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Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];
" input data "
" load F [N] "
" major diameter d [mm] "
" screw pitch p [mm] "
" coefficient of friction for thread  $\mu$  "
" collar diameter dc [mm] "
" coefficient of friction for collar  $\mu_c$  "
data = {F  $\rightarrow$  8000, d  $\rightarrow$  40, p  $\rightarrow$  6,  $\mu$   $\rightarrow$  0.08, dc  $\rightarrow$  45,  $\mu_c$   $\rightarrow$  0.1}
"solution"
l = 2 p;
Print["lead for double thread l = ", l, " = ", l /. data, " [mm]"]
dr = d - p;
Print["minor diameter dr = ", dr, " = ", dr /. data, " [mm]"]
dm = d - p / 2;
Print["mean (pitch) diameter dm = ", dm, " = ", dm /. data, " [mm]"]
Mr = 0.5 F dm ( 1 +  $\pi \mu$  dm ) / (  $\pi$  dm -  $\mu$  l ) + 0.5 F dc  $\mu_c$ ;
Print["moment to raise load
Mr = 0.5 F dm ( 1 +  $\pi \mu$  dm ) / (  $\pi$  dm -  $\mu$  l ) + 0.5 F dc  $\mu_c$  "]
Print["Mr = ", Mr /. data, " [N mm] = ", 10(-3) Mr /. data, " [N m]"]
Ml = 0.5 F dm (  $\pi \mu$  dm - l ) / (  $\pi$  dm +  $\mu$  l ) + 0.5 F dc  $\mu_c$ ;
Print[
"moment to lower load Ml = 0.5 F dm (  $\pi \mu$  dm - l ) / (  $\pi$  dm +  $\mu$  l ) + 0.5 F dc  $\mu_c$  "]
Print["Ml = ", Ml /. data, " [N mm] = ", 10(-3) Ml /. data, " [N m]"]
sf = (  $\pi \mu$  dm - l );
"self-locking condition: (  $\pi \mu$  dm - l ) > 0 "
Print["(  $\pi \mu$  dm - l ) = ", sf /. data]
If[(sf /. data) > 0, Print["the screw is self-locking"],
Print["the screw is not self-locking"]]
e = F l / ( 2  $\pi$  Mr );
Print["efficiency e = F l / ( 2  $\pi$  Mr ) = ", e /. data]

input data

load F [N]

major diameter d [mm]

screw pitch p [mm]

coefficient of friction for thread  $\mu$ 

collar diameter dc [mm]

coefficient of friction for collar  $\mu_c$ 

{F  $\rightarrow$  8000, d  $\rightarrow$  40, p  $\rightarrow$  6,  $\mu$   $\rightarrow$  0.08, dc  $\rightarrow$  45,  $\mu_c$   $\rightarrow$  0.1}

solution

lead for double thread l = 2 p = 12 [mm]

minor diameter dr = d - p = 34 [mm]

mean (pitch) diameter dm = d -  $\frac{p}{2}$  = 37 [mm]

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moment to raise load  $M_r = 0.5 F d_m ( 1 + \pi \mu d_m ) / ( \pi d_m - \mu l ) + 0.5 F d_c \mu c$

$M_r = 45344.7 \text{ [N mm]} = 45.3447 \text{ [N m]}$

moment to lower load  $M_l = 0.5 F d_m ( \pi \mu d_m - 1 ) / ( \pi d_m + \mu l ) + 0.5 F d_c \mu c$

$M_l = 14589.3 \text{ [N mm]} = 14.5893 \text{ [N m]}$

self-locking condition:  $( \pi \mu d_m - 1 ) > 0$

$( \pi \mu d_m - 1 ) = -2.70089$

the screw is not self-locking

efficiency  $e = F l / ( 2 \pi M_r ) = 0.336949$

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(* Acme double thread power screw *)
Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

(*Given data*)
d = 2.; (* in *) (* major diameter *)
F = 2000.; (* lb *) (* weight of the load *)
dc = 3.; (* in *) (* mean diameter of plain thrust collar *)
μ = 0.12; μc = 0.09; (* coefficients of friction *)

(*Assumption: coefficient of starting friction is about one-
third higher than the coefficient of friction (running friction)
  μs=(4/3) μ ; μcs=(4/3) μc
*)

(* a *)
p = 1/4.; (* in *) (* screw pitch; Standard sizes of power screw threads) for d=
  2 in. there are 4 (four) threads per inch, p=(1 in.)/4=0.25 in. *)
l = 2 p; (* lead, because of the double thread *)
dm = d - p/2; (* mean diameter of thread contact from figure *)
λ = ArcTan[l / (π dm)]; (* rad *) (* lead angle *)

(* b *)
(*For starting, increase the coefficients of running friction by one third*)
μs = (4/3) μ;
μcs = (4/3) μc;
(* coefficients of static friction *)
α = 14.5 * π / 180; (* rad *)
(* from Figure , 2 α=29 deg yields α=14.5 deg (Acme)*)
αn = ArcTan[Tan[α] * Cos[λ]]; (* thread angle in the normal plane *)
(* the torque for raising the load: *)
(* Moment using coefficients of static friction *)
Mrs = 0.5 F dm (π μs dm + l Cos[αn]) / (π dm Cos[αn] - μs l) + 0.5 F dc μcs;
(* Moment using coefficients of running friction *)
Mr = 0.5 F dm (π μ dm + l Cos[αn]) / (π dm Cos[αn] - μ l) + 0.5 F dc μc;
(* for lowering the load*)
Mls = 0.5 F dm (π μs dm - l Cos[αn]) / (π dm Cos[αn] + μs l) + 0.5 F dc μcs;
Ml = 0.5 F dm (π μ dm - l Cos[αn]) / (π dm Cos[αn] + μ l) + 0.5 F dc μc;

(* c *)
(* Efficiency (the ratio of friction-free moment to actual moment) *)
Mr0 = F l / (2 π);
e = F l / (2 π Mr);

Print["Screw pitch p = (1 in.)/4 = ", p, " in"];
Print["Lead l = 2 p = ", l, " in"];
Print["Mean pitch diameter dm = d-p/2 = ", dm, " in"];
Print["Lead angle λ=ArcTan[l/(π dm)] = ", N[λ] * 180 / Pi, " deg"];
Print["Thread angle αn=ArcTan[Tan[α]*Cos[λ]] = ", αn * 180 / Pi, " deg"];
Print["Moment for raising the load with starting friction = ", Mrs, " lb in"];
Print["Moment for lowering the load with starting friction = ", Mls, " lb in"];
Print["Moment for raising the load with running friction = ", Mr, " lb in"];
Print["Moment for raising the load with no friction = ", N[Mr0], " lb in"];
Print["Efficiency (the ratio of friction-free moment to actual moment) e = ", e, " %"];

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deg = 180 / N[Pi];

μ = 0.05;
graph1 = Plot[(Cos[αn] - μ * Tan[λ]) / (Cos[αn] + μ * Cot[λ]) * 100,
  {λ, 0.0, 1.5192087045738232}, PlotLabel → "Efficiency e(λ) (%) for μ =0.05"];

μ = 0.12;
graph2 = Plot[(Cos[αn] - μ * Tan[λ]) / (Cos[αn] + μ * Cot[λ]) * 100,
  {λ, 0.0, 1.4475046197126569}, PlotLabel → "Efficiency e(λ) (%) for μ =0.12"];

μ = 0.15;
graph3 =
  Plot[(Cos[αn] - μ * Tan[λ]) / (Cos[αn] + μ * Cot[λ]) * 100, {λ, 0.0, 1.4171173868835931},
  PlotLabel → "Efficiency e(λ) (%) for μ =0.15"];

Show[graph1, graph2, graph3];

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Screw pitch  $p = (1 \text{ in.})/4 = 0.25 \text{ in}$

Lead  $l = 2 p = 0.5 \text{ in}$

Mean pitch diameter  $d_m = d - p/2 = 1.875 \text{ in}$

Lead angle  $\lambda = \text{ArcTan}[l / (\pi d_m)] = 4.85179 \text{ deg}$

Thread angle  $\alpha_n = \text{ArcTan}[\text{Tan}[\alpha] * \text{Cos}[\lambda]] = 14.4502 \text{ deg}$

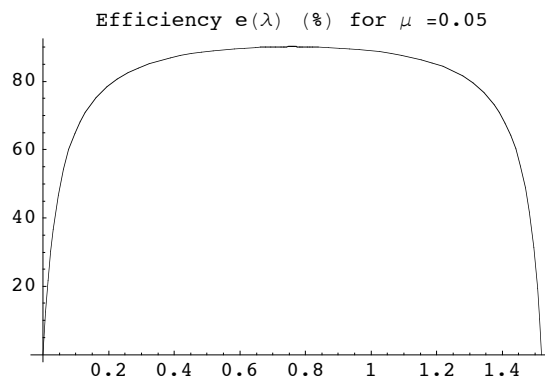
Moment for raising the load with starting friction = 835.626 lb in

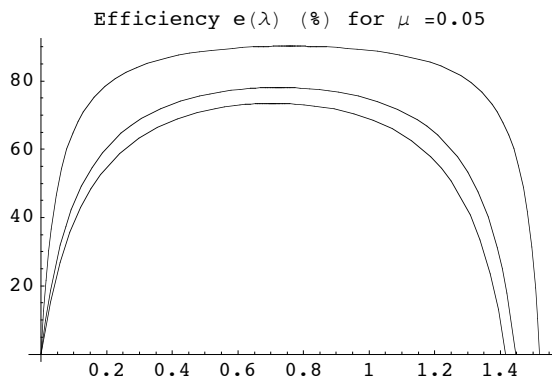
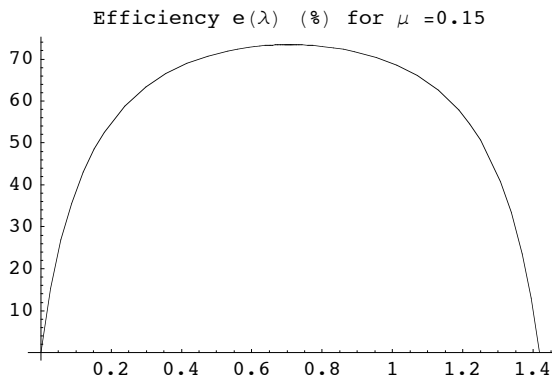
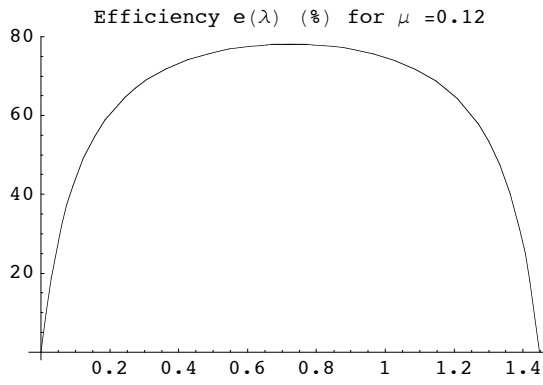
Moment for lowering the load with starting friction = 508.562 lb in

Moment for raising the load with running friction = 665.667 lb in

Moment for raising the load with no friction = 159.155 lb in

Efficiency (the ratio of friction-free moment to actual moment)  $e = 0.239091 \%$





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Apply[Clear,Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

d=14. 10^(-3); (* m *) (* major diameter *)
dr=12. 10^(-3); (* m *) (* minor diameter *)
p=2. 10^(-3); (* m *) (* pitch *)
l=50. 10^(-3); (* m *) (* joint axial length *)

Print["major thread diameter d=",d," m"];
Print["minor thread diameter dr=",dr," m"];
Print["pitch p=",p," m"];
Print["joint axial length l=",l," m"];

"modulus of elasticity [GPa] E=206.8 for steel"
Eb=206.8 10^9;
Print["modulus of elasticity of the bolt Eb=",Eb," Pa"];

"a. Bolt stiffness kb"
lt=l/2;
lte=lt+0.4 dr;
Print["effective length of threaded portion of the bolt lte=lt+0.4 dr=",lte," m"];
ls=l/2;
lse=ls+0.4 d;
Print["effective length of the unthreaded portion of the bolt lse=ls+0.4 d=",lse," m"];

Ar=N[Pi] dr^2/4.;
Print["minor diameter area Ar=Pi dr^2/4=",Ar," m^2"];
As=N[Pi] d^2/4.;
Print["major diameter area As=Pi d^2/4=",As," m^2"];

kt=Ar Eb/lte;
Print["stiffness of the threaded portion kt=Ar Eb/lte=",kt," N/m"];
ks=As Eb/lse;
Print["stiffness of the unthreaded portion ks=As Eb/lse=",ks," N/m"];
kb=kt ks/(kt+ks);
Print["bolt stiffness kb=kt ks/(kt+ks)=",kb," N/m"];

Print["joint axial length l=",l," m"];
l1=l2=l/2;
"axial length of the parts l1=l2=l/2"
"modulus of elasticity [GPa] E=100.0 for gray cast iron"
Ec=100 10^9;
Print["modulus of elasticity of the part Ec=",Ec," Pa"];
"Joint Stiffness kc"

"b. Stiffness of clamped parts (Shigley)"
Print["J.E.Shigley & C.R.Mischke, Mechanical Engineering Design, McGraw-Hill 1989"];
kc=0.577 N[Pi] Ec d/(2 Log[5 (0.577 l + 0.5 d)/(0.577 l + 2.5 d)]);
Print["kc=0.577 Pi Ec d/(2 Log[5 (0.577 l + 0.5 d)/(0.577 l + 2.5 d)])"];
Print["stiffness of clamped parts (Shigley) kc=",kc," N/m"];

"c. Stiffness of clamped parts (Wileman)"
Print["J.Wileman et al.,1990"];
"constants A=0.78715; B=0.62873 for steel"
"constants A=0.77871; B=0.61616 for cast iron"
"joint stiffness: kc = Ec d A e^(B d/l)"
"parts of the joint: cast iron "
A=0.77871;
B=0.61616;
kcw=Ec d A E^(B d/l);
Print["joint stiffness (Wileman) kc=",kcw," N/m"];

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major thread diameter  $d=0.014$  m

minor thread diameter  $d_r=0.012$  m

pitch  $p=0.002$  m

joint axial length  $l=0.05$  m

modulus of elasticity [GPa]  $E=206.8$  for steel

modulus of elasticity of the bolt  $E_b=2.068 \times 10^{11}$  Pa

a. Bolt stiffness  $k_b$

effective length of threaded portion of the bolt  $l_{te}=l+0.4 d_r=0.0298$  m

effective length of the unthreaded portion of the bolt  $l_{se}=l+0.4 d=0.0306$  m

minor diameter area  $A_r=\pi d_r^2/4=0.000113097$  m<sup>2</sup>

major diameter area  $A_s=\pi d^2/4=0.000153938$  m<sup>2</sup>

stiffness of the threaded portion  $k_t=A_r E_b/l_{te}=7.8485 \times 10^8$  N/m

stiffness of the unthreaded portion  $k_s=A_s E_b/l_{se}=1.04034 \times 10^9$  N/m

bolt stiffness  $k_b=k_t k_s/(k_t+k_s)=4.47357 \times 10^8$  N/m

joint axial length  $l=0.05$  m

axial length of the parts  $l_1=l_2=l/2$

modulus of elasticity [GPa]  $E=100.0$  for gray cast iron

modulus of elasticity of the part  $E_c=100000000000$  Pa

Joint Stiffness  $k_c$

b. Stiffness of clamped parts (Shigley)

J.E.Shigley & C.R.Mischke, Mechanical Engineering Design, McGraw-Hill 1989

$k_c=0.577 \pi E_c d / (2 \log[5 (0.577 l + 0.5 d) / (0.577 l + 2.5 d)])$

stiffness of clamped parts (Shigley)  $k_c=1.22925 \times 10^9$  N/m

c. Stiffness of clamped parts (Wileman)

J.Wileman et al., 1990

constants  $A=0.78715$ ;  $B=0.62873$  for steel

constants  $A=0.77871$ ;  $B=0.61616$  for cast iron

joint stiffness:  $k_c = E_c d A e^{(B d/l)}$

parts of the joint: cast iron

joint stiffness (Wileman)  $k_c=1.29548 \times 10^9$  N/m

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Apply[Clear,Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

"coarse thread"
d=5/8.; (* in *) (* major diameter *)
dr=0.5135; (* in *) (* minor diameter *)
n=11; (* threads per inch *)
p=1./n;
(* At=226 in^2 *)
l=1.5 ; (* in *) (* joint axial length *)

Print["major thread diameter d=",d," in.];
Print["minor thread diameter dr=",dr," in.];
Print["pitch p=1/n=",p," in.];
Print["joint axial length l=",l," in.];

l1=l2=l/2;
"axial length of the parts l1=l2=l/2"

"modulus of elasticity [Mpsi] E=30 for steel"
"modulus of elasticity [Mpsi] E=12 for cast iron"

Eb=30;
Ec=12;
Print["modulus of elasticity of the bolt Eb=",Eb," Mpsi"];
Print["modulus of elasticity of the part Ec=",Ec," Mpsi"];

Print["-----"];
"a. Bolt stiffness kb"
Print["-----"];
Print["J.E.Shigley & C.R.Mischke, Mechanical Engineering Design, McGraw-Hill 1989"];
lt=l/2;
Print["length of threaded portion of the bolt lt=",lt];
ls=l/2;
Print["length of unthreaded portion of the bolt ls=",ls];
As=N[Pi] d^2/4.;
Print["major diameter area As=Pi d^2/4=",As];
At=0.7854(d-0.9743 p)^2;
Print["tensile strength area At=0.7854(d-0.9743 p)^2=",At];
ks=As Eb/ls;
Print["stiffness of the unthreaded portion ks=As Eb/ls=",ks," Mlb/in"];
kt=At Eb/lt;
Print["stiffness of the threaded portion kt=At Eb/lt=",kt," Mlb/in"];
kb=ks kt/(ks+kt);
Print["bolt stiffness kb=kt ks/(kt+ks)=",kb," Mlb/in"];

Print["-----"];
Print["Stiffness of the parts kc"];
Print["-----"];

Print["b. J.E.Shigley & C.R.Mischke, Mechanical Engineering Design, McGraw-Hill 1989"];

k1=0.577 N[Pi] Ec d/(2 Log[5 (0.577 l1+0.5 d)/(0.577 l1+2.5 d)]);
Print["k1=0.577 Pi Ec d/(2 Log[5 (0.577 l1 +0.5 d)/(0.577 l1 +2.5 d)])"];
Print["stiffness of the cast iron part k1=",k1," Mlb/in"];

k2=0.577 N[Pi] Eb d/(2 Log[5 (0.577 l2+0.5 d)/(0.577 l2+2.5 d)]);
Print["k2=0.577 Pi Eb d/(2 Log[5 (0.577 l2 +0.5 d)/(0.577 l2 +2.5 d)])"];
Print["stiffness of the steel part k2=",k2," Mlb/in"];
kc=k1 k2/(k1+k2);
Print["resulting stiffness of clamped parts: kc=k1 k2/(k1+k2)=",kc," Mlb/in"];

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Print["c. J.Wileman et al,1990"];
"constants A=0.78715; B=0.62873 for steel"
"constants A=0.77871; B=0.61616 for cast iron"
"joint stiffness: kc = Ec d A e^(B d/L)"

"parts of the joint: cast iron "
A1=0.77871;
B1=0.61616;
kw1=Ec d A1 E^(B1 d/l1);
Print["stiffness k1=Ec d1 A1 E^(B1 d1/l1)=",kw1," Mlb/in"];

"parts of the joint: steel "
A2=0.78715;
B2=0.62873;
kw2=Eb d A2 E^(B2 d/l2);
Print["joint stiffness: k2=Eb d A2 E^(B2 d/l2)=",kw2," Mlb/in"];

kwc=kw1 kw2/(kw1+kw2);
Print["resulting stiffness of clamped parts: kc=k1 k2/(k1+k2)=",kwc," Mlb/in"];

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coarse thread

major thread diameter d=0.625 in.

minor thread diameter dr=0.5135 in.

pitch p=1/n=0.0909091 in.

joint axial length l=1.5 in.

axial length of the parts l1=l2=l/2

modulus of elasticity [Mpsi] E=30 for steel

modulus of elasticity [Mpsi] E=12 for cast iron

modulus of elasticity of the bolt Eb=30 Mpsi

modulus of elasticity of the part Ec=12 Mpsi

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a. Bolt stiffness kb  
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J.E.Shigley & C.R.Mischke, Mechanical Engineering Design, McGraw-Hill 1989

length of threaded portion of the bolt lt=0.75

length of unthreaded portion of the bolt ls=0.75

major diameter area As=Pi d^2/4=0.306796

tensile strength area At=0.7854(d-0.9743 p)^2=0.226002

stiffness of the unthreaded portion ks=As Eb/ls=12.2718 Mlb/in

stiffness of the threaded portion kt=At Eb/lt=9.04009 Mlb/in

bolt stiffness kb=kt ks/(kt+ks)=5.20547 Mlb/in

-----  
 Stiffness of the parts  $k_c$   
 -----

b. J.E.Shigley & C.R.Mischke, Mechanical Engineering Design, McGraw-Hill 1989

$$k_1 = 0.577 \pi E_c d / (2 \log[5 (0.577 l_1 + 0.5 d) / (0.577 l_1 + 2.5 d)])$$

stiffness of the cast iron part  $k_1 = 10.8826 \text{ Mlb/in}$

$$k_2 = 0.577 \pi E_b d / (2 \log[5 (0.577 l_2 + 0.5 d) / (0.577 l_2 + 2.5 d)])$$

stiffness of the steel part  $k_2 = 27.2065 \text{ Mlb/in}$

resulting stiffness of clamped parts:  $k_c = k_1 k_2 / (k_1 + k_2) = 7.77327 \text{ Mlb/in}$

c. J.Wileman et al, 1990

constants  $A = 0.78715$ ;  $B = 0.62873$  for steel

constants  $A = 0.77871$ ;  $B = 0.61616$  for cast iron

joint stiffness:  $k_c = E_c d A e^{(B d/L)}$

parts of the joint: cast iron

$$\text{stiffness } k_1 = E_c d_1 A_1 E^{(B_1 d_1/l_1)} = 9.75962 \text{ Mlb/in}$$

parts of the joint: steel

$$\text{joint stiffness: } k_2 = E_b d A_2 E^{(B_2 d/l_2)} = 24.9232 \text{ Mlb/in}$$

resulting stiffness of clamped parts:  $k_c = k_1 k_2 / (k_1 + k_2) = 7.0133 \text{ Mlb/in}$