Contents

5 Lubrication and Sliding Bearings

Lubrication reduces the friction, wear, and heating of machine parts in relative motion. A lubricant is a substance that is inserted between the moving parts.

5.1 Viscosity

Newton's law of viscous flow
A surface of area A is moving with the linear velocity V on a film of lubricant as shown in Fig. 5.1(a). The thickness of the lubricant is s and the deforming force acting on the film is F. The layers of the fluid in contact with the moving surface have the velocity $v = V$ and the layers of the fluid in contact with the fixed surface have the velocity $v = 0$.

Newton's law of viscous flow states that the shear stress τ in a fluid is proportional to the rate of change of the velocity v with respect to the distance y from the fixed surface,

$$\tau = \frac{F}{A} = \mu \frac{\partial v}{\partial y}, \tag{5.1}$$

where μ is a constant, the *absolute viscosity*, or the *dynamic viscosity*. The derivate $\frac{\partial v}{\partial y}$ is the rate of change of velocity with distance and represents the rate of shear, or the velocity gradient. Thicker oils have a higher viscosity value causing relatively higher shear stesses at the same shear rate. For a constant velocity gradient Eq. (5.1) can be written as [Fig. 5.1(b)]:

$$\tau = \mu \frac{V}{s}. \tag{5.2}$$

The unit of viscosity μ, for U.S. Customary units, is pound-second per square inch (lb·s/in^2) or reyn (from Osborne Reynolds).
In SI units the viscosity is expressed as newton-seconds per square meter (N·s/m^2) or pascal seconds (Pa·s).
The conversion factor between the two is the same as for stress

$$1\,\text{reyn} = 1\,\text{lb·s/in}^2 = 6890\,\text{N·s/m}^2 = 6890\,\text{Pa·s}.$$

The reyn and pascal-second are such large units that microreyn (μreyn) and millipascal second (mPa·s) are more commonly used. The former standard metric unit of viscosity was the poise (shortening of Jean Louis Marie

Poiseuille, French physician and physiologist). One centipoise, cp, is equal to one millipascal-second:

$$1\,\text{cp} = 1\,\text{mPa·s}.$$

Dynamic viscosities are usually measured under high shear conditions, for example, the cylinder viscometer in which the viscous shear torque is measured between two cylinders. The *kinematic viscosity* is defined as absolute viscosity, μ, divided by mass density, ρ

$$\nu = \frac{\mu}{\rho}. \tag{5.3}$$

The units for kinematic viscosity are length^2/time, as cm^2/s, which is the Stoke (St). Using SI units: 1 m^2/s = 10^4 St and 1 cSt (centistoke) = 1 mm^2/s.

The physical principle of measurement is based on the rate at which a fluid flows vertically downward under gravity through a small-diameter tube. Liquid viscosities are determined by measuring the time required for a given quantity of the liquid to flow by gravity through a precision opening. For lubricating oils, the Saybolt Universal Viscometer, shown in Fig. 5.2, is an instrument used to measure the viscosity. The viscosity measurements are *Saybolt seconds*, or *SUS* (Saybolt Universal Seconds), *SSU* (Saybolt Seconds Universal), and *SUV* (Saybolt Universal Viscosity).

With a Saybolt Universal Viscometer one can measure the kinematic viscosity, ν. Absolute viscosities can be obtained from Saybolt viscometer measurements by the equations

$$\mu\,(\text{mPa·s or cp}) = \left(0.22\,t - \frac{180}{t}\right)\rho, \tag{5.4}$$

and

$$\mu\,(\mu\text{reyn}) = 0.145\left(0.22\,t - \frac{180}{t}\right)\rho, \tag{5.5}$$

where ρ is the mass density in grams per cubic centimeter, g/cm^3 (which is also called specific gravity) and t is the time in seconds. For petroleum oils the mass density at different temperatures is

$$\rho = 0.89 - 0.00063\left({}^0\text{C} - 15.6\right)\ \text{g/cm}^3, \tag{5.6}$$

or

$$\rho = 0.89 - 0.00035\left({}^{0}\mathrm{F} - 60\right) \;\; \mathrm{g/cm^3}. \tag{5.7}$$

The Society of Automotive Engineers (SAE) classifies oils according to viscosity. Any viscosity grade should be proceeded by the initials SAE. It should be noted that SAE is not a performance category it only refers to the viscosity of the oil.

Two series of SAE viscosity are defined (Table 5.1): monograde oils or single viscosity oils and multigrade oils (those with the suffix W, which are essentially for winter conditions).

For the monograde oils (grades without the W), the viscosity is measured in cSt at 100^0C (212^0F). A monograde oil only has one part, such as SAE 30, or SAE 40. The number after SAE gives a measure of the viscosity of the oil at high temperature (100^0C). The lower the number the thinner the oil is at high temperature. So an SAE 30 is a thinner, or less viscous, oil than an SAE 40.

A multigrade oil (multi-viscosity graded motor oil) must meet the viscosity standard at the W temperature and the SAE viscosity requirement at 100^0C. For the W grades (W from winter), viscosity is measured by two test methods, one in the cold cranking simulator and the other in a pumping test that evaluates borderline pumping temperature. The cold cranking simulator reports dynamic viscosity in cp at temperatures that depend upon the grade. Additionally, the viscosity of motor oils with a W suffix is measured in cSt at 100^0C. A multigrade oil is an oil that has two parts, such as SAE 15W-40 or 20W-50. For the multigrade oil SAE-10W20, the first number (10W) refers to the viscosity grade at low temperatures (W), whereas the second number (20) refers to the viscosity grade at high temperatures. The lower the W number the lower the viscosity of the oil. Therefore an SAE 5W oil is a lower viscosity oil than an SAE 10W oil.

The multigrade oils SAE 10W-30 and SAE 15W-30 have a similar high temperature viscosity as indicated by the 30. The 10W-30 oil is a thinner oil than the SAE 15W-30 at cold temperatures as indicated by the W number ($10 < 15$). Therefore, in cold temperatures, the SAE 10W-30 oil is better than the SAE 15W-30 oil. In winter it is beneficial to move from an SAE 15W-30 oil to an SAE 10W-30 oil.

During summer the ambient temperatures are high and the oil tends to be thinner, so a more viscous oil should be used. The SAE 10W-20 is a thinner oil than the SAE 10W-30 or SAE 15W-30 at high temperatures as indicated by the second number ($20 < 30$). Therefore in warm temperatures, a thicker oil (either an SAE 10W-30 or SAE 15W-30) could offer better engine protection than SAE 10W-20. In summer SAE 10W type oil is not required which is why the SAE 15W-40 is favored.

There is much discussion about mineral oils versus synthetic oils and the relative performance of each type. The synthetic oils offer certain advantages over mineral oils in terms of low temperature performance and high temperature oxidation stability. Synthetic oils are very expensive, and properly formulated mineral oils are more than suitable for most engine applications. A synthetic oil can be considered for very cold temperatures, or for applications that may need a high level of oxidation protection. The manufacturer's recommendations should be followed.

Petroleum products can be graded according to the ISO Viscosity Classification System, approved by the International Standards Oreganization (ISO). Each ISO viscosity grade number corresponds to the mid-point of a viscosity range expressed in centistokes (cSt) at 40^0C (the viscosity of the ISO grades, however, is measured at 40^0C instead of $100^0\text{F} = 37.8^0$C, which results in a slightly more viscous lubricant for each corresponding grade). For the ISO 3448 viscosity classification system the ISO VG 22 lubricant refers to a viscosity grade of 22 cSt $\pm$ 10 % at 40^0C. The kinematic viscosity limits are 19.8 cSt (min.) and 24.2 cSt (max.), and the mid-point viscosity is 22 cSt.

In Figs. 5.3, 5.4, and 5.5 the absolute viscosity function of temperature for typical SAE numbered oils is shown. Grease is a non-Newtonian material that does not begin to flow until a shear stress exceeding a yield point is applied.

The *viscosity index* (VI) measures the variation in viscosity with temperature. The viscosity index, on the Dean and Davis scale, of Pennsylvania oils is VI=100. The viscosity index, on the same scale, of Gulf Coast oils is VI=0. Other oils are rated intermediately. Nonpetroleum-base lubricants have widely varying viscosity indices. Silicone oils have relatively little variation of viscosity with temperature. The viscosity index improvers (additives) can increase viscosity index of petroleum oils.

5.2 Petroff's Equation

Hydrodynamic lubrication is defined when the surfaces of the bearing are separated by a film of lubricant and does not depend upon the introduction of the lubricant under pressure. The pressure is created by the motion of the moving surface. Hydrostatic lubrication is defined when the lubricant is introduced at a pressure sufficiently high to separate the surfaces of the bearing.

A hydrodynamic bearing (hydrodynamic lubrication) is considered in Fig. 5.6. There is no lubricant flow in the axial direction and the bearing carries a very small load. The radius of the shaft is R, the radial clearance is c, and the length of the bearing is L (Fig. 5.6). The shaft rotates with the angular speed n rev/s and its surface velocity is $V = 2\pi Rn$.
From Eq. (5.2) the shearing stress is

$$\tau = \mu \frac{V}{s} = \frac{2\pi R\mu n}{c}. \tag{5.8}$$

The force required to shear the film is the stress times the area,

$$F = \tau A,$$

where $A = 2\pi RL$.
The friction torque is the force times the lever arm:

$$T_f = FR = (\tau A)\, R = \left(\frac{2\pi R\mu n}{c} 2\pi RL\right) R = \frac{4\pi^2 \mu n L R^3}{c}. \tag{5.9}$$

If a small radial load W is applied on the bearing, the pressure P (the radial load per unit of projected bearing area) is

$$P = \frac{W}{2RL}.$$

The friction force is $f\,W$, where f is the coefficient of friction, and the friction torque is

$$T_f = f\,W\,R = f\,(2RLP)\,R = 2R^2 f LP. \tag{5.10}$$

Equations. (5.9) and (5.10) can be equated and the coefficient of friction is

$$f = 2\pi^2 \left(\frac{\mu\, n}{P}\right) \left(\frac{R}{c}\right). \tag{5.11}$$

This is called Petroff's law or Petroff's equation. In Petroff's equation there are two important bearing parameters: the dimensionless variable, $\left(\frac{\mu\, n}{P}\right)$ and the clearance ratio $\left(\frac{R}{c}\right)$ with the order between 500 to 1000.
The bearing *characteristic number*, or Sommerfeld number S, is given by

$$S = \frac{\mu\, n}{P}\left(\frac{R}{c}\right)^2. \tag{5.12}$$

where R is the journal radius (in.), c is the radial clearance (in.), μ is the absolute viscosity (reyn), n is the speed (rev/s), and P is the pressure (psi). The power loss in SI units is calculated with the relation

$$H = 2\,\pi\, T_f\, n, \quad \text{W} \tag{5.13}$$

where H = power (W), n = shaft speed (rev/s), T_f = torque (N·m), or

$$H = \frac{T_f\, n}{9549}, \quad \text{kW} \tag{5.14}$$

where H = power (kW), n = shaft speed (rpm), and T_f = torque (N·m).

The power loss in British units is

$$H = \frac{T_f\, n}{5252}, \quad \text{hp} \tag{5.15}$$

where H = power (hp), n = shaft speed (rpm), and T_f = torque (lb·ft).

5.3 Hydrodynamic Lubrication Theory

In Fig. 5.7 a small element of lubricant film of dimensions dx, dy, and dz is shown. The normal forces, due the pressure, act upon right and left sides of the element. The shear forces, due to the viscosity and to the velocity, act upon the top and bottom sides of the element. The equilibrium of forces give

$$p\ dx\ dz + \tau\ dx\ dz - \left(p + \frac{dp}{dx}dx\right)\ dy\ dz - \left(\tau + \frac{\partial \tau}{\partial y}dy\right)\ dx\ dz = 0, \tag{5.16}$$

which reduces to

$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y}. \tag{5.17}$$

In Eq. (5.17) the pressure of the film p is constant in y direction and depends only on the coordinate x, $p = p(x)$. The shear stress τ is calculated from Eq. (5.1):

$$\tau = \mu \frac{\partial v(x,y)}{\partial y}. \tag{5.18}$$

The velocity v of any particle of lubricant depends on both coordinates x and y, $v = v(x,y)$.
From Eqs. (5.17) and (5.18), it results

$$\frac{dp}{dx} = \mu \frac{\partial^2 v}{\partial^2 y}, \tag{5.19}$$

or

$$\frac{\partial^2 v}{\partial^2 y} = \frac{1}{\mu}\frac{dp}{dx}. \tag{5.20}$$

Holding x constant and integrating twice with respect to y gives

$$\frac{\partial v}{\partial y} = \frac{1}{\mu}\left(\frac{dp}{dx} y + C_1\right), \tag{5.21}$$

and

$$v = \frac{1}{\mu}\left(\frac{dp}{dx}\frac{y^2}{2} + C_1 x + C_2\right). \tag{5.22}$$

The constants C_1 and C_2 are calculated using the boundary conditions:
for $y = 0 \Longrightarrow v = 0$, and for $y = s \Longrightarrow v = V$.
With C_1 and C_2 values computed, Eq.(5.18) gives the equation for the velocity distribution of the lubricant film across any yz plane:

$$v = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - s\,y\right) + \frac{V}{s} y. \tag{5.23}$$

Equation (5.23) gives the velocity distribution of the lubricant in the film as a function of the coordinate y and the pressure gradient dp/dx. The velocity distribution is a superposition of a parabolic distribution, the first term, onto a linear distribution, the second term (and shown as a dashed line in Fig. 5.8).

The volume of lubricant Q flowing across the section for width of unity in the z direction is

$$Q = \int_0^s v(x,y)dy = \frac{Vs}{2} - \frac{s^3}{12\mu}\frac{dp}{dx}. \tag{5.24}$$

For incompressible lubricant the flow is the same for any section:

$$\frac{dQ}{dx} = 0.$$

By differentiating Eq. (5.24), one can write

$$\frac{dQ}{dx} = \frac{V}{2}\frac{ds}{dx} - \frac{d}{dx}\left(\frac{s^3}{12\mu}\frac{dp}{dx}\right),$$

or

$$\frac{d}{dx}\left(\frac{s^3}{\mu}\frac{dp}{dx}\right) = 6V\frac{ds}{dx}, \tag{5.25}$$

which is the classical Reynolds equation for one-dimensional flow.
The following assumptions were made:

- the fluid is Newtonian, incompressible, of constant viscosity, and experiences no inertial or gravitational forces;
- the fluid has a laminar flow, with no slip at the boundary surfaces;
- the fluid experiences negligible pressure variation over its thickness;
- the journal radius can be considered infinite.

The Reynolds equation for two-dimensional flow is (the z direction is included)

$$\frac{\partial}{\partial x}\left(\frac{s^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{s^3}{\mu}\frac{\partial p}{\partial z}\right) = 6V\frac{\partial s}{\partial x}. \tag{5.26}$$

For short bearings, one can neglect the x term in the Reynolds equation

$$\frac{\partial}{\partial z}\left(\frac{s^3}{\mu}\frac{\partial p}{\partial z}\right) = 6V\frac{\partial h}{\partial x}. \tag{5.27}$$

Equation (5.27) can be used for analysis and design.

5.4 Design Charts

Raimondi and Boyd have transformed the solutions of the Reynolds Eq. (5.27) to chart form. The charts provide accurate solutions for bearings of all proportions. Some charts are shown in Figs. 5.9 to 5.15. The quantities given in the charts are shown in Fig. 5.16. The Raimondi and Boyd charts give plots of dimensionless bearing parameters as functions of the bearing characteristic number, or Sommerfeld variable, S. The S scale on the charts is logarithmic except for a linear portion between 0 and 0.01.

5.5 Examples

Example 5.1. The Saybolt kinematic viscosity of an oil corresponds to 60 seconds at 90^0C (Fig. 5.2). What is the corresponding absolute viscosity in millipascal-seconds (or centipoises) and in microreyns?

Solution.

From Eq. (5.6), the mass density of the oil is
$\rho = 0.89 - 0.00063\ (^0\text{C} - 15.6) = 0.89 - 0.00063\ (90 - 15.6) = 0.843$ g/cm^3.
From Eq. (5.4), the absolute viscosity in centipoise is
$\mu = \left(0.22\,t - \dfrac{180}{t}\right)\rho = \left[(0.22)(60) - \dfrac{180}{60}\right] 0.843 = 8.598$ cp (or 8.598 mPa·s).
From Eq. (5.5), the absolute viscosity in microreyns, μreyn, is
$\mu = 0.145\left(0.22t - \dfrac{180}{t}\right)\rho = 0.145\left[(0.22)(60) - \dfrac{180}{60}\right] 0.843 = 1.246\ \mu$reyn.

Example 5.2. A shaft with a 120 mm diameter (Fig. 5.17) is supported by a bearing of 100 mm length with a diametral clearance of 0.2 mm and is lubricated by oil having a viscosity of 60 mPa·s. The shaft rotates at 720 rpm. The radial load is 6000 N. Find the bearing coefficient of friction and the power loss.

Solution.

The pressure is calculated with the relation

$$P = \frac{W}{2\,R\,L} = \frac{6000}{2(0.06)\,(0.1)} = 500\,000\ \text{N/m}^2 = 500\,000\ \text{Pa},$$

where $W = 6000$ N, $R = 0.06$ m, and $L = 0.1$ m.

From Eq.(5.11), the coefficient of friction is

$$f = 2\pi^2 \left(\frac{\mu\, n}{P}\right)\left(\frac{R}{c}\right) = 2\pi^2 \left[\frac{(0.06)(12)}{500\,000}\right]\left(\frac{60}{0.1}\right) = 0.017,$$

where $\mu = 60$ mPa·s $= 0.06$ Pa·s, $n = 720$ rev/min $= 12$ rev/s, $R = 60$ mm, and $c = 0.1$ mm.

The friction torque is calculated with

$$T_f = f\, W\, R = (0.017)\,(6000)\,(0.06) = 6.139 \text{ N}\cdot\text{m}$$

The power loss is
$H = 2\,\pi\, T_f\, n = 2\,\pi\;(6.139)\,(12) = 462.921$ N·m/s $=$ 462.921 W.

Example 5.3. A journal bearing has the diameter $D = 2.5$ in., the length $L = 0.625$ in., and the radial clearance $c = 0.002$ in., as shown in Fig. 5.18. The shaft rotates at 3600 rpm. The journal bearing supports a constant load, $W = 1500$ lb. The lubricant film is SAE 40 oil at atmospheric pressure. The average temperature of the oil film is $T_{avg} = 140^0$F.
Find the minimum oil film thickness, h_0, the bearing coefficient of friction, f, the maximum pressure, p_{max}, the position angle of minimum film thickness, ϕ, the angular position of the point of maximum pressure $\theta_{p_{max}}$, the terminating position of the oil film θ_{po}, the total oil flow rate, Q, and the flow ratio (side flow / total flow) Q_s/Q.

Solution.

The pressure is $P = \dfrac{W}{L\,D} = \dfrac{1500}{(0.625)\,(2.5)} = 960$ psi.
The dynamic viscosity, is $\mu = 5 \times 10^{-6}$ reyn (SAE 40, $T_{avg} = 140^0 F$), from Fig. 5.4.

The Sommerfeld number is

$$S = \left(\frac{R}{c}\right)^2 \frac{\mu\, n}{P} = \left(\frac{1.25}{0.002}\right)^2 \left[\frac{(5 \times 10^{-6})\,(60)}{960}\right] = 0.12.$$

For all charts $S = 0.12$ and $L/D = 0.25$ are used.
From Fig. 5.9, the minimum film thickness variable is $h_o/c = 0.125$ and the minimum film thickness is $h_0 = 0.125\, c = 0.00025$ in.
From Fig. 5.11, the friction variable is $(R/c)\, f = 5$ and the coefficient of friction is $f = 5\, c/R = 0.00832$.
From Fig. 5.14, the pressure ratio is $P/p_{max} = 0.2$ and the maximum film

pressure is $p_{max} = P/0.2 = 4800$ psi.
From Fig. 5.10, the position angle of minimum film thickness is $\phi = 24°$ (see Fig. 5.16).
From Fig. 5.15, the terminating position of the oil film is $\theta_{po} = 33°$ and the angular position of the point of maximum pressure is $\theta_{p_{max}} = 9.5°$ (see Fig. 5.16)
From Fig. 5.12, the flow variable is $\dfrac{Q}{R\,c\,n\,L} = 5.9$ and the total flow is $Q = 0.553$ in^3/s.
From Fig. 5.13, the flow ratio (side leakage flow / total flow) is $Q_s/Q = 0.94$. Of the volume of oil Q pumped by the rotating journal, an amont Q_s flows out the ends. The side leakage that must be made up by the oil represents 94% of the flow. The remaining 6% of the flow is recirculated.

Example 5.4. A journal shaft of a gear train has a rotational speed of 2200 rpm and a radial load of 2200 lb. The shaft is lubricated with a SAE 30 oil and the average film temperature is 180°F. Determine the values of the clearance c for the two edges of the optimum zone for the bearing characteristic number for $L/D = 1$ (Fig. 5.9).

Solution.

From Fig. 5.9 the optimum values for the bearing characteristic number are [7]

L/D	S for min. friction	S for max. load
1	0.082	0.21

where D is the diameter and L is the length of the bearing.

The absolute viscosity of an SAE 30 oil at 180°F is [7] (see Fig. 5.4):

$$\mu = 1.87\,\mu\text{reyn} = 1.87 \times 10^{-6}\text{reyn}.$$

From Table 5.2, the representative unit sleeve bearing load, for gear reducers, arbitrarily is selected $P = 250$ psi.
With $L = D$ the bearing length is

$$L = D = \sqrt{\frac{W}{P}} = \sqrt{\frac{2200}{250}} = 2.966 \text{ in.}$$

A diameter of $D = 3$ in. is selected.
The clearance is calculated from the relation $S = \left(\dfrac{\mu\,n}{P}\right)\left(\dfrac{R}{c}\right)^2$,

where $n = 2200/60 = 36.666$ rps and $R = 3/2 = 1.5$ in.
For minimum friction $S_{min} = 0.082$ the clearance is

$$c = R\sqrt{\frac{\mu\, n}{P\, S_{min}}} = (1.5)\sqrt{\frac{(1.87 \times 10^{-6})\,(36.666)}{(250)\,(0.082)}} = 0.002743 \text{ in.}$$

and for maximum load $S_{max} = 0.21$ the clearance is

$$c = R\sqrt{\frac{\mu\, n}{P\, S_{max}}} = (1.5)\sqrt{\frac{(1.87 \times 10^{-6})\,(36.666)}{(250)\,(0.21)}} = 0.001714 \text{ in.}$$

Example 5.5. The oil lubricated bearing of a steam turbine has the diameter $D = 160$ mm (Fig. 5.19). The angular velocity of the rotor shaft is $n = 2400$ rpm. The radial load is $W = 18$ kN. The lubricant is SAE 20, controlled to an average temperature of 78°C.
Find the bearing length, the radial clearance, the corresponding values of the minimum oil film thickness, the coefficient of friction and the friction power loss.

Solution.

From Table 5.2, for steam turbine (1 to 2 MPa range), the unit load P=1.5 MPa is arbitrarily selected. The bearing length is

$$L = \frac{W}{P\,D} = \frac{18\,000}{(1.5)(160)} = 75 \text{ mm.}$$

Arbitrarily round this up to $L = 80$ mm to give $L/D = 1/2$ for convenient use of the Raimondi and Boyd charts.
With $L = 80$ mm, P is given by the relation

$$P = \frac{W}{L\,D} = \frac{18\,000}{(80)\;(160)} = 1.406 \text{ MPa.}$$

From Fig. 5.3 the viscosity of SAE 20 oil at 78°C is $\mu = 9.75$ mPa·s.

From Fig. 5.9 the optimum values for the bearing characteristic number are [7]:

L/D	S for min. friction	S for max. load
1/2	0.037	0.35

For minimum friction $S_{min} = 0.037$ the clearance is

$$c = R\sqrt{\frac{\mu\, n}{P\, S_{min}}} = (0.08)\sqrt{\frac{(9.75 \times 10^{-3})\,(40)}{(1.406 \times 10^{6})\,(0.037)}} = 0.219 \times 10^{-3} \text{ m.}$$

and for maximum load $S_{max} = 0.35$ the clearance is

$$c = R\sqrt{\frac{\mu\, n}{P\, S_{max}}} = (0.08)\sqrt{\frac{(9.75 \times 10^{-3})\,(40)}{(1.406 \times 10^{6})\,(0.35)}} = 0.071 \times 10^{-3} \text{ m.}$$

The minimum oil film thickness, h_o, is calculated from the ratio, h_o/c obtained from Fig. 5.9, and the coefficient of friction, f, is calculated from the ratio $R\,f/c$ obtained from Fig. 5.11.

The values of S, h_o, and f function of c, (0.048 mm $\leq c \leq$ 0.243 mm), with c extending to either side of the optimum range are listed below.

c mm	S	h_0 mm	f
0.0482629	0.762	0.0284751	0.00965258
0.0712126	0.350	0.0302654	0.00774437
0.1125970	0.140	0.0292752	0.00619284
0.1445050	0.085	0.0281784	0.00559955
0.2080650	0.041	0.0249678	0.00494154
0.2190230	0.037	0.0240926	0.00479113
0.2432370	0.030	0.0243237	0.00486475

Figure 5.20 shows h_o and f function of c, and indicates a good operation.

For the minimum acceptable oil film thickness, h_o, the following empirical relations are given (Trumpler empirical equation) [7]:

$$h_o \geq h_{omin} = 0.0002 + 0.00004D \quad (h_o \text{ and } D \text{ in inches}),$$
$$h_o \geq h_{omin} = 0.005 + 0.00004D \quad (h_o \text{ and } D \text{ in millimeters}). \quad (5.28)$$

For $D = 160$ mm, the minimum acceptable oil film thickness is
$h_{omin} = 0.005 + 0.00004\,(160) = 0.0114$ mm.
The minimum film thickness using a safety factor of C_s=2 applied to the load, and assuming an "extreme case" of $c = 0.243$ mm, is calculated as

follows:

• the Sommerfeld number is

$$S = \left(\frac{\mu\, n}{C_s P}\right)\left(\frac{R}{c}\right)^2 = \frac{(9.75 \times 10^{-3})(40)}{(2)(1.406 \times 10^6)}\left(\frac{80}{0.243}\right)^2 = 0.015.$$

• from Fig. 5.9 using S=0.015, $\dfrac{h_o}{c} = 0.06$ is obtained, and the minimum film thickness is $h_o = 0.0145$ mm.

This value satisfies the condition $h_o = 0.0145 \geq h_{omin} = 0.0114$.

For the tightest bearing fit, where $c = 0.048$ mm and f=0.009, the friction torque is

$$T_f = \frac{W\, f\, D}{2} = \frac{(18\,000)(0.009)(0.16)}{2} = 13.899 \text{ N} \cdot \text{m},$$

and the friction power is

$$\text{friction power} = \frac{n\, T_f}{9549} = \frac{(2400 \text{ rpm})(13.899 \text{ N} \cdot \text{m})}{9549} = 3.493 \text{ kW}.$$

The *Mathematica*™ program for this example is given in Program 5.1.

5.6 Problems

5.1 Determine the density of a SAE 40 oil at 160°F.

5.2 From a Saybolt viscometer the kinematic viscosity of an oil corresponds to 60 seconds at 120°C. Find the corresponding absolute viscosity in millipascal-seconds and in microreyns.

5.3 An engine oil has a kinematic viscosity at 93°C corresponding to 50 seconds, as determined from a Saybolt viscometer. Find the corresponding SAE number.

5.4 A 100 mm diameter shaft is supported by a bearing of 100 mm length with a diametral clearance of 0.075 mm. It is lubricated by SAE 20 oil at the operating temperature of 70°C. The shaft rotates 3000 rpm and carries a radial load of 5000 N. Estimate the bearing coefficient of friction and power loss using the Petroff approach.

5.5 A journal bearing 4 in. in diameter and 6 in. long is lubricated with an SAE 10 oil with the average temperature of 130°F. The diametral clearance of the bearing is 0.0015 in. The shaft rotates at 2000 rpm. Find the friction torque and the power loss.

5.6 A shaft with the diameter D, rotational speed n, and radial load W is supported by an oil lubricated bearing of length L and radial clearance c. There is no eccentricity between the bearing and the journal, and no lubricant flow in the axial direction. Determine the bearing coefficient of friction and the power loss. Numerical data is as follows: a) $D =$ 0.2 m, $L = 0.15$ m, c=0.075 mm, $n = 1200$ rpm, $W = 6.5$ kN, and $\mu = 32$ mPa·s (for SAE 10 oil at 40°C); b) $D = 1.5$ in., $L = 1.5$ in., c=0.0015 in., $n = 30$ rps, $W = 500$ lb, and $\mu = 4$ μreyn.

5.7 A journal bearing of 200 mm diameter, 100 mm length, and 0.1 mm radial clearance carries a load of 20 kN. The shaft rotates at 1000 rpm. The bearing is lubricated by SAE 20 oil and the average temperature of the oil film is estimated at 70°C. Determine the minimum oil film thickness bearing coefficient of friction, maximum pressure within the oil film, angles ϕ, $\theta_{p\max}$, θ_{po}, total oil flow rate through the bearing, and side leakage.

5.8 A journal bearing of 2 in. diameter, 2 in. length, and 0.001 in. radial clearance supports a load of 1500 lb when the shaft rotates 1000 rpm. The lubrication oil is SAE 30 supplied at atmospheric pressure. The average temperature of the oil film is 140°F. Using the Raimondi and Boyd charts, determine the minimum oil film thickness bearing coefficient of friction, maximum pressure within the oil film, angles ϕ, $\theta_{p\,\max}$, θ_{po}, total oil flow rate, and fraction of the flow rate that is recirculated oil flow.

5.9 A full journal bearing has the diameter of 60 mm and an L/D ratio of unity and runs at a speed of 200 rpm. The radial clearance is 0.04 mm and the oil supply is SAE 30 at the temperature of 60°C. The radial load is 3000 N. Determine the minimum oil film thickness bearing coefficient of friction and its angular location, the maximum pressure within the oil film and its angular location, and the side flow.

5.10 A 5 kN load is applied to a 100 mm diameter shaft rotating at 2000 rpm. A journal bearing is used to carry the load. The journal bearing with a diameter to length ratio of 0.25 is lubricated with an SAE 40 oil with an inlet temperature of 40°C. Determine the minimum oil film thickness bearing coefficient of friction and its angular location, the maximum pressure within the oil film and its angular location, and the side flow.

5.11 A shaft rotates at 1000 rpm and is lubricated with an SAE 30 oil at 80°C. The radial load is 30 kN. Determine the values of the clearance c for the two edges of the optimum zone for the bearing characteristic number for the ratio $L/D = 1$.

5.12 A journal bearing for a gear train rotates at 1000 rpm and applies a force of 10 kN to the bearing. An SAE 20 oil is used and the average temperature is expected to be 60°C. A proportion of $L/D = 1/2$ is desired. Find: a) the value of D, and b) the values of c corresponding for the two edges of the optimum zone for the bearing characteristic number.

5.13 The oil lubricated journal bearing of a gear reducer has the diameter $D = 100$ mm. The angular speed of the shaft is 1200 rpm and the radial load is 20 kN. The lubricant is SAE 30, controlled to an average temperature of 65^0C. Find the bearing length, the radial clearance, and the corresponding values of the minimum oil film thickness, the coefficient of friction, the friction power loss, and the oil flows.

5.7 Programs

Program 5.1

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Table 5.2. Representative unit sleeve bearing loads in current practice.

Application	Units load $P = \dfrac{W_{\max}}{LD}$	
Relatively steady load		
Electric motors	0.8-1.5 MPa	120-250 psi
Steam turbines	1.0-2.0 MPa	150-300 psi
Gear reducers	0.8-1.5 MPa	120-250 psi
Centrif. pumps	0.6-1.2 MPa	100-180 psi
Rapidly fluctuating loads		
Diesel engines		
Main bearings	6-12 MPa	900-1700 psi
Connecting rod bearings	8-15 MPa	1150-2300 psi
Automotive gasoline engines		
Main bearings	4-5 MPa	600-750 psi
Connecting rod bearings	10-15 MPa	1700-2300 psi

Source: R.C. Juvinall and K.M. Marshek, *Fundamentals of machine component design*, John Wiley & Sons, New York, 1991.

Table 5.3. Numerical values for Example 4.

c mm	S	h_0 mm	f	Q $\frac{\text{mm}^3}{\text{s}}$	Q_s $\frac{\text{mm}^3}{\text{s}}$
0.0300	1.7453	0.0231	0.0147	26730	9620
0.0400	0.9817	0.0256	0.0111	39980	19990
0.0500	0.6283	0.0270	0.0093	52980	31790
0.0670	0.3500	0.0288	0.0075	77450	54220
0.0900	0.1939	0.0288	0.0068	110540	88430
0.1100	0.1298	0.0275	0.0062	140400	117940
0.1300	0.0929	0.0260	0.0060	169060	145390
0.1500	0.0698	0.0255	0.0055	198680	176830
0.1800	0.0485	0.0234	0.0050	242750	220910
0.2000	0.0370	0.0220	0.0047	272140	253090
0.2200	0.0325	0.0220	0.0047	300940	281380
0.2300	0.0297	0.0218	0.0046	315730	296780

Figure captions

Fig. 5.1. Slider bearing.
Fig. 5.2. Saybolt universal viscometer.
Fig. 5.3. Absolute viscosity (mPa·s) function of temperature (°C).
Fig. 5.4. Absolute viscosity (μreyn) function of temperature (°F).
Fig. 5.5. Absolute viscosity function of temperature for multigrade oils.
Fig. 5.6. Hydrodynamic bearing.
Fig. 5.7. Pressure and forces on an element of lubricant film.
Fig. 5.8. Velocity distribution.
Fig. 5.9. Minimum film thickness variable.
Fig. 5.10. Position angle of minimum film thickness.
Fig. 5.11. Friction variable.
Fig. 5.12. Flow variable.
Fig. 5.13. Flow ratio (side leakage flow / total flow).
Fig. 5.14. Pressure ratio.
Fig. 5.15. Terminating position of the oil film and position of the maximum pressure.
Fig. 5.16. Notation for Raimondi and Boyd charts.
Fig. 5.17. Journal bearing for Example 5.2.
Fig. 5.18. Journal bearing for Example 5.3.
Fig. 5.19. Journal bearing for Example 5.5.
Fig. 5.20. Variation of h_o and f function of c for Example 5.5.