```
In[42]:= Apply[Clear, Names["Global`*"]];
         Off[General::spell];
         Off[General::spell1];

         (* helical compression spring
            hard-drawn spring steel
            ends are plain and ground *)

         d = 2. * 10^-3; (* m *)
         D0 = 24. * 10^-3; (* m *)
         Nt = 9.;

         Na = Nt - 1;
         Ls = d Nt;
         Print["Ls = ", Ls, " m"];

         mD = D0 - d; (* mean diameter *)
         G = 79 * 10^9;
         k = (d^4 G) / (8 mD^3 Na);
         Print["k = ", k, " N/m"];

         m = 0.201;
         A = 1510 * 10^6;
         Cind = mD / d;
         Sut = A / d^m;
         Ssy = 0.5 Sut;
         Ks = (2 Cind + 1) / (2 Cind);
         Fs = (Ssy N[Pi] d^3) / (8 mD Ks);
         Print["Fs = ", Fs, " N"];

         def = Fs / k;

         L0 = Ls + def;
         Print["L0 = ", L0, " m"];

         Rx = L0 / mD;
         Ry = def / L0;

         Print["Rx = ", Rx];
         Print["Ry = ", Ry];
         Print["unstable"];
```

```
Ls = 0.018 m

k = 1854.81 N/m

Fs = 359.635 N

L0 = 0.211893 m

Rx = 9.63152
```

```
Ry = 0.915052

unstable
```

```
In[111]:= Apply[Clear, Names["Global`*"]];
          Off[General::spell];
          Off[General::spell1];

          (* helical compression spring made of steel
             closed ends *)

          d = 3 * 10^-3; (* m *)
          D0 = 56 * 10^-3; (* m *)
          L0 = 0.1; (* m *)
          Nt = 13;

          mD = D0 - d;
          Cind = mD / d;

          "a)"
          G = 79. * 10^9; (* Pa *)
          Na = Nt - 2;
          k = (G d) / (8 Cind^3 Na);
          Print["k = ", k, " N/m"];

          "b)"
          Ls = d (Nt + 1);
          def = L0 - Ls;
          Fs = k def;
          Print["Fs = ", Fs, " N"];

          Ks = (2 Cind + 1) / (2 Cind);
          ts = (8 Ks Fs mD) / (N[Pi] d^3);

          Print["ts = ", ts, " N"];

Out[120]= a)

        k = 488.429 N/m

Out[124]= b)

        Fs = 28.3289 N

        ts = 1.45614×10^8 N
```

Problem II.7.5

The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the contraints.

$$\delta U = 0 \tag{1}$$

$$F\delta y - kx\delta x - mg\delta y_1 - mg\delta y_2 = 0 \tag{2}$$

Remark: Work is positive when the working component of the force is in the same direction as the displacement.

$$y = l\sin\theta \qquad \delta y = l\cos\theta\delta\theta \tag{3}$$

$$x = 2l - 2l\cos\theta \qquad \delta x = 2l\sin\theta\delta\theta \tag{4}$$

$$y_1 = l\cos\theta \qquad \delta y_1 = -l\sin\theta\delta\theta \tag{5}$$

$$y_2 = 3l\cos\theta \qquad \delta y_2 = -3l\sin\theta\delta\theta \tag{6}$$

$$Fl\cos\theta\delta\theta - 4l^2k(1-\cos\theta)\sin\theta\delta\theta + 4lmg\sin\theta\delta\theta = 0 \tag{7}$$

$$F = 4[kl(1-\cos\theta) - mg]\tan\theta \tag{8}$$

Problem II.7.7

The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the contraints.

$$\delta U = 0 \tag{1}$$

$$F\delta y - mg\delta y_1 - mg\delta y_2 - kx\delta x = 0 \tag{2}$$

Remark: Work is positive when the working component of the force is in the same direction as the displacement.

$$y = 2l\cos\theta \qquad \delta y = -2l\sin\theta\delta\theta \tag{3}$$

$$x = 2l\sin\theta \qquad \delta x = 2l\cos\theta\delta\theta \tag{4}$$

$$y_2 = \frac{l}{2}\cos\theta \qquad \delta y_2 = -\frac{l}{2}\sin\theta\delta\theta \tag{5}$$

$$y_1 = l\cos\theta \qquad \delta y_1 = -l\sin\theta\delta\theta \tag{6}$$

$$-2Fl\sin\theta\delta\theta + \frac{l}{2}mg\sin\theta\delta\theta + lmg\sin\theta\delta\theta = 4l^2k\cos\theta\sin\theta\delta\theta \tag{7}$$

$$k = \frac{3mg - 4F}{8l\cos\theta} \tag{8}$$

```
In[132]:= Apply[Clear, Names["Global`*"]];
          Off[General::spell];
          Off[General::spell1];

          init = {l → 0.4, m -> 10, g -> 9.81, F → 70, k → 1.8};

          x2 = l / 2 Sin[phi];
          x1 = l Cos[phi];
          x3 = l / 2 Cos[phi];
          x = l / 2 - l / 2 Cos[phi];

          dx2 = D[x2, phi];
          dx1 = D[x, phi];
          dx3 = D[x3, phi];
          dx = D[x, phi];

          Fx2 = F Cos[phi];
          Fx1 = F Sin[phi];
          G = m g;

          dU = (Fx2 dx2 - Fx1 dx1 - k x dx - G dx3) /. init;

          sol = Solve[dU == 0, phi]
          Print["phi = ", -0.454 / Degree, "°"];

     Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

Out[148]= {{phi → -2.68583}, {phi → -0.45433},
           {phi → 1.57265 - 0.518841 i}, {phi → 1.57265 + 0.518841 i}}

     phi = -26.0123°
```

Problem II.7.10

The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the contraints.

$$\delta U = 0 \tag{1}$$

$$kx\delta x - ky\delta y + \frac{l}{2}mg\cos\theta = 0 \tag{2}$$

Remark: Work is positive when the working component of the force is in the same direction as the displacement.

$$y = \frac{L}{2} + l\sin\theta \qquad \delta y = l\cos\theta\delta\theta \tag{3}$$

$$x = \frac{L}{2} - l\sin\theta \qquad \delta x = -l\cos\theta\delta\theta \tag{4}$$

$$-\frac{L}{2}kl\cos\theta\delta\theta + \frac{1}{2}kl^2\sin 2\theta\delta\theta + \frac{L}{2}kl\cos\theta\delta\theta + \frac{1}{2}kl^2\sin 2\theta\delta\theta - \frac{1}{2}mgl\sin\theta\delta\theta = 0 \tag{5}$$

$$k = \frac{mg\frac{l}{2}\sin\theta}{2l^2\sin\theta\cos\theta} \qquad \theta = 0 \qquad k = \frac{mg}{4l} \tag{6}$$

Problem II.7.11

The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the contraints.

$$\delta U = 0 \tag{1}$$

$$M\delta\theta + G\delta x = 0 \tag{2}$$

Remark: Work is positive when the working component of the force is in the same direction as the displacement.

$$x = 2l\cos\frac{\theta}{2} \qquad \delta x = -l\sin\frac{\theta}{2}\delta\theta \tag{3}$$

$$Mk\theta\delta\theta - mgl\sin\frac{\theta}{2}\delta\theta = 0 \tag{4}$$

$$k = \frac{mgl\sin\frac{\theta}{2}}{Mk\theta} \tag{5}$$

$$k_{min} = \lim_{\theta\to 0}\frac{mgl\sin\frac{\theta}{2}}{M\theta} = \frac{mgl}{2M} \tag{6}$$

```
Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

init = {l → 15, m -> 10, g -> 9.81, F → 90, k → 15};

ss = 2 l Sin[phi / 2] - L0; (* spring strech *)
Ve = 1 / 2 k ss^2;
Vg = m l / 2 Cos[phi] + m (l + l / 2 Cos[phi]) + F l Cos[phi];
V = Ve + Vg;
eq = D[V, phi] /. init;
sol = Solve[eq == 0, phi]
```

```
Solve::ifun : Inverse functions are being used by Solve, so
   some solutions may not be found; use Reduce for complete solution information. More…
```

$\{\{phi \to -\pi\}, \{phi \to \pi\}, \{phi \to 2\,\mathrm{ArcSin}[\frac{3\,L0}{50}]\}\}$