S_H = 150(0.8)(1) = 120 \) ksi

3. For SF = 1.5:
   \( 120,000 = 6980.5 \sqrt{1.5 \ F_i} \ ; \ F_t = 197 \) lb
   \[ \dot{W} = \frac{F_i V}{33,000} = \frac{148(720)}{33,000} = 4.3 \) hp

For the gear:

4. From Eq. (15.24): \( \sigma_H = C_p \sqrt{\frac{F_i}{b d_i I}} \ K_i K_o K_m \)
   \( d_g = 40/8 \). Hence, \( \sigma_H = 4936 \sqrt{F_i} \)

5. From Eq. (15.25): \( S_H = S_{fe} C_{Li} C_R \)
   \( S_{fe} = 0.4 \) (Bhn) - 10 ksi = 0.4(350) - 10 = 130 ksi.
   Hence, \( S_H = 130 (0.8)(1) = 104 \) ksi.

6. For SF = 1.5:
   \( 104,000 = 4936 \sqrt{1.5 \ F_i} ; \ F_t = 295.95 \) lb
   \[ \dot{W} = \frac{F_i V}{33,000} = \frac{295.95(720)}{33,000} = 6.46 \) hp

It is evident that the gear is stronger than the pinion based on surface durability. Therefore, the horsepower rating with respect to surface durability is approximately 4.3 hp.

For bending fatigue:

For the pinion:

1. From Eq. (15.17): \( \sigma = \frac{F_i P}{b J} \ K_v \ K_o \ K_m \)
   \( K_v \) requires finding the pitch line velocity as,
   \[ V = \frac{\pi d n}{12} = \frac{\pi(20/8)1100}{12} = 720 \) fpm
   from Curve C of Fig. 15.24,
   \( K_v = 50 + \sqrt{720} \)
   \[ \frac{50}{50} = 1.54 \]
   from Fig. 15.23(a), \( J = 0.24 \) (for the pinion - and with no load sharing)
   Also, \( K_m = 1.6 \) (from Table 15.2 - probably best judgement)
and \( K_0 = 1.0 \) (from Table 15.1- uniform driving and driven torque)

Therefore,

\[
\sigma = \frac{F_t(8)}{(1.0)(0.28)}(1.54)(1.0)(1.6) = 70.37 \text{ F}_t
\]

2. From Eq. (15.18):

\[
S_n = S_n' C_L C_G C_s k_r k_t k_{ms}
\]

\[
= (250 \times 350)(1)(1)(0.66)(0.814)(1)(1.4)
\]

\[
= 65,812 \text{ psi}
\]

where

\( S_n' = 250 \text{ (Bhn)} = 250 \times 350 \text{ psi for infinite life,} \)

since design life = 5 yr \( \times \) (50 wk/yr) \( \times \) (60 hr/wk) \( \times \) (60 min/hr) \( \times \) 1100 rpm

\[
= 9.9 \times 10^8 > 10^6 \text{ cycles}
\]

\( C_L = 1.0, \)

\( C_G = 1.0 \text{ since } P > 5 \)

\( C_s = 0.66 \text{ from Fig. 8.13} \)

\( k_r = 0.814 \text{ from Table 15.3} \)

\( k_t = 1 \text{ and } k_{ms} = 1.4 \text{ since the pinion is not an idler} \)

3. For SF = 1.5: \( 70.37(1.5F_t) = 65,812 \)

hence, \( F_t = 623.5 \text{ lb} \)

\[
\dot{W} = \frac{F_t V}{33,000} = \frac{(623.5)(720)}{33,000} = 13.6 \text{ hp}
\]

For the gear:

4. From Eq. (15.17): \( \sigma = \frac{F_t P}{b J K_v K_0 K_m} \)

\( K_v \) requires finding the pitch line velocity as,

\[
V = \frac{\pi d n}{12} = \frac{\pi (20/8) 1100}{12} = 720 \text{ fpm}
\]

from Curve C of Fig. 15.24,

\[
K_v = \frac{50 + \sqrt{720}}{50} = 1.54
\]

from Fig. 15.23(a), \( J = 0.28 \) (for the gear - and with no load sharing)

Also, \( K_m = 1.6 \) (from Table 15.2 - probably best judgement)

and \( K_0 = 1.0 \) (from Table 15.1- uniform driving and driven torque)

Therefore,

\[
\sigma = \frac{F_t(8)}{(1.0)(0.24)}(1.54)(1.0)(1.6) = 82.1 \text{ F}_t
\]

5. From Eq. (15.18):

\[
S_n = S_n' C_L C_G C_s k_r k_t k_{ms}
\]

\[
= (250 \times 400)(1)(1)(0.66)(0.814)(1)(1.4)
\]

\[
= 75,214 \text{ psi}
\]
where
\[ S_n' = 250 \text{ (Bhn)} = 250 \times 400 \text{ psi for infinite life,} \]
since design life = 5 yr \times (50 \text{ wk/yr}) \times (60 \text{ hr/wk}) \times (60 \text{ min/hr}) \times 1100 \text{ rpm} 
= 9.9 \times 10^8 \times 10^6 \text{ cycles} 
\[ C_L = 1.0, \]
\[ C_G = 1.0 \text{ since } P > 5 \]
\[ C_S = 0.66 \text{ from Fig. 8.13} \]
\[ k_r = 0.814 \text{ from Table 15.3} \]
\[ k_t = 1 \text{ and } k_{ms} = 1.4 \text{ since the pinion is not an idler} \]
6. For SF = 1.5 : 82.1(1.5 \text{ } F_t) = 75,214
\[ \text{hence, } F_t = 610.75 \text{ lb} \]
\[ \dot{W} = \frac{F_t V}{33,000} = \frac{(610.75)(720)}{33,000} \approx 13.325 \text{ hp} \]

It is evident that the gear is stronger than the pinion based on the surface durability. Therefore, the horsepower rating with respect to bending fatigue failure is approximately 13.325 hp.

Comments:
1. The horsepower rating with respect to surface durability is much less than with respect to bending fatigue. This homework problem illustrates the need to consider both bending fatigue and surface durability in the design and analysis of gears. It also shows the need for calculating the strengths of both the gear and the pinion for comparison when the hardness of the gear materials differ, unlike the case where the hardness of both the materials is the same, and we could carry out our design calculations for the smaller of the two gears.
2. The bending stresses can be reduced for the specified rpm by decreasing P or increasing b. But these parameters as well as the factors K_V and J are closely interrelated. Decreasing P for the same number of teeth increases pitch diameter which leads to larger pitch line velocity and hence to larger values of K_V and s. Decreasing P for the same pitch diameter decreases the number of teeth resulting in a smaller value of J and a larger value of s. Increasing the value of b requires accurate mounting and manufacturing to utilize the entire face width and ultimately tends to increase the value of K_m. Thus choice of suitable values for gear geometry parameters for specific applications requires balancing the parameter values with other side effects.
3. It is evident that the gear is stronger than the pinion based on surface durability. Therefore, the horsepower rating based on surface fatigue is 4.3 hp.
4. The horsepower rating of the gear pair is much lower when analyzed with respect to surface durability than with respect to bending fatigue (Problem 15.29). With other choices of material and geometry the opposite result can also occur. This problem illustrates the need for considering both bending fatigue and surface durability in the design and analysis of gears.
SOLUTION (15.37)

**Known:** A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

**Find:** (a) Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based on surface durability. (b) Estimate the value to which the gear hardness can be reduced without making the gear teeth weaker than the pinion teeth based on surface fatigue.

**Schematic and Given Data:**

![Schematic Diagram]

**Material:** Steel, heat treated to 350 Bhn for the gear, and heat treated to 400 Bhn for the pinion.

**Standard full depth teeth**

**Accurate mounting**

**P = 8**

**φ = 20°**

**b = 1 in.**

**Design life:** 5 yrs, 60 hr/wk,

50 wk/yr operation

**Top quality hobbing operation for manufacturing**

**Assumptions:**

1. The gears mesh along their pitch circles.
2. The gear tooth loads are transmitted at the pitch point.
3. Tooth contact surfaces are approximated by cylinders.
4. Surface contact stresses are unaffected by lubricant and sliding friction.

**Analysis:**

1. For the pinion: From Eq. (15.24): $\sigma_H = C_p \sqrt{\frac{F_t}{b d_p I}} K_v K_o K_m$

   
   
   \[
   I = \frac{\sin \phi \cos \phi}{2} \frac{R}{R + 1} = \frac{\sin 20^\circ \cos 20^\circ}{2} \cdot \frac{2}{2 + 1} = 0.107
   \]

   and $b = 1$ in., $K_v = 1.54$, $K_o = 1.0$, $K_m = 1.6$,

   $d_p = N_p / P = (20/8)$ in. (from the analysis of Problem 15.29),

   Therefore, $\sigma_H = 2300 \sqrt{\frac{F_t (1.54)(1)(1.6)}{(1.0)(20/8)(0.107)}} = 6980.5 \sqrt{F_t}$
2. From Eq. (15.25): $S_H = S_{fe} C_{Li} C_R$

$S_{fe} = 0.4 \text{ (Bhn)} - 10 \text{ ksi} = (0.4)(350) - 10 = 130 \text{ ksi}$

design life = 1100 cyl/min $\times$ 60 min/hr $\times$ 60 hr/wk
$\times 50 \text{ wk/yr} \times 5 \text{ yr} = 9.9 \times 10^8 \text{ cycles}$

hence, $C_{Li} = 0.8$

$S_H = 130(0.8)(1) = 104 \text{ ksi}$

3. For SF = 1.5:

$104,000 = 6980.5\sqrt{1.5F_t}; F_t = 148 \text{ lb}$

$\dot{W} = \frac{F_t V}{33,000} = \frac{148(720)}{33,000} = 3.23 \text{ hp}$

Therefore, the horsepower rating with respect to surface durability is approximately 3.2 hp.

4. For the gear: From Eq. (15.24); $\sigma_H = C_p \sqrt{\frac{F_t}{b_d g I}} K_v K_o K_m$

$I = \frac{\sin \phi \cos \phi}{2} \quad \frac{R}{R + 1} = \frac{\sin 20^\circ \cos 20^\circ}{2} \quad \frac{2}{2 + 1} = 0.107$

$b = 1 \text{ in.}, \quad K_v = 1.54, \quad K_o = 1.0, \quad K_m = 1.6,$

$d_g = N_g/P = 40/8 \text{ in.} \quad \text{(from the analysis of Problem 15.29)}. \quad \therefore$

$\sigma_H = 2300 \sqrt{\frac{148(1.54)(1)(1.6)}{1.0(40/8)(0.107)}} = 60049 \text{ psi} = 60 \text{ ksi}$

5. From Eq. (15.25); $S_H = S_{fe} C_{Li} C_R$

$S_{fe} = 0.4 \text{ (Bhn)} - 10 \text{ ksi}$

$S_H = [0.4 \text{ (Bhn)} - 10 \text{ ksi}](0.8)(1) = 0.32 \text{ (Bhn)} - 8 = 60 \text{ ksi}. \quad \text{Solving for Bhn,}$

$\text{Bhn} = 187.65.$

Comment: The gear hardness can be reduced to 187.65 Bhn without making the gear teeth weaker than the pinion teeth based on surface fatigue.
SOLUTION (15.38)

**Known:** A spur gear speed reducer is driven by an electric motor and drives a load involving "moderate shock". The gear teeth are standard full depth and of given geometry and material. Required life is $10^6$ pinion revolutions for a specified transmitted load.

**Find:** Determine an estimate of the reliability of the speed reducer with respect to surface durability.

**Schematic and Given Data:**

![Schematic](image)

- $N_p = 18$
- $P = 10$
- $b = 1.0$ in.
- $\phi = 20^\circ$
- $N_g = 36$
- $K_m = 1.8$
- $k_t = 1$
- $F_i = 100$ lb
- Tooth cut with low-cost average quality cutting process

**Assumptions:**
1. The gears mesh at the pitch circles.
2. Load sharing is not expected since the cutting process is of average quality.
3. The effects corrected by the velocity factor, $K_v$, correspond to the middle of the range in Fig. 15.24 with manufacture by form cutters.
4. The pinion is driven by a uniform power motor while the gear drives a load involving "moderate shock" (given).
5. The surfaces at the contact region of the teeth can be approximated by cylinders.
6. The surface stress distribution is unaffected by the presence of the lubricant.
7. Surface loads due to sliding in the tooth contact region are negligible.

**Analysis:**
1. The pitch diameters of the gears are,
   - $d_g = N_g/P = 36/10 = 3.6$ in.
   - $d_p = N_p/P = 18/10 = 1.8$ in.

2. Ratio of pitch diameters, $R = \frac{d_g}{d_p} = \frac{3.6}{1.8} = 2$.

3. Pitch line velocity,
   \[
   V = \frac{\pi d n}{12} = \frac{\pi N_p n_p}{12P} = \frac{\pi(18)(1500)}{12(10)} = 706.8 \text{ ft/min}
   \]
from Fig. 15.24, with $V = 706.8$ ft/min, $K_v = 2.0$
from Table 15.1, $K_o = 1.25$

4. From Eq. (15.23): $I = \frac{\sin 20^\circ \cos 20^\circ \cdot 2}{3} = 0.107$

5. From Eq. (15.24):

$$\sigma_H = C \sqrt{\frac{F_i}{b d_p I}} K_v K_o K_m$$

$$= 2300 \sqrt{\frac{100}{(1.0)(1.8)(0.107)}} (2.0)(1.25)(1.8)$$
$$= 111,174 \text{ psi} = 111.2 \text{ ksi}$$

6. From Table 15.5, an estimate of surface fatigue strength,

$$S_{fe} = 0.4 \left( \frac{\text{Bhn of gear} + \text{Bhn of pinion}}{2} \right) - 10 \text{ ksi}$$

and from Fig. 15.27, $C_{L1} = 1.12$

7. Therefore, from Eq. (15.25):

$$S_H = S_{fe} C_{Li} C_R$$

$$= \left[ 0.4 \left( \frac{235 + 260}{2} \right) - 10 \right] (1.12) C_R = 99.68 C_R$$

Equating stress and strength $111.2 = 99.68 C_R$; hence $C_R = 1.11$

8. Interpolating the rough data from page 489 of the text:

<table>
<thead>
<tr>
<th>$C_R$</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>50%</td>
</tr>
<tr>
<td>1.00</td>
<td>99%</td>
</tr>
<tr>
<td>0.80</td>
<td>99.9%</td>
</tr>
</tbody>
</table>

A rough approximation of the reliability for $C_R = 1.11$ is reliability $\approx 80\%$.

**Comments:**

1. The estimate of reliability from the value of $C_R$ is very approximate. As the textbook suggests, data on reliability factors for surface durability are scarce.

2. In comparing the reliability obtained in Problem 15.31 for bending fatigue for the same speed reducer, we find that the estimate of reliability of the gears is much lower for surface fatigue. This indicates that the speed reducer is more likely to fail due to surface damage than bending fatigue.

3. The change in contact pressure distribution due to the presence of a lubricant and sliding loads on the tooth are not explicitly considered in the Hertz equation used for calculating maximum stress in the contact region. These considerations are by implication absorbed in the surface strength data obtained from experiments. It is thus important to judge if the lubrication and sliding effects in the application are similar to those in experiments from which this data is obtained.

**SOLUTION (15.39)**

**Known:** An identical pair of standard full depth spur gears of given geometry and material rotate at a given rpm.
**Find:** Determine an estimate of the horsepower that can be transmitted for $10^9$ cycles with 90% reliability based on surface fatigue.

**Schematic and Given Data:**

![Diagram of spur gears with specifications]

- Identical spur gears:
  - $N = 60$
  - $\phi = 20^\circ$
  - $P = 12$
  - $b = 1.0$ in.
- Alloy steel material, case hardness 680 Bhn, core hardness 500 Bhn.
- Tooth profiles finished and ground to requirement of curve A in Fig. 15.24.
- Overload factor, $K_o = 1.1$

**Assumptions:**

1. The spur gears mesh at the pitch circles using accurate mountings.
2. Loading on the gears involves only mild shock (given).
3. High precision gears with fine ground tooth profiles allow the use of curve A in Fig. 15.24 to estimate velocity factor $K_v$ (given).
4. Load sharing between teeth is expected in these high precision gears but the full transmitted load will be applied to a tooth to make a conservative estimate of horsepower.
5. The surfaces at the contact region is approximated by cylinders.
6. The surface stress distribution is unaffected by the presence of lubricant.
7. Surface loads due to sliding in the tooth contact region are negligible.
8. The operating temperature for the gears is below 160°F.

**Analysis:**

1. Pitch line velocity,
   \[ V = \frac{\pi \cdot d \cdot n}{12} = \frac{\pi (60/12)5000}{12} = 6545 \text{ fpm} \]

2. Velocity factor, $K_v = \sqrt{\frac{78 + \sqrt{6545}}{78}} = 1.43$

3. Pitch diameters are, $d = \frac{N}{P} = \frac{60}{12} = 5$ in.
   Ratio of pitch diameters, $R = 1$

4. From Table 15.2, the mounting factor, $K_m = 1.3$
   and $K_o = 1.1$ (given)
5. From Eq. (15.23): the geometry factor,
\[ I = \frac{\sin 20^\circ \cos 20^\circ}{2} \left( \frac{1}{1 + 1} \right) = 0.080 \]

6. From Eq. (15.24): the surface stress,
\[ \sigma_H = 2300\sqrt{\frac{F_t}{(1.0)(5)(0.08)(1.43)(1.1)(1.3)}} = 5200\sqrt{F_t} \]

7. From Table 15.6: \( C_R = 1.06 \) for 90% reliability
From Fig. 15.27, \( C_L = 0.8 \)
8. From Eq. (15.25) and Table 15.5:
\[ S_H = [0.4(680) - 10](0.8)(1.06) = 222.222 \text{ ksi} \]
Equating stress and strength \( 5200\sqrt{F_t} = 222,222 \text{ psi} : F_t = 1826.3 \text{ lb} \)

9. Horsepower that can be transmitted,
\[ \dot{W} = \frac{F_t V}{33,000} = \frac{1826.3 \text{ lb}(6545 \text{ fpm})}{33,000} = 362.2 \text{ hp} \]

**Comments:**

1. Comparing the horsepower rating of the drive unit with respect to bending fatigue estimated in Problem 15.32, it is evident that the horsepower rating determined by consideration of surface fatigue is more critical. While the horsepower rating is 379 hp with a 99% reliability with respect to bending fatigue failure, the horsepower rating is only 362 hp with a 90% reliability with respect to surface fatigue failure. However, the process of failure and its consequence are substantially different in bending fatigue and surface fatigue. Bending fatigue failure is sudden and drastic while surface fatigue failure is gradual and provides easily observable indications of failure.

2. If the surface of the teeth were not made harder than the core, the durability of the teeth with respect to surface fatigue would be further reduced and the horsepower rating would drop by a factor of approximately 1.36 \( \left( = \frac{\text{surface hardness}}{\text{core hardness}} \right) \) to about 266 hp.

3. The horsepower rating is unaffected whether the gears act as idlers or not as far as surface fatigue failure is concerned. This is in contrast to the case of bending fatigue where the strength is less by a factor of 1.4 for idler gears due to two way tooth bending.

4. If the solution of SAMPLE PROBLEM 5.5 is followed rather than the solution of SAMPLE PROBLEM 5.4, Figure 9.21 gives \( S_H = 120,000 \text{ psi} \), and a smaller transmitted horsepower rating is calculated (estimated).

**SOLUTION (15.40)**

**Known:** A two stage gear speed reducer is given which uses a countershaft and has identical gear pairs in each stage. Gear and shaft geometry is specified such that the input and output shafts are collinear.

**Find:** Determine the relative strengths of the gears for serving in the high-speed and low-speed positions considering both bending fatigue and surface durability with \( 10^7 \) cycles life for the high speed gear.

15-60
Schematic and Given Data:

Assumptions:
1. The high-speed and low-speed gears are mounted to mesh identically.
2. Load sharing need not be considered.
3. The high speed and low speed gears operate at the same temperature.
4. The countershaft can be considered to be rigid so that both gears have the same overload conditions determining the value of the overload factor, \( K_o \).
5. The friction in the gears and bearings can be neglected.

Analysis:
1. Pitch line velocity of gear in high speed position is,

\[
V_{\text{high}} = \frac{\pi d_p n_p}{12} = \frac{\pi d_p n_p}{12P} = \frac{\pi(15)(1200)}{12(5)} = 942.4 \text{ ft/min}
\]

Pitch line velocity of gear in low speed position is,

\[
V_{\text{low}} = \frac{\pi d_p n_p}{12} = \frac{\pi n_{p} n_{g}}{12P} = \frac{\pi(45)(1200/9)}{12(5)} = 314.1 \text{ ft/min}
\]

Ratio of tangential tooth loads in low-speed and high-speed positions for the gears is,

\[
\frac{F_{t, \text{low}}}{F_{t, \text{high}}} = \frac{V_{\text{high}}}{V_{\text{low}}} = 3
\]

2. For bending fatigue, using Eq. (15.17):

\[
\frac{\sigma_{\text{low, sp}}}{\sigma_{\text{hi, sp}}} = \frac{F_{t, \text{low}}}{F_{t, \text{high}}} \cdot \frac{K_{v, \text{low}}}{K_{v, \text{high}}}
\]
\[ = (3) \left( \frac{1.10}{1.18} \right) = 2.8 \text{ with } K_v \text{ from curve A in Fig. 15.24.} \]

\[ = (3) \frac{1.26}{1.78} = 2.12 \text{ with } K_v \text{ from curve D in Fig. 15.24.} \]

3. Eq. (15.18) for \( S_n \) is the same for both applications
   Thus for bending fatigue, the low speed application is more severe by factor
   of 2.12 to 2.8 depending on the manufacturing accuracy.

4. For surface fatigue, using Eq. (15.24):
   \[ \frac{\sigma_{H\text{ low sp}}}{\sigma_{H\text{ hi sp}}} = \sqrt{2.8} = 1.67 \text{ with } K_v \text{ from curve A and} \]

\[ \frac{\sigma_{H\text{ low sp}}}{\sigma_{H\text{ hi sp}}} = \sqrt{2.12} = 1.46 \text{ with } K_v \text{ from curve D} \]

5. From Eq. (15.25):
   \[ \frac{S_{H\text{ low sp}}}{S_{H\text{ hi sp}}} = \frac{C_{Li\text{ low}}}{C_{Li\text{ high}}} \approx \frac{1.03}{1.0} = 1.03 \]
   because the low speed gears accumulate fatigue cycles only a third as rapidly
   at the high speed gears and the high speed gear must have a life of \( 10^7 \)
   cycles.

6. Thus, for surface fatigue, the low speed application is more severe by a
   factor of about \( \frac{1.46}{1.03} = 1.42 \) to \( \frac{1.67}{1.03} = 1.62 \), depending on manufacturing
   accuracy.

Comments:
1. Unlike the surface fatigue strength, the bending strength is unaffected by the fact
   that the low-speed gear accumulates fatigue cycles only a third as rapidly as the
   high-speed gear. This is because, while the endurance limit for bending stress is
   reached at \( 10^6 \) cycles the surface strength continues to decrease well past \( 10^7 \)
   cycles.

2. Since the surface fatigue stress is proportional to the root of the velocity factor,
   \( K_v \), the relative severity of service for the low-speed gear is not as high as in the
   case of bending fatigue.

3. If friction forces in the gears and bearings were taken into consideration, the
   tangential tooth loads for the low-speed and high-speed gears would not be
   precisely in the inverse ratio of their speeds.

SOLUTION (15.41D)
Known: A two stage spur gear speed reducer is given which uses a countershaft and
has identical gear pairs in each stage. Gear and shaft geometry is specified such that
the input and output shafts are collinear. Shafts and mountings correspond to good
industrial practice but not "high precision".

Find: Determine a design of the gears for \( 10^7 \) cycles with 99\% reliability and safety
factor of 1.2.
Schematic and Given Data:

Decisions:
1. Choose steel for gear material with pinion material 10% harder than the gear material.
2. Choose standard full depth teeth with pressure angle, $\phi = 20^\circ$.
3. Select number of teeth for pinion, $N_p = 20$.
4. Choose manufacturing precision between curves C and D in Fig. 15.24 to estimate velocity factor, $K_v$.
5. Choose face width for gears as $b = \frac{12}{P}$.
6. The surface hardness and core hardness for the teeth are equal.
7. The tooth fillet radius is 0.35/P (to enable use of Fig. 15.23(a) to estimate J).

Assumptions:
1. The gears are mounted at their theoretical center distance.
2. Friction losses in gears and bearings can be neglected.
3. No load sharing is expected and all tooth loads are transmitted at the pitch point.
4. The operating temperature for the gears is below 160°F.
5. Surface stress can be estimated by approximating tooth contact region by cylinders.
6. Surface stress distribution is unaffected by lubricant.
7. Surface stress due to sliding friction is negligible.

Design Analysis:
1. With a center distance of 8 in., the 9:1 reduction requires 3:1 reduction by each gear set.
   Hence, $d_p = 4$ in., $d_g = 12$ in.
2. With 20 pinion teeth, $P = \frac{20}{4} = 5$
3. With $b = \frac{12}{P}$, $b = 2.4$ in.
4. Pitch line velocities are,
   $V = (4/12)\pi \times 2700 = 2827$ fpm (high speed set)
   and similarly $V = 942$ fpm (low speed set)
5. With manufacturing precision between curves C and D, from Fig. 15.24, $K_v = 1.7$ (low speed) and 2.7 (high speed)
6. Since \( F_t \cdot K_v \) is the greatest on the low speed set, we design the gears for this application.

\[
\dot{W} = \frac{F_t \cdot V}{33,000} : 10 = \frac{F_t(942)}{33,000} : F_t = 350 \text{ lb},
\]

with a safety factor of 1.2, \( F_t = 350(1.20) = 420 \text{ lb} \)

7. Having chosen steel gears with \( \phi = 20^\circ \), we find hardness needed for surface fatigue criterion:

from Eq. (15.24):

\[
\sigma_H = 2300 \sqrt{\frac{420}{2.4 (4) (0.12)}} (1.7)(1.5)(1.6)
\]

= 88,707 psi

where, from Eq. (15.23),

\[
I = \frac{\sin 20^\circ \cos 20^\circ \cdot \frac{3}{4}}{2} = 0.12
\]

since, from Table 15.4a, \( C_p = 2300 \text{ psi} \),

from Table 15.1, \( K_o = 1.5 \),

from Table 15.2, \( K_m = 1.6 \) and \( K_v = 1.7 \).

8. Equating stress and strength,

88.7 ksi = \( S_H = S_{fe} C_{Li} C_R \)

88.7 = (0.4 Bhn - 10)(1)(1) : hence, Bhn = 247

specify gear hardness as 250 Bhn, pinion hardness as 275 Bhn

9. To verify that the above solution is adequate for bending fatigue:

from Eq. (15.17):

\[
\sigma = \frac{420(5)}{2.4(0.24)} (1.7)(1.5)(1.6) = 14,875 \text{ psi}
\]

where, from Fig. 15.23(a), \( J = 0.24 \),

and \( K_v = 1.7, K_o = 1.5, K_m = 1.6 \).

from Eq. (15.18):

\[
S_n = \frac{275}{2} (1)(0.85)(0.71)(0.814)(1.4) = 47.283 \text{ ksi} = 47,283 \text{ psi}
\]

where, \( S_n' = \frac{1}{4}(Bhn) \text{ ksi} \)

\( C_L = 1.0, C_G = 0.85, C_s = 0.71 \)

for \( S_u = \frac{275}{2} = 137.5 \text{ ksi}, k_f = 0.814 \) from Table 15.3,

\( k_l = 1, k_{ms} = 1.4 \) from Eq. (15.19)

hence the gears are more than adequate to resist bending fatigue.

10. Design results:

c = 8 in., \( b = 2.4 \) in.

\( N_p = 20, N_g = 60 \), standard full depth teeth.

\( P = 5, \phi = 20^\circ \), steel gears, pinion hardness 275 Bhn, gear hardness 250 Bhn.

\( d_p = 4 \text{ in.}, d_g = 12 \text{ in.} \)
Tooth fillet radius \( = \frac{0.35}{P} = 0.07 \text{ in.} \)

Manufacturing precision between curves C and D in Fig. 15.24. Shafts and
mountings of ordinary good engineering practice.

Comments:
1. Specifying a higher surface hardness and a lower core hardness for the gear teeth
would have resulted in a more balanced factor of safety for bending fatigue and
surface fatigue.
2. By specifying a material of 250 Bhn using design calculations which required a
hardness of 247 Bhn for the pinion, separate design calculations for the gear were
avoided. Since the pinion is always more severely stressed than the
corresponding gear. By selecting pinion material 10\% harder than the gear an
additional factor of safety is provided.
3. If a larger number of teeth for the pinion were selected, the diametral pitch would
have been larger and a proportionately smaller face width could be selected.
These decisions would result in a higher value of bending stress as well as a
higher value of surface stress thus requiring harder gear material specifications.

**SOLUTION** (15.42D)

**Known:** A pair of standard spur gears are to transmit a specified hp from an electric
motor to a machine with minimum size and weight. Case hardened alloy steel gears of
specified Bhn are to be used.

**Find:** Determine a design of the spur gears for \(10^7\) pinion revolutions at full load and
99\% reliability and a factor of safety of 1.2.

**Schematic and Given Data:**

```
Motor
60 hp
5200 rpm

Case hardened alloy steel gears.
Pinion: steel, 660 Bhn
Gears: steel, 600 Bhn

Machine
1300 rpm
```

**Decisions:**
1. Select curve A in Fig. 15.24 for high precision manufacturing accuracy with
shaved and ground teeth.
2. Select standard full depth teeth with pressure angle, \( \phi = 20^\circ \).
3. Choose number of teeth on pinion, \( N_p = 18 \).
4. Choose face width, \( b = 14/P \).
5. Choose velocity factor, \( K_v = 1.4 \), \( K_m = 1.3 \) (to be verified later).
6. Core hardness of the teeth will be specified from bending fatigue considerations.
7. Choose tooth fillet radius as \( 0.35/P \) (to enable use of Fig. 15.22(a) to estimate J).
Assumptions:
1. No significant shock load is present and thus the overload factor, $K_o = 1$.
2. The operating temperature of the gear set is less than 160°F.
3. The gears are mounted at the theoretical center distance.
4. Load sharing between the teeth can be expected since the gears are of high precision.
5. Surface stress can be estimated by approximating tooth contact region by cylinders.
6. Surface stresses are unaffected by sliding friction and presence of lubricant.

Design Analysis:
1. With $N_p = 18$, $N_g = N_p \left( \frac{N_p}{n} \right) = 18 \left( \frac{5200}{1300} \right) = 72$
2. Pitch line velocity,
   \[ V = \pi d_p (5200)/12 = \pi \left( \frac{18}{12} \right) (5200)/12 = \frac{24504}{P} \text{ ft/min} \]
3. Tangential tooth load,
   \[ F_t = \frac{60 \text{ hp}(33,000)}{V} = 96.96P \]
   at "design overload"
4. We solve for $P$ with $\sigma_H = S_H$ at design overload conditions:
   \[ C_p \sqrt{\frac{F_t}{b d_p I}} K_v K_o K_m = S_f e C_{Li} C_R \]
   with $I = (\sin 20^\circ \cos 20^\circ/2)(4/5) = 0.128$ from Eq. (15.23)
   $K_v = 1.4$, $K_m = 1.3$ (must be checked later)
   $K_o = 1$
   \[ S_f e = 0.4 \left( \frac{600 + 660}{2} \right) - 10 = 242 \text{ ksi} \]
   $C_{Li} = 1$, $C_R = 1$, $C_p = 2300$, $b = \frac{14}{P}$, $d_p = \frac{18}{P}$
   Therefore, \[ 2300 \sqrt{\frac{96.96P}{(14/P)(18/P)(0.128)}} (1.4)(1)(1.3) \]
   = 242,000 from which $P = 12$. (note: choosing $P = 12.6$. We choose $P = 12$. (note: choosing $P = 14$ would require $b > 14/P$)
5. Then, $V = \frac{24504}{12} = 2042 \text{ fps}$
   for which $K_v = 1.25$ (therefore, decision of $K_v = 1.4$ is conservative)
6. To solve for $b$:
   \[ 2300 \sqrt{\frac{96.96(12)}{b(18/12)(0.128)}} (1.4)(1)(1.3) = 242,000 \]
   $b = 0.996$ in. specify $b = 1$ in.
   (note: $K_m = 1.3$ is satisfactory, and $b = \frac{12}{P}$ which is satisfactory)
7. To check contact ratio:

\[ r_p = \frac{1}{2} \frac{N_p}{P} = \frac{9}{12} = 0.75 \text{ in. and similarly } r_g = 3.0 \text{ in.} \]

adding addendum = \( \frac{1}{P} \): \( r_{ap} = 0.833 \text{ in.} \),

\( r_{ag} = 3.083 \text{ in.} \)

from Eq. (15.11): \( r_{bp} = 0.75 \cos 20^\circ = 0.7048 \text{ in.} \)

\( r_{bg} = 3.0 \cos 20^\circ = 2.8190 \text{ in.} \)

from Eq. (15.11): \( p_b = \frac{\pi}{12} \cos 20^\circ = 0.2460 \text{ in.} \)

center distance, \( c = r_p + r_g = 3.75 \text{ in.} \)

from Eq. (15.9):

\[
CR = \frac{\sqrt{0.833^2 - 0.7048^2} + \sqrt{3.083^2 - 2.8190^2} - 3.75 \sin 20^\circ}{0.2460} = 1.66
\]

CR = 1.66 is satisfactory.

8. To check core hardness for bending fatigue

from Eq. (15.17):

\[
\sigma = \frac{(96.96 \times 12)(12)}{(1)(0.34)} \frac{(1.25)(1.0)(1.3)}{1} = 66731.3 \text{ psi}
\]

where from Fig. 15.23(a), \( J = 0.34 \).

Evaluating this value to \( S_n \) in Eq. (15.18),

\[ 66731 = S_n \times (1)(1)(1)(0.814)(1)(1.4) \]

where \( C_L = 1, C_G = 1, C_S = 1, k_f = 0.814, k_l = 1, \text{ and } k_{ms} = 1.4 \)

\( S_n = 58,556 \text{ psi} \)

This requires \( S_u = 117 \text{ ksi, or approximately } 235 \text{ Bhn.} \)

We specify core hardness \( \geq 235 \text{ Bhn} \)

9. Design results:
Case hardened alloy steel gear and pinion.
Pinion: surface hardness is 660 Bhn
Gear: surface hardness is 600 Bhn
Core hardness for gears is at least 235 Bhn.

\( N_p = 18, N_p = 72, \phi = 20^\circ \text{ FD.} \)

\( P = 12, b = 1 \text{ in.}, c = 3.75 \text{ in.} \),

\( d_p = 1.5 \text{ in.}, d_g = 6.0 \text{ in.} \)

Tooth fillet radius = 0.35/P = 0.03 in.
Manufacturing accuracy is high precision with shaved and ground teeth.

Comment: Choice of a lower surface hardness for the pinion and gear would have resulted in a lower surface strength and consequently a lower value of diametral pitch. A smaller diametral pitch implies thicker teeth and a smaller contact ratio leading to less quieter and less smoother operation. A smaller diametral pitch also implies larger pitch diameters for the same numbers of teeth requiring a larger center distance.
SOLUTION (15.43)

**Known:** A simple planetary gear train is used as an automotive overdrive unit. Speed ratio when the overdrive is engaged is specified and number of teeth on the planet is given.

**Find:**
(a) Determine the number of teeth on the sun and ring.
(b) Determine whether four equally spaced planets can be used.
(c) Determine whether three equally spaced planets can be used.

**Schematic and Given Data:**

![Diagram of a planetary gear train]

- Planet, 20 teeth
- Ring, output
- Arm, input
- Sun, fixed

**Overdrive engaged:** speed ratio = 1.43
**Overdrive disengaged:** sun, arm, ring rotate as one unit
speed ratio is 1:1

**Assumptions:**
1. The gears are mounted to mesh at the pitch circles.
2. The planets are equally spaced (given).

**Analysis:**
1. From Eq. (g): \[ 1.43 = 1 + \frac{S}{R} \]
   where \( R = S + 2P = S + 40 \)
   Therefore, \( 0.43 = \frac{S}{S + 40} \), or \( S = 30.175 \), or \( S = 30 \) teeth
   \[ R = 70 \text{ teeth (exact ratio} = 1 + \frac{30}{70} = 1.4286) \]
2. For four planets:

- 20 tooth planet (top position) engages with ring tooth and with sun tooth in position shown.
- Bottom position requires planet engagement with ring tooth and sun tooth; Side position requires engagement with ring space and sun space. This works with 20 tooth (even number) planets.
- Use of four equally spaced planets is workable.

3. For three planets:
An even-numbered planet (as 20T) will not fit the 120° position shown. Three equally spaced planets is not workable.

**Comment:** It is evident from this problem that if equally spaced planets are chosen from manufacturing and maintenance considerations, the speed ratios available are limited by geometric constraints.

### SOLUTION (15.44)

**Known:** There is a need for a planetary gear train providing a speed ratio of 2.0.

**Find:** Determine an explanation of why it is theoretically impossible to provide a speed ratio of 2 with a planetary gear train.

### Schematic and Given Data:

![Planetary Gear Diagram](image)

### Assumptions:
1. The gears mesh at their pitch circles.
2. At least one among ring, arm and sun is fixed with the other gears acting as input or output.
3. The speed ratio required is exactly 2.

### Analysis:

1. With the sun fixed: \( \omega_R/\omega_A = 1 + \frac{S}{R} \) which can only equal 2 if \( S = R \), and this requires planets of zero size.
2. With the ring fixed: a vector diagram like Fig. 15.32 shows that again, a ratio of 2 requires planets of zero size.
3. With the arm fixed: (not a true planetary train, as all gear axes are fixed.) A vector diagram like Fig. 15.32 shows that the ratio is negative, and numerically equal to 2 only if planets have zero size.

**Comment:** This problem provides an important lesson in design problem solving: it is sometimes revealing to analyze a configuration of design using a symbolic model to establish the limits of its performance rather than iteratively choosing values of parameters to satisfy design requirements or estimate sensitivity.
SOLUTION (15.45)

**Known:** A planetary gear train for a bicycle is capable of three operating states:
(i) annulus (ring) is connected to input, sun is fixed and arm is connected to output (low gear).
(ii) direct engagement between input and output.
(iii) arm is connected to input, sun is fixed and annulus is connected to output (high gear).

There are four planet gears of specified pitch diameters and numbers of teeth.

**Find:**
(a) Determine the number of teeth on the annulus and the diametral pitch of the gears.
(b) Determine the ratio of wheel rpm to sprocket rpm for each operating state using two of the three methods given in text.
(c) Determine an explanation of operation when the bicycle coasts in each operating state.

**Schematic and Given Data:**

![Schematic Diagram]
**Assumption:** The gears mesh along their pitch circles.

**Analysis:**

1. Annulus dia. = sun dia. + 2(planet dia.)
   Since all the gears must have the same \( P \).
   teeth on annulus = teeth on sun
   \[ + 2(\text{teeth on planet}) \]
   \[ = 25 + 2(25) = 75 \text{ teeth} \]
   \[ P = \frac{25}{5/8} = 40 \]

2. Free Body Diagram Method for Low Gear:
   annulus = input, carrier = output
   let \( T = \text{input torque} \)

   carrier torque = \( 4\left(\frac{T}{75}\right)25 = \frac{4}{3}T \)
   hence, output speed = 3/4 of input speed
3. Velocity Vector Method for Low Gear:

\[
\text{output (carrier) } \omega = \frac{V}{25}
\]

\[
\text{input (annulus) } \omega = \frac{2V}{37.5}
\]

\[
\frac{\omega_{\text{out}}}{\omega_{\text{in}}} = \frac{V}{25} \times \frac{37.5}{2V} = \frac{3}{4}
\]

From Eq. (15.30):

\[
e = -\frac{S}{R} = \frac{\omega_R - \omega_A}{\omega_S - \omega_A} = -\frac{25}{75} = \frac{\omega_R - \omega_A}{0 - \omega_A} = -\frac{1}{3}
\]

\[
\omega_R - \omega_A = \frac{1}{3} \omega_A \quad \text{hence, } \omega_R \text{ (in)} = \frac{4}{3} \omega_A \text{ (out)}, \text{ or}
\]

\[
\frac{\omega_{\text{out}}}{\omega_{\text{in}}} = \frac{3}{4}
\]

4. neutral gear: \(\frac{\omega_{\text{out}}}{\omega_{\text{in}}} = 1.0\), and high gear: \(\frac{\omega_{\text{out}}}{\omega_{\text{in}}} = 4/3\)

5. While coasting since the drive is always through a 1-way clutch, the sprocket stops and the wheel continues to rotate. The 1-way clutch over-runs, causing a clicking noise:
   low gear: pawls 9 is over-run
   neutral gear: pawls 3 and 9 are both over-run
   high gear: pawls 3 and 9 are both over-run

Comments:
1. The application described in this problem demonstrates the compactness with which a speed-torque changing transmission can be implemented using planetary gear trains. A significant advantage of using planetary gear trains over ordinary spur gear trains in this application is the retention of circular symmetry of the transmission unit.
2. For applications in which more speed ratios are required it is possible to arrange sets of planetary gear trains in serial and parallel arrangements.
SOLUTION (15.46)

**Known:** A planetary gear train operates with the ring fixed, the sun driven at a given speed and torque and the arm driving a machine. Gears are of a specified module and pressure angle. There are two planets with a specified number of teeth.

**Find:** Determine:
(a) the circular pitch, \( p \) of the gears
(b) the free-body diagram of each member in equilibrium.
(c) the output torque
(d) the rpm and direction of rotation of the arm
(e) the pitch line velocity of each gear
(f) the nominal radial loads on bearings supporting each gear.
(g) the torque to be applied to the ring to keep it fixed.

**Schematic and Given Data:**

![Planetary Gear Diagram]

- Ring fixed
- Number of teeth = 70
- Sun, input
- Torque = 16 N·m
- Planets
- Number of teeth = 20
- Arm, output
- Module, \( m = 2.0 \) (mm/tooth)
- Pressure angle = 20°

**Assumptions:**
1. All gears mesh along their pitch circles.
2. Friction losses in gears and bearings are negligible.
3. All the tooth loads are transferred at the pitch point.
4. Centripetal forces will not be considered in this analysis (i.e., the gears are considered massless).

**Analysis:**

(a) From Eq. (15.6): \( p = \pi m : p = 2.0\pi \text{ mm} \)
(b) Since pitch diameter, \( d = Nm \),
   - planet diameter = \( 20(2) = 40 \) mm
   - ring diameter = \( 70(2) = 140 \) mm
   - sun diameter = \( R - 2P = 60 \) mm
(c) \( T_{\text{out}} = T_A = 53 \ 1/3 \ \text{N} \cdot \text{m} \)

(d) \( \omega_A = \omega_A(T_s/T_A) = 800 \ \text{rpm} \ (16/53.33) \):
\[ \omega_A = 240 \ \text{rpm clockwise} \]

(e) With respect to the arm, the sun rotates 560 rpm.
\[ V = \pi d n = \pi (0.06)(560/60) = 1.76 \ \text{m/s} \]

(f) The bearings supporting each planet carry a total radial load of 533 1/3 N.
Bearings supporting the other members carry zero nominal load.

(g) \( T_{\text{brake}} = T_r = 37 \ 1/3 \ \text{N} \cdot \text{m} \)

Comments:
1. Choice of a higher module for the gears would have resulted in larger diameter gears and correspondingly smaller tooth loads and smaller radial bearing loads for the planets (other parameter values remaining the same).
2. If friction forces in the gears and bearings were included then the output torque, the braking torque as well as the planet radial bearing loads would be lower.
3. Consideration of masses and centripetal forces would tend to reduce the radial tooth loads between the planets and the sun and increase the radial tooth loads between the planets and the ring. Further, there will be an additional radial load on the planet bearings. However, the input, output and ring braking torque would remain unaffected.

SOLUTION (15.47)
**Known:** A planetary gear train with double planets, two suns and no ring gear is given. Numbers of teeth on the planets and one of the suns are specified. One sun is the input member, the other sun is fixed and the arm is the output member.

**Find:** Determine the input-output speed ratio.
Schematic and Given Data:

Assumption: The gears mesh along their pitch circles.

Analysis:
1. **Free body analysis method**
   For a unit clockwise torque applied to S1, forces on the planet-pair are as follows. 
   (Let S1, P1, etc. represent relative radii of the members):

   ![Diagram showing forces on a gear system]

   Summing moments to zero:
   \[ \sum M_0 = 0 : F_2 = \frac{1}{S_1} \left| \frac{P_1}{P_2} \right| \]

   Summing forces to zero:
   \[ \sum F = 0 : \text{hence, } F_0 = \frac{1}{S_1} \left| 1 - \frac{P_1}{P_2} \right| \]

   Therefore, arm torque = \( F_0 \) (arm radius)
   \[ \text{=} \frac{1}{S_1} \left| 1 - \frac{P_1}{P_2} \right| (S_1 + P_1) \]
\[
\frac{1}{30} \left(1 - \frac{40}{32}\right)(70) = -0.5833
\]

For 100% efficiency, \( \frac{T_A}{T_{S1}} = \frac{\omega_{S1}}{\omega_A} \)

hence, \( \frac{\omega_A}{\omega_{S1}} = \frac{1}{-0.5833} = -1.714 \)

2. **Velocity vector solution**

![Diagram of velocity vector solution]

We assign unit velocity, as shown in the figure.
let \( P1, S1, \) etc. represent relative radii.

From known point of zero velocity, we determine planet velocity = \( -\frac{P2}{P1 - P2} \)

From \( \omega = V/r \):

\[
\frac{\omega_A}{\omega_{S1}} = \frac{-\frac{P2}{P1 - P2}}{S1 + P1} \cdot S1 = \frac{-\frac{32}{40 - 32}}{1} \cdot \frac{30}{30 + 40} = 1.714
\]

3. Input-output speed ratio,

\[
\frac{\omega_A}{\omega_{S1}} = -1.714
\]

**Comments:**
1. As this problem illustrates, it is not essential to have a ring gear to achieve speed changes in planetary gear trains. Here, the ring (an internal gear) is replaced by a second sun (an external gear) to perform the same function.
2. The use of two suns and two planets allows more flexibility in achieving speed ratio than if a ring gear were used because the pairs \( S1, P1 \) and \( S2, P2 \) can be independently chosen as long as \( S1 + P1 = S2 + P2 \).
SOLUTION (15.48)
Known: A planetary gear train with double planets, two suns and no ring gear is given. Numbers of teeth on the planets and one of the suns are specified. One sun is the input member, the other sun is fixed and the arm is the output member.

Find: Determine the input-output speed ratio.

Schematic and Given Data:

Assumption: The gears mesh along their pitch circles.

Analysis:
1. Free body analysis method
For a unit clockwise torque applied to S1, forces on the planet-pair are as follows. (Let S1, P1, etc. represent relative radii of the members):

Unit torque
\[
\frac{S1}{S1} = \frac{1}{S1}
\]

Summing moments to zero:
\[
\Sigma M_0 = 0 : F_2 = \frac{1}{S1} \frac{P1}{P2}
\]
Summing forces to zero:

\[ \Sigma F = 0 : \text{hence, } F_o = \frac{1}{S_1} \left[ 1 - \frac{P_1}{P_2} \right] \]

Therefore, arm torque = \( F_o \) (arm radius)

\[ = \frac{1}{S_1} \left[ 1 - \frac{P_1}{P_2} \right] (S_1 + P_1) \]

\[ = \frac{1}{28} \left( 1 - \frac{30}{24} \right) \frac{58}{58} = -0.5178 \]

For 100% efficiency, \( \frac{T_A}{T_{s1}} = \frac{\omega_{s1}}{\omega_A} \)

hence, \( \frac{\omega_A}{\omega_{s1}} = -\frac{1}{0.5178} = -1.931 \)

2. **Velocity vector solution**

![Diagram showing velocity vector](image)

We assign unit velocity, as shown in the figure. Let P1, S1, etc. represent relative radii.

From known point of zero velocity, we determine planet velocity = \( -\frac{P_2}{P_1 - P_2} \)

From \( w = V/r \):

\[ \frac{\omega_A}{\omega_{s1}} = \left( \frac{P_2}{P_1 - P_2} \right) \cdot \frac{S_1}{S_1 + P_1} = \left( \frac{24}{30 - 24} \right) \cdot \frac{28}{28 + 30} = -1.931 \]

3. **Input-output speed ratio**

\[ \frac{\omega_A}{\omega_{s1}} = -1.931 \]

**Comments:**

1. As this problem illustrates, it is not essential to have a ring gear to achieve speed changes in planetary gear trains. Here, the ring (an internal gear) is replaced by a second sun (an external gear) to perform the same function.
2. The use of two suns and two planets allows more flexibility in achieving speed ratio than if a ring gear were used because the pairs S1, P1 and S2, P2 can be independently chosen as long as S1 + P1 = S2 + P2.

SOLUTION (15.49)

**Known:** A planetary gear train with double planets, two suns and no ring gear is given. Numbers of teeth on the planets and one of the suns are specified. One sun is the input member, the other sun is fixed and the arm is the output member.

**Find:** Determine the input-output speed ratio.

**Schematic and Given Data:**

![Diagram of planetary gear train]

**Assumption:** The gears mesh along their pitch circles.

**Analysis:**

1. **Free body analysis method**
   For a unit clockwise torque applied to S1, forces on the planet-pair are as follows. (Let S1, P1, etc. represent relative radii of the members):
Summing moments to zero:

$$\Sigma M_0 = 0 : F_2 = \frac{1}{S_1} \left| \frac{P_1}{P_2} \right|$$

Summing forces to zero:

$$\Sigma F = 0 : \text{hence, } F_0 = \frac{1}{S_1} \left( 1 - \frac{P_1}{P_2} \right)$$

Therefore, arm torque = $F_0$ (arm radius)

$$= \frac{1}{S_1} \left( 1 - \frac{P_1}{P_2} \right) (S_1 + P_1)$$

$$= \frac{1}{32} \left( 1 - \frac{36}{30} \right) (68) = -0.425$$

For 100% efficiency, $\frac{T_A}{T_{s1}} = \frac{\omega_{s1}}{\omega_A}$

hence, $\frac{\omega_A}{\omega_{s1}} = \frac{1}{0.425} = -2.353$

2. Velocity vector solution

We assign unit velocity, as shown in the figure.

let $P_1$, $S_1$, etc. represent relative radii.

From known point of zero velocity, we determine planet velocity $= \frac{-P_2}{P_1 - P_2}$

From $w = V/r$:

$$\frac{\omega_A}{\omega_{s1}} = \left( \frac{-P_2}{P_1 - P_2} \right) \cdot \frac{S_1}{S_1 + P_1} = \left( \frac{-30}{36 - 30} \right) \cdot \frac{32}{32 + 36} = -2.353$$

3. Input-output speed ratio,

$$\frac{\omega_A}{\omega_{s1}} = -2.353$$
Comments:
1. As this problem illustrates, it is not essential to have a ring gear to achieve speed changes in planetary gear trains. Here, the ring (an internal gear) is replaced by a second sun (an external gear) to perform the same function.
2. The use of two suns and two planets allows more flexibility in achieving speed ratio than if a ring gear were used because the pairs \( S_1, P_1 \) and \( S_2, P_2 \) can be independently chosen as long as \( S_1 + P_1 = S_2 + P_2 \).

SOLUTION (15.50)

Known: A planetary gear train with double planets, two suns, and no ring gear is given. Numbers of teeth on planets and suns are specified. The arm acts as the input member, one of the suns is fixed and the other sun acts as the output member.

Find: Determine the input-output speed ratio.

Schematic and Given Data:

Assumption: All gears mesh along their pitch circles.

Analysis:
1. Let the center distance be unity; i.e.,

\[
\frac{1}{2}(P_1 + S_1) = \frac{1}{2}(P_2 + S_2) = 1; \\
\frac{P_1}{P_1 + S_1} = \text{radius of } P_1, \quad \frac{S_1}{P_1 + S_1} = \text{radius of } S_1, \text{ etc.}
\]

2. Rotate input member (arm A) so as to give the unit linear velocity vector shown; thus

\[
\omega_{in} = \omega_A = \frac{\text{velocity}}{\text{radius}} = \frac{1}{1} = 1
\]
3. Using the velocity vector method,

\[ \omega_{out} = \omega_{S1} = \frac{V}{S1 + P1} \]

where the pitch line velocity of S1 is

\[ V = \frac{P2}{P2 + S2} - \frac{P1}{P1 + S1} = \frac{P2 - P1}{P2} \]

Thus,

\[ \omega_{out} = \frac{P2 - P1}{S1 + P1} = \frac{(P2 - P1)(S1 + P1)}{P2(S1)} = \frac{(102 - 101)(100 + 101)}{(102)(100)} = 0.0197 \]

and \( \frac{\omega_{out}}{\omega_{in}} = +0.0197 \)

**Comment:** Increasing the number of teeth on all the gears while keeping the difference in number of teeth, \( S1 - S2 = P2 - P1 = 1 \), will produce an even lower speed ratio:

\[ \frac{\omega_{out}}{\omega_{in}} = \frac{(P2 - P1)(S1 + P1)}{(P2)(S1)} \approx \frac{S1 + P1}{P1(S1)} = \frac{1}{P1} + \frac{1}{S1} \]
SOLUTION (15.51)

**Known:** A planetary gear transmission has three suns, three planets (all planets mounted on the same arm), and no ring gear. Numbers of teeth on all gears are specified. The arm is the input. In each of two operating states, exactly one of two sun gears is fixed and the third sun gear acts as output.

**Find:** Determine the transmission ratios for each operating state.

**Schematic and Given Data:**

![Planetary Gear Schematic](image)

**Analysis:**

1. **Velocity Vector Method for Low Gear:**

   Assigning unit linear velocity as shown

   \[ \omega_{in} = \omega_A = \frac{1}{S1 + P1} = \frac{1}{27 + 27} = \frac{1}{54} \]

   on the same basis:

   \[ \omega_{out} = \omega_{S1} = \frac{\text{vector } "V"}{S1} = \frac{(P3 - P1)/P3}{S1} = \frac{(33 - 27)/33}{27} \]

   Therefore, speed ratio \[\frac{\omega_{in}}{\omega_{out}} = \left(\frac{1}{54}\right) \left(\frac{(33)(27)}{(33 - 27)}\right) = 2.75\]

   Therefore, transmission ratio \[\frac{T_{out}}{T_{in}} = +2.75\]
2. **Free Body Analysis Method for Reverse Gear:**

A given $T_{in} = T_A$ produces the vector $F_{in}$ shown. Summing moments to zero gives:

$$F_{out} = \frac{F_{in}(P2)}{P1 - P2}$$

correspondingly, $T_{out} = \frac{F_{in}(P2)}{P1 - P2}$ (S1)

Therefore, Torque ratio,

$$\frac{T_{out}}{T_{in}} = \frac{\left(\frac{T_{in}}{S1 + P1}\right)(P2)(S1)}{T_{in}(P1 - P2)} = \frac{(P2)(S1)}{(S1 + P1)(P1 - P2)}$$

$$= \frac{(24)(27)}{(27 + 27)(27 - 24)} = 4.00$$

The diagram shows a clockwise input torque, producing $F_{in}$ acting to the right. Vector $F_{out}$ acting on the output member (S1) is equal and opposite to vector $F_{out}$ shown acting on the planet. Thus $T_{out}$ is counterclockwise, and the transmission ratio in reverse gear $= -4.00$
Comment: The idea illustrated in this problem can be extended if more operating states and speed ratios are required, i.e., more sun gears and/or ring gears can be incorporated and other speed ratios can be obtained by fixing or freeing the suns/rings.