\textbf{In[42]}:= \texttt{Apply[Clear, Names["Global\`*"]];}
\texttt{Off[General::spell];}
\texttt{Off[General::spell1];}

(* helical compression spring
  hard-drawn spring steel
  ends are plain and ground *)

d = 2. \times 10^{-3}; (* m * )
D0 = 24. \times 10^{-3}; (* m *)
Nt = 9.;

Na = Nt - 1;
Ls = \texttt{dNt};
\texttt{Print["Ls = ", Ls, " m"]};

mD = D0 - d; (* mean diameter *)
G = 79 \times 10^9;
k = (d^4 G) / (8 mD^3 Na);
\texttt{Print["k = ", k, " N/m"]};

m = 0.201;
A = 1510 \times 10^6;
\texttt{Cind = mD/d};
\texttt{Sut = A/d^m};
\texttt{Ssy = 0.5 Sut};
\texttt{Ks = (2 Cind + 1) / (2 Cind)};
\texttt{Fs = (Ssy N[Pi] d^3) / (8 mD Ks)};
\texttt{Print["Fs = ", Fs, " N"]};

\texttt{def = Fs / k};

L0 = Ls + def;
\texttt{Print["L0 = ", L0, " m"]};

\texttt{Rx = L0 / mD};
\texttt{Ry = def / L0};

\texttt{Print["Rx = ", Rx]};
\texttt{Print["Ry = ", Ry]};
\texttt{Print["unstable"]};

\texttt{Ls = 0.018 m}
\texttt{k = 1854.81 N/m}
\texttt{Fs = 359.635 N}
\texttt{L0 = 0.211893 m}
\texttt{Rx = 9.63152}
Ry = 0.915052

unstable
In[111]:= Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

(* helical compression spring made of steel
closed ends *)

d = 3 \times 10^{-3}; (* m *)
\(D_0 = 56 \times 10^{-3}; \) (* m *)
\(L_0 = 0.1; \) (* m *)
\(N_t = 13;\)

\(mD = D_0 - d;\)
\(C_{in} = mD / d;\)

"a)"
\(G = 79. \times 10^9; \) (* Pa *)
\(Na = N_t - 2;\)
\(k = (G d) / (8 C_{in}^3 Na);\)
Print["k = ", k, " N/m";]

"b)"
\(L_s = d \times (N_t + 1);\)
\(\text{def} = L_0 - L_s;\)
\(Fs = k \text{def};\)
Print["Fs = ", Fs, " N";]

\(K_s = (2 C_{in} + 1) / (2 C_{in});\)
\(ts = (8 K_s Fs mD) / (N[\Pi] d^3);\)
Print["ts = ", ts, " N";]

Out[120]= a)
\(k = 488.429 \text{ N/m}\)

Out[124]= b)
\(Fs = 28.3289 \text{ N}\)
\(ts = 1.45614 \times 10^8 \text{ N}\)
Problem II.7.5

The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the contraints.

\[ \delta U = 0 \] (1)

\[ F \delta y - kx \delta x - mg \delta y_1 - mg \delta y_2 = 0 \] (2)

Remark: Work is positive when the working component of the force is in the same direction as the displacement.

\[ y = l \sin \theta \quad \delta y = l \cos \theta \delta \theta \] (3)

\[ x = 2l - 2l \cos \theta \quad \delta x = 2l \sin \theta \delta \theta \] (4)

\[ y_1 = l \cos \theta \quad \delta y_1 = -l \sin \theta \delta \theta \] (5)

\[ y_2 = 3l \cos \theta \quad \delta y_2 = -3l \sin \theta \delta \theta \] (6)

\[ Fl \cos \theta \delta \theta - 4l^2 k(1 - \cos \theta) \sin \theta \delta \theta + 4lmg \sin \theta \delta \theta = 0 \] (7)

\[ F = 4[kl(1 - \cos \theta) - mg] \tan \theta \] (8)
Problem II.7.7

The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the contraints.

\[ \delta U = 0 \]  \hspace{1cm} (1)

\[ F \delta y - mg \delta y_1 - mg \delta y_2 - kx \delta x = 0 \]  \hspace{1cm} (2)

Remark: Work is positive when the working component of the force is in the same direction as the displacement.

\[ y = 2l \cos \theta \quad \delta y = -2l \sin \theta \delta \theta \]  \hspace{1cm} (3)

\[ x = 2l \sin \theta \quad \delta x = 2l \cos \theta \delta \theta \]  \hspace{1cm} (4)

\[ y_2 = \frac{l}{2} \cos \theta \quad \delta y_2 = -\frac{l}{2} \sin \theta \delta \theta \]  \hspace{1cm} (5)

\[ y_1 = l \cos \theta \quad \delta y_1 = -l \sin \theta \delta \theta \]  \hspace{1cm} (6)

\[ -2Fl \sin \theta \delta \theta + \frac{l}{2} mg \sin \theta \delta \theta + lmg \sin \theta \delta \theta = 4l^2 k \cos \theta \sin \theta \delta \theta \]  \hspace{1cm} (7)

\[ k = \frac{3mg - 4F}{8l \cos \theta} \]  \hspace{1cm} (8)
In[132]:= Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

init = {l -> 0.4, m -> 10, g -> 9.81, F -> 70, k -> 1.8};

x2 = l/2 Sin[phi];
x1 = l Cos[phi];
x3 = l/2 Cos[phi];
x = l/2 - l/2 Cos[phi];

dx2 = D[x2, phi];
dx1 = D[x, phi];
dx3 = D[x3, phi];
dx = D[x, phi];

Fx2 = F Cos[phi];
Fx1 = F Sin[phi];
G = m g;

dU = (Fx2 dx2 - Fx1 dx1 - k x dx - G dx3) /. init;

sol = Solve[dU == 0, phi]
Print["phi = ", -0.454 / Degree, "^\"°\"];

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

Out[148]= {{phi -> -2.68583}, {phi -> -0.45433},
{phi -> 1.57265 - 0.518841 i}, {phi -> 1.57265 + 0.518841 i}}

phi = -26.0123^°
Problem II.7.10

The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the contraints.

\[
\delta U = 0 \tag{1}
\]

\[
kx \delta x - ky \delta y + \frac{l}{2} mg \cos \theta = 0 \tag{2}
\]

Remark: Work is positive when the working component of the force is in the same direction as the displacement.

\[
y = \frac{L}{2} + l \sin \theta \quad \delta y = l \cos \theta \delta \theta \tag{3}
\]

\[
x = \frac{L}{2} - l \sin \theta \quad \delta x = -l \cos \theta \delta \theta \tag{4}
\]

\[
-\frac{L}{2} kl \cos \theta \delta \theta + \frac{1}{2} kl^2 \sin 2 \theta \delta \theta + \frac{L}{2} kl \cos \theta \delta \theta + \frac{1}{2} kl^2 \sin 2 \theta \delta \theta - \frac{1}{2} mgl \sin \theta \delta \theta = 0 \tag{5}
\]

\[
k = \frac{mg l}{2l^2 \sin \theta \cos \theta} \quad \theta = 0 \quad k = \frac{mg}{4l} \tag{6}
\]
Problem II.7.11

The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the contraints.

\[ \delta U = 0 \]  

(1)

\[ M\delta \theta + G\delta x = 0 \]

(2)

Remark: Work is positive when the working component of the force is in the same direction as the displacement.

\[ x = 2l \cos \frac{\theta}{2}, \quad \delta x = -l \sin \frac{\theta}{2} \delta \theta \]

(3)

\[ Mk\theta \delta \theta - mgl \sin \frac{\theta}{2} \delta \theta = 0 \]

(4)

\[ k = \frac{mgl \sin \frac{\theta}{2}}{M\theta} \]

(5)

\[ k_{\text{min}} = \lim_{\theta \to 0} \frac{mgl \sin \frac{\theta}{2}}{M\theta} = \frac{mgl}{2M} \]

(6)
Apply[Clear, Names["Global`}*)];
Off[General::spell];
Off[General::spell1];

init = {l \rightarrow 15, m \rightarrow 10, g \rightarrow 9.81, F \rightarrow 90, k \rightarrow 15};

ss = 2 l \text{Sin}[, phi / 2] - L0; (* spring strech *)
Ve = l / 2 k ss^2;
Vg = m l / 2 \text{Cos}[phi] + m (l + l / 2 \text{Cos}[phi]) + F l \text{Cos}[phi];
V = Ve + Vg;
eq = D[V, phi] /. init;
sol = Solve[eq == 0, phi]

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. More...

\{
{phi \rightarrow -\pi}, {phi \rightarrow \pi}, {phi \rightarrow 2 \text{ArcSin}[\frac{3 L0}{50}]}\}