

6 Force Analysis

Homework 9: Joint reaction forces and equilibrium moment

Problem 6.1: four-bar mechanism

The four-bar mechanism shown in Fig. P6.1 has the dimensions: $AB=CD=0.04$ m and $AD=BC=0.09$ m. The driver link AB rotates with a constant angular speed of 120 rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m³ and the gravitational acceleration is $g = 9.807$ m/s². The external moment applied on the link CD is opposed to the motion of the link and has the value $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}| \frac{\boldsymbol{\omega}_3}{|\boldsymbol{\omega}_3|}$, where $|\mathbf{M}_{ext}| = 500$ N·m. Find the equilibrium moment on link AB and the joint forces for $\phi = 30^\circ$ using: a) free-body diagram of individual links and b) dyad method.

For $\phi=30^\circ$ the kinematics of the mechanism are given by:

$$\begin{aligned}
 x_B &= 0.034641 \text{ m}, & y_B &= 0.02 \text{ m}, \\
 x_C &= 0.103859 \text{ m}, & y_C &= -0.0375222 \text{ m}, \\
 \phi_2 &= -39.7274^\circ, & \phi_3 &= -69.7274^\circ, \\
 \dot{x}_B &= -0.251327 \text{ m/s}, & \dot{y}_B &= 0.435312 \text{ m/s}, \\
 \ddot{x}_B &= -5.47029 \text{ m/s}^2, & \ddot{y}_B &= -3.15827 \text{ m/s}^2, \\
 \dot{x}_C &= -0.884619 \text{ m/s}, & \dot{y}_C &= -0.32675 \text{ m/s}, \\
 \ddot{x}_C &= 3.84741 \text{ m/s}^2, & \ddot{y}_C &= 25.1222 \text{ m/s}^2, \\
 \omega_2 &= \dot{\phi}_2 = -11.0095 \text{ rad/s}, & \alpha_2 &= \ddot{\phi}_2 = 307.84 \text{ rad/s}^2, \\
 \omega_3 &= \dot{\phi}_3 = -23.5759 \text{ rad/s}, & \alpha_3 &= \ddot{\phi}_3 = 307.84 \text{ rad/s}^2.
 \end{aligned}$$

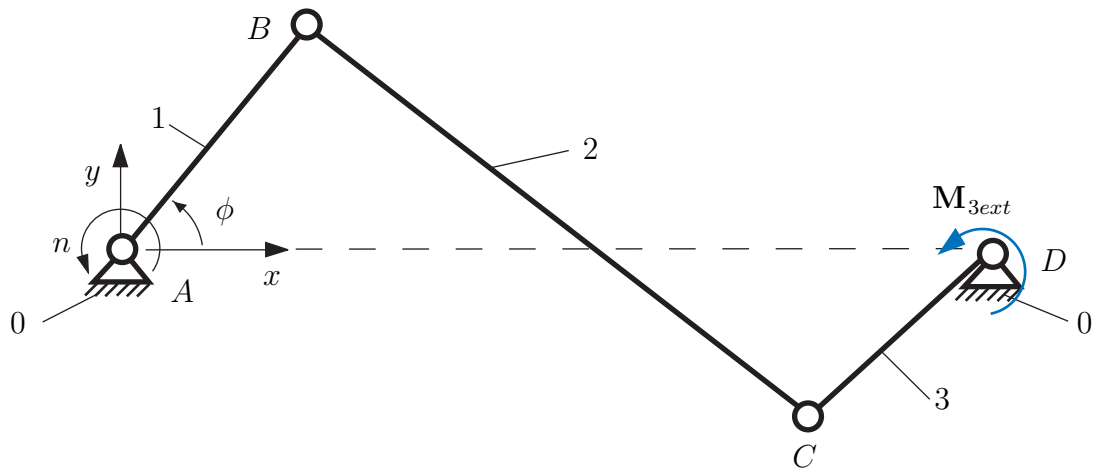


Figure P6.1

Problem 6.2: R-RRR-RRT mechanism

A planar mechanism is shown in Fig. P6.2. The following data are given: $AB=0.150$ m, $BC=0.400$ m, $CD=0.370$ m, $CE=0.230$ m, $EF=CE$, $L_a=0.300$ m, $L_b=0.450$ m, and $L_c=CD$. The constant angular speed of the driver link 1 is 60 rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The steel slider 5 has the width $w_{Slider} = 0.05$ m, the height $h_{Slider} = 0.02$ m, and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m³ and the gravitational acceleration is $g = 9.807$ m/s².

The external force on 5 is opposed to the motion of the link

$$\mathbf{F}_{ext} = \mathbf{F}_{5ext} = -|\mathbf{F}_{ext}| \frac{\mathbf{v}_F}{|\mathbf{v}_F|}, \text{ where } |\mathbf{F}_{ext}| = 500 \text{ N.}$$

Find the equilibrium moment on link AB and the joint reaction forces for $\phi = \phi_1 = 30^\circ$.

For $\phi = 30^\circ$ the kinematics of the mechanism are given by:

$$\begin{aligned} x_B &= 0.129904 \text{ m}, & y_B &= 0.075 \text{ m}, \\ x_C &= -0.0689445 \text{ m}, & y_C &= 0.422073 \text{ m}, \\ x_E &= -0.298288 \text{ m}, & y_E &= 0.404712 \text{ m}, \\ x_F &= -0.37 \text{ m}, & y_F &= 0.186177 \text{ m}, \\ \phi_2 &= -1.05052 \text{ rad}, & \phi_3 &= 0.0755515 \text{ rad}, & \phi_4 &= 1.25372 \text{ rad}. \\ \dot{x}_B &= -0.471239 \text{ m/s}, & \dot{y}_B &= 0.81621 \text{ m/s}, \\ \ddot{x}_B &= -5.1284 \text{ m/s}^2, & \ddot{y}_B &= -2.96088 \text{ m/s}^2, \\ \dot{x}_C &= -0.0788027 \text{ m/s}, & \dot{y}_C &= 1.04105 \text{ m/s}, \\ \ddot{x}_C &= 2.87595 \text{ m/s}^2, & \ddot{y}_C &= 1.03567 \text{ m/s}^2, \\ \dot{x}_E &= -0.127788 \text{ m/s}, & \dot{y}_E &= 1.68819 \text{ m/s}, \\ \ddot{x}_E &= 4.66371 \text{ m/s}^2, & \ddot{y}_E &= 1.67947 \text{ m/s}^2, \\ \dot{x}_F &= 0 \text{ m/s}, & \dot{y}_F &= 1.64625 \text{ m/s}, \\ \ddot{x}_F &= 0 \text{ m/s}^2, & \ddot{y}_F &= 3.29262 \text{ m/s}^2, \\ \omega_2 &= \dot{\phi}_2 = -1.1307 \text{ rad/s}, & \alpha_2 &= \ddot{\phi}_2 = -22.33 \text{ rad/s}^2, \\ \omega_3 &= \dot{\phi}_3 = -2.82169 \text{ rad/s}, & \alpha_3 &= \ddot{\phi}_3 = -2.20443 \text{ rad/s}^2, \\ \omega_4 &= \dot{\phi}_4 = 0.58475 \text{ rad/s}, & \alpha_4 &= \ddot{\phi}_4 = -21.453 \text{ rad/s}^2. \end{aligned}$$

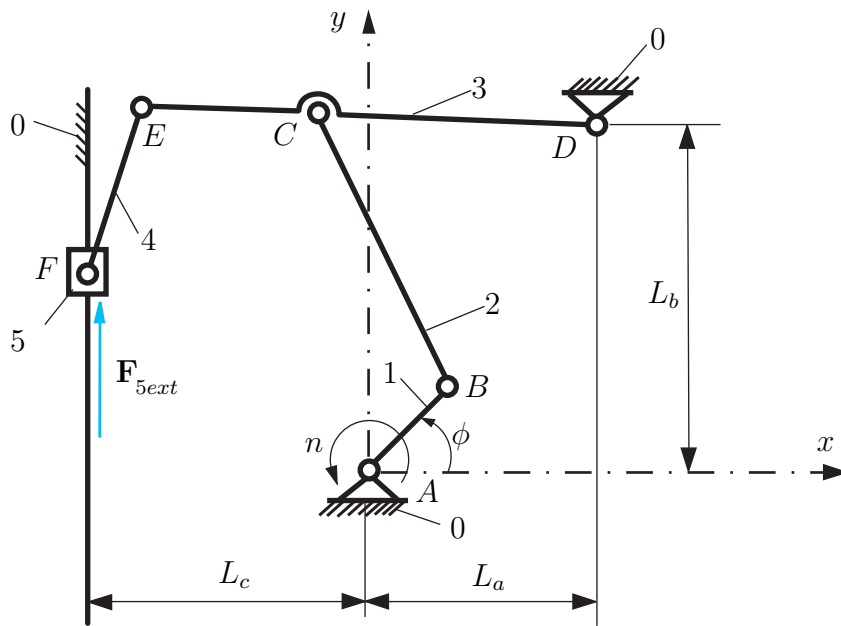


Figure P6.2