

6 Force Analysis

Homeworks: Joint reaction forces and equilibrium moment

Problem 6.1: four-bar mechanism

The four-bar mechanism shown in Fig. P6.1 has the dimensions: $AB=CD=0.04$ m and $AD=BC=0.09$ m. The driver link AB rotates with a constant angular speed of 120 rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m³ and the gravitational acceleration is $g = 9.807$ m/s². The external moment applied on the link CD is opposed to the motion of the link and has the value $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}| \frac{\boldsymbol{\omega}_3}{|\boldsymbol{\omega}_3|}$, where $|\mathbf{M}_{ext}| = 500$ N·m. Find the equilibrium moment on link AB and the joint forces for $\phi = 30^\circ$ using: a) free-body diagram of individual links and b) dyad method.

For $\phi=30^\circ$ the kinematics of the mechanism are given by:

$$\begin{aligned}
 x_B &= 0.034641 \text{ m}, & y_B &= 0.02 \text{ m}, \\
 x_C &= 0.103859 \text{ m}, & y_C &= -0.0375222 \text{ m}, \\
 \phi_2 &= -39.7274^\circ, & \phi_3 &= -69.7274^\circ, \\
 \dot{x}_B &= -0.251327 \text{ m/s}, & \dot{y}_B &= 0.435312 \text{ m/s}, \\
 \ddot{x}_B &= -5.47029 \text{ m/s}^2, & \ddot{y}_B &= -3.15827 \text{ m/s}^2, \\
 \dot{x}_C &= -0.884619 \text{ m/s}, & \dot{y}_C &= -0.32675 \text{ m/s}, \\
 \ddot{x}_C &= 3.84741 \text{ m/s}^2, & \ddot{y}_C &= 25.1222 \text{ m/s}^2, \\
 \omega_2 &= \dot{\phi}_2 = -11.0095 \text{ rad/s}, & \alpha_2 &= \ddot{\phi}_2 = 307.84 \text{ rad/s}^2, \\
 \omega_3 &= \dot{\phi}_3 = -23.5759 \text{ rad/s}, & \alpha_3 &= \ddot{\phi}_3 = 307.84 \text{ rad/s}^2.
 \end{aligned}$$

Problem 6.2: R-RRR-RRT mechanism

A planar mechanism is shown in Fig. P6.2. The following data are given: $AB=0.150$ m, $BC=0.400$ m, $CD=0.370$ m, $CE=0.230$ m, $EF=CE$, $L_a=0.300$ m, $L_b=0.450$ m, and $L_c=CD$. The constant angular speed of the driver link 1 is 60 rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The steel slider 5 has the width $w_{Slider} = 0.05$ m, the height $h_{Slider} = 0.02$ m, and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m³ and the gravitational acceleration is $g = 9.807$ m/s².

The external force on 5 is opposed to the motion of the link

$$\mathbf{F}_{ext} = \mathbf{F}_{5ext} = -|\mathbf{F}_{ext}| \frac{\mathbf{v}_F}{|\mathbf{v}_F|}, \text{ where } |\mathbf{F}_{ext}| = 500 \text{ N.}$$

Find the equilibrium moment on link AB and the joint reaction forces for $\phi = \phi_1 = 30^\circ$.

For $\phi = 30^\circ$ the kinematics of the mechanism are given by:

$$\begin{aligned} x_B &= 0.129904 \text{ m}, & y_B &= 0.075 \text{ m}, \\ x_C &= -0.0689445 \text{ m}, & y_C &= 0.422073 \text{ m}, \\ x_E &= -0.298288 \text{ m}, & y_E &= 0.404712 \text{ m}, \\ x_F &= -0.37 \text{ m}, & y_F &= 0.186177 \text{ m}, \\ \phi_2 &= -1.05052 \text{ rad}, & \phi_3 &= 0.0755515 \text{ rad}, & \phi_4 &= 1.25372 \text{ rad}. \\ \dot{x}_B &= -0.471239 \text{ m/s}, & \dot{y}_B &= 0.81621 \text{ m/s}, \\ \ddot{x}_B &= -5.1284 \text{ m/s}^2, & \ddot{y}_B &= -2.96088 \text{ m/s}^2, \\ \dot{x}_C &= -0.0788027 \text{ m/s}, & \dot{y}_C &= 1.04105 \text{ m/s}, \\ \ddot{x}_C &= 2.87595 \text{ m/s}^2, & \ddot{y}_C &= 1.03567 \text{ m/s}^2, \\ \dot{x}_E &= -0.127788 \text{ m/s}, & \dot{y}_E &= 1.68819 \text{ m/s}, \\ \ddot{x}_E &= 4.66371 \text{ m/s}^2, & \ddot{y}_E &= 1.67947 \text{ m/s}^2, \\ \dot{x}_F &= 0 \text{ m/s}, & \dot{y}_F &= 1.64625 \text{ m/s}, \\ \ddot{x}_F &= 0 \text{ m/s}^2, & \ddot{y}_F &= 3.29262 \text{ m/s}^2, \\ \omega_2 &= \dot{\phi}_2 = -1.1307 \text{ rad/s}, & \alpha_2 &= \ddot{\phi}_2 = -22.33 \text{ rad/s}^2, \\ \omega_3 &= \dot{\phi}_3 = -2.82169 \text{ rad/s}, & \alpha_3 &= \ddot{\phi}_3 = -2.20443 \text{ rad/s}^2, \\ \omega_4 &= \dot{\phi}_4 = 0.58475 \text{ rad/s}, & \alpha_4 &= \ddot{\phi}_4 = -21.453 \text{ rad/s}^2. \end{aligned}$$

Problem 6.3

The mechanism in Fig. P6.3 has the dimensions: $AB=200$ mm, $AC=600$ mm, $BD=1000$ mm, $L_a=150$ mm, $L_b=250$ mm, and $EF=600$ mm. The driver link 1 rotates with a constant angular speed of $n=60$ rpm.

The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The steel sliders 3 and 4 have the width $w_{Slider} = 0.05$ m, the height $h_{Slider} = 0.02$ m, and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m³ and the gravitational acceleration is $g = 9.807$ m/s².

The external moment on 5 is opposed to the motion of the link $\mathbf{M}_{ext} = \mathbf{M}_{5ext} = -|\mathbf{M}_{ext}| \frac{\boldsymbol{\omega}_5}{|\boldsymbol{\omega}_5|}$, where $|\mathbf{M}_{ext}| = 900$ N·m.

Find the equilibrium moment on link AB and the joint reaction forces for $\phi=\phi_1=120^\circ$.

For $\phi=120^\circ$ the kinematics of the mechanism are given by:

$$\begin{aligned}
 x_B &= -0.1 \text{ m}, & y_B &= 0.173205 \text{ m}, \\
 x_C &= -0.6 \text{ m}, & y_C &= 0 \text{ m}, \\
 x_D &= -1.04491 \text{ m}, & y_D &= -0.154122 \text{ m}, \\
 \phi_2 &= 0.333473 \text{ rad}, & \phi_5 &= 0.940376 \text{ rad}, \\
 \dot{x}_{B_1} &= -1.08828 \text{ m/s}, & \dot{y}_{B_1} &= -0.628319 \text{ m/s}, \\
 \ddot{x}_{B_1} &= 3.94784 \text{ m/s}^2, & \ddot{y}_{B_1} &= -6.83786 \text{ m/s}^2, \\
 v_{C_2C_3} &= -1.23399 \text{ m/s}, & \omega_2 &= -0.448799 \text{ rad/s}, \\
 \dot{x}_{D_2} &= -1.23518 \text{ m/s}, & \dot{y}_{D_2} &= -0.204243 \text{ m/s}, \\
 a_{C_2C_3x}^{cor} &= -0.362557 \text{ m/s}^2, & a_{C_2C_3y}^{cor} &= 1.04661 \text{ m/s}^2, \\
 a_{C_2C_3} &= 1.59873 \text{ m/s}^2, & \alpha_3 &= -16.7458 \text{ rad/s}^2, \\
 \ddot{x}_{D_2} &= -1.34318 \text{ m/s}^2, & \ddot{y}_{D_2} &= 9.05135 \text{ m/s}^2, \\
 v_{D_5D_4} &= 0.893105 \text{ m/s}, & \omega_5 &= -1.75371 \text{ rad/s}, \\
 a_{D_5D_4x}^{cor} &= 2.53037 \text{ m/s}^2, & a_{D_5D_4y}^{cor} &= -1.84656 \text{ m/s}^2, \\
 a_{D_5D_4} &= -4.98108 \text{ m/s}^2, & \alpha_5 &= -6.57248 \text{ rad/s}^2.
 \end{aligned}$$

Problem 6.4

The mechanism in Fig. P6.4 has the dimensions: $AB=150$ mm, $AC=350$ mm, $BD=530$ mm, $DE=300$ mm, $EF=200$ mm, $L_a=55$ mm, and $L_b=125$ mm. The driver link 1 rotates with a constant angular speed of $n=30$ rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The slider 3 has the width $w_{Slider} = 0.05$ m, the height $h_{Slider} = 0.02$ m, and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m³ and the gravitational acceleration is $g = 9.807$ m/s².

The external moment on 5 is opposed to the motion of the link $\mathbf{M}_{ext} = \mathbf{M}_{5ext} = -|\mathbf{M}_{ext}| \frac{\boldsymbol{\omega}_5}{|\boldsymbol{\omega}_5|}$, where $|\mathbf{M}_{ext}| = 1500$ N·m.

Find the equilibrium moment on link AB and the joint reaction forces for $\phi=\phi_1=120^\circ$.

For $\phi=\phi_1=120^\circ$ the kinematics of the mechanism are given by:

$$\begin{aligned}
 x_B &= -0.075 \text{ m}, & y_B &= 0.129904 \text{ m}, \\
 x_D &= -0.554223 \text{ m}, & y_D &= -0.0964704 \text{ m}, \\
 x_E &= -0.482421 \text{ m}, & y_E &= 0.19481 \text{ m}, \\
 \phi_2 &= 0.441306 \text{ rad}, & \phi_4 &= 1.32911 \text{ rad}, & \phi_5 &= -0.356559 \text{ rad}, \\
 \dot{x}_B &= -0.408105 \text{ m/s}, & \dot{y}_B &= -0.235619 \text{ m/s}, \\
 \ddot{x}_B &= 0.74022 \text{ m/s}^2, & \ddot{y}_B &= -1.2821 \text{ m/s}^2, \\
 v_{C_3C_2x} &= 0.42465 \text{ m/s}, & v_{C_3C_2y} &= 0.200595 \text{ m/s}, & \omega_2 &= -0.127362 \text{ rad/s}, \\
 \omega_4 &= -1.16943 \text{ rad/s}, & \omega_5 &= 1.37953 \text{ rad/s}, \\
 \dot{x}_E &= -0.0963053 \text{ m/s}, & \dot{y}_E &= -0.258552 \text{ m/s}, \\
 a_{C_3C_2x}^{cor} &= 0.0510963 \text{ m/s}^2, & a_{C_3C_2y}^{cor} &= -0.108168 \text{ m/s}^2, \\
 \alpha_2 &= -5.24453 \text{ rad/s}^2, & a_{C_3C_2x} &= -0.114494 \text{ m/s}^2, & a_{C_3C_2y} &= -0.0540842 \text{ m/s}^2, \\
 \ddot{x}_D &= -0.439231 \text{ m/s}^2, & \ddot{y}_D &= 1.23487 \text{ m/s}^2, \\
 \alpha_4 &= -3.94679 \text{ rad/s}^2, & \alpha_5 &= -3.66019 \text{ rad/s}^2, \\
 \ddot{x}_E &= 0.612199 \text{ m/s}^2, & \ddot{y}_E &= 0.553139 \text{ m/s}^2.
 \end{aligned}$$

Problem 6.5

The dimensions of the mechanism shown in Fig. P6.5 are: $AB=200$ mm, $AC=300$ mm, $CD=500$ mm, $DE=250$ mm, $CF = AB+AC$, and $L_a=400$ mm. The constant angular speed of the driver link 1 is $n=40$ rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The sliders 2 and 5 have the width $w_{Slider} = 0.05$ m, the height $h_{Slider} = 0.02$ m, and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m³ and the gravitational acceleration is $g = 9.807$ m/s².

The external force on 5 is opposed to the motion of the link

$$\mathbf{F}_{ext} = \mathbf{F}_{5ext} = -|\mathbf{F}_{ext}| \frac{\mathbf{v}_E}{|\mathbf{v}_E|}, \text{ where } |\mathbf{F}_{ext}| = 1000 \text{ N.}$$

Find the equilibrium moment on link AB and the joint reaction forces for $\phi=\phi_1=60^\circ$.

Find the velocities and the accelerations of the mechanism when the angle of the driver link 1 with the horizontal axis is $\phi=60^\circ$.

For $\phi=60^\circ$ the kinematics of the mechanism are given by:

$$\begin{aligned} x_B &= 0.1 \text{ m}, & y_B &= 0.173205 \text{ m}, \\ x_D &= -0.758831 \text{ m}, & y_D &= -0.19868 \text{ m}, \\ x_E &= -0.7 \text{ m}, & y_E &= 0.0442993 \text{ m}, \\ \phi_2 &= 0.408638 \text{ rad}, & \phi_4 &= 1.33324 \text{ rad}, \\ \dot{x}_{B_1} &= -0.72552 \text{ m/s}, & \dot{y}_{B_1} &= 0.418879 \text{ m/s}, \\ \ddot{x}_{B_1} &= -1.7546 \text{ m/s}^2, & \ddot{y}_{B_1} &= -3.03905 \text{ m/s}^2, \\ \dot{x}_D &= 0.30661 \text{ m/s}, & \dot{y}_D &= -0.708086 \text{ m/s}, \\ \ddot{x}_D &= 0.841861 \text{ m/s}^2, & \ddot{y}_D &= 1.05257 \text{ m/s}^2, \\ \dot{x}_E &= 0 \text{ m/s}, & \dot{y}_E &= -0.633848 \text{ m/s}, \\ \ddot{x}_E &= 0 \text{ m/s}^2, & \ddot{y}_E &= 0.846817 \text{ m/s}^2, \\ \omega_2 = \dot{\phi}_2 &= 1.54324 \text{ rad/s}, & \alpha_2 = \ddot{\phi}_2 &= -1.26276 \text{ rad/s}^2, \\ \omega_4 = \dot{\phi}_3 &= 1.26188 \text{ rad/s}, & \alpha_4 = \ddot{\phi}_3 &= 3.0792 \text{ rad/s}^2. \end{aligned}$$

Problem 6.6

The dimensions of the mechanism shown in Fig. P6.7 are: $AB=180$ mm, $AC=90$ mm, $CD=200$ mm, $CF = CD$, and $AG = CD + AC$. The constant angular speed of the driver link 1 is $n=180$ rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The sliders 2 and 4 have the width $w_{slider} = 0.05$ m, the height $h_{slider} = 0.02$ m, and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m³ and the gravitational acceleration is $g = 9.807$ m/s².

The external moment on 5 is opposed to the motion of the link

$$\mathbf{M}_{ext} = \mathbf{M}_{5ext} = -|\mathbf{M}_{ext}| \frac{\boldsymbol{\omega}_5}{|\boldsymbol{\omega}_5|}, \text{ where } |\mathbf{M}_{ext}| = 1500 \text{ N}\cdot\text{m}.$$

Find the equilibrium moment on link AB and the joint reaction forces for $\phi=\phi_1=60^\circ$.

For $\phi=60^\circ$ the kinematics of the mechanism are given by:

$$\begin{aligned} x_B &= 0.09 \text{ m}, y_B = 0.155885 \text{ m}, \\ x_D &= -0.16138 \text{ m}, y_D = -0.0281381 \text{ m}, \\ \phi_2 &= 0.631914 \text{ rad}, \phi_4 = 0.172624 \text{ rad}, \\ \dot{x}_{B_1} &= -2.93835 \text{ m/s}, \dot{y}_{B_1} = 1.69646 \text{ m/s}, \\ \ddot{x}_{B_1} &= -31.9775 \text{ m/s}^2, \ddot{y}_{B_1} = -55.3867 \text{ m/s}^2, \\ \dot{x}_{D_3} &= 3.28823 \text{ m/s}, \dot{y}_{D_3} = -4.4918 \text{ m/s}, \\ \ddot{x}_{D_3} &= 178.405 \text{ m/s}^2, \ddot{y}_{D_3} = 18.6037 \text{ m/s}^2, \\ \omega_2 = \dot{\phi}_2 &= 27.8338 \text{ rad/s}, \alpha_2 = \ddot{\phi}_2 = 451.854 \text{ rad/s}^2, \\ \omega_4 = \dot{\phi}_3 &= 30.4604 \text{ rad/s}, \alpha_4 = \ddot{\phi}_3 = 992.942 \text{ rad/s}^2. \end{aligned}$$