Homework 8: Accelerations of CM, mass moments of inertia

Problem 6.1: four-bar mechanism

The four-bar mechanism shown in Fig. P6.1 has the dimensions: AB=CD=0.04 m and AD=BC=0.09 m. The driver link AB rotates with a constant angular speed of 120 rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2.

For ϕ=ϕ_1=30° find:

1. the accelerations of the mass centers of the links $\mathbf{a}_{C_i}$, $i = 1$, 2, 3;

2. the terms $m_i\,\mathbf{a}_{C_i}$ and $I_{C_i}\,\boldsymbol{\alpha}_i$, where m_i is the mass, I_{C_i} is the mass moment of inertia, and $\boldsymbol{\alpha}_i$ is the angular acceleration of the link $i = 1$, 2, 3.

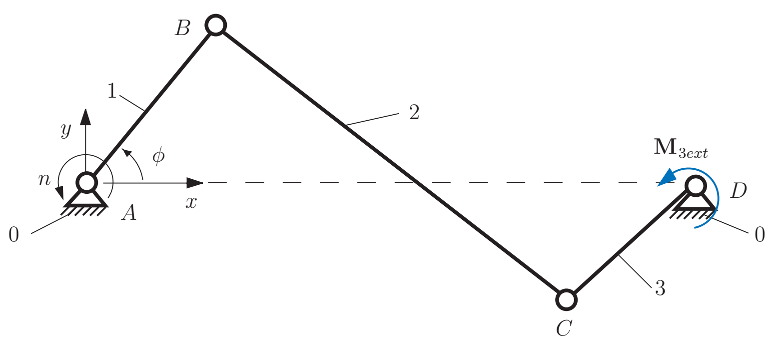

Figure P6.1: Mechanism P6.1

For ϕ=30° the kinematics of the mechanism are given by:
x_B=0.034641 m, y_B=0.02 m,
x_C=0.103859 m, y_C=-0.0375222 m,
ϕ_2=-39.7274°, ϕ_3=-69.7274°,
$\dot{x}_B$=-0.251327 m/s, $\dot{y}_B$=0.435312 m/s,
$\ddot{x}_B$=-5.47029 m/s^2, $\ddot{y}_B$=-3.15827 m/s^2,
$\dot{x}_C$=-0.884619 m/s, $\dot{y}_C$=-0.32675 m/s,
$\ddot{x}_C$=3.84741 m/s^2, $\ddot{y}_C$=25.1222 m/s^2,
ω_2=$\dot{\phi}_2$=-11.0095 rad/s, α_2=$\ddot{\phi}_2$=307.84 rad/s^2,
ω_3=$\dot{\phi}_3$=-23.5759 rad/s, α_3=$\ddot{\phi}_3$=307.84 rad/s^2.

Problem 6.2: R-RRR-RRT mechanism

A planar mechanism is shown in Fig. P6.2. The following data are given: AB=0.150 m, BC=0.400 m, CD=0.370 m, CE=0.230 m, EF=CE, L_a=0.300 m, L_b=0.450 m, and L_c=CD. The constant angular speed of the driver link 1 is 60 rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. The steel slider 5 has the width $w_{Slider} = 0.05$ m, the height $h_{Slider} = 0.02$ m, and the depth $d = 0.001$ m.

For ϕ=ϕ_1=30° find:

1. the accelerations of the mass centers of the links $\mathbf{a}_{C_i}$, $i = 1, \ldots 5$;

2. the terms $m_i\,\mathbf{a}_{C_i}$ and $I_{C_i}\,\boldsymbol{\alpha}_i$, where m_i is the mass, I_{C_i} is the mass moment of inertia, and $\boldsymbol{\alpha}_i$ is the angular acceleration of the link $i = 1, \ldots 5$.

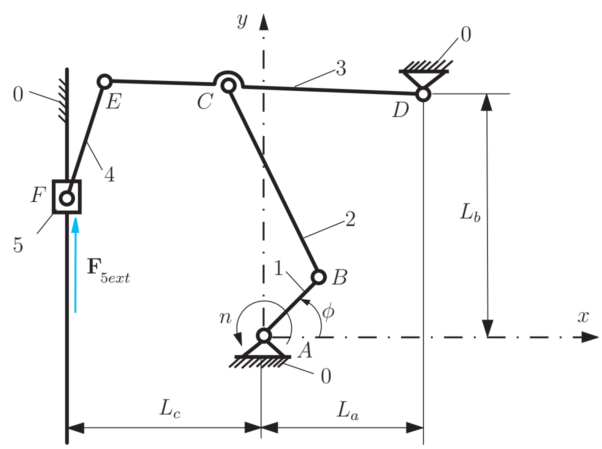

Figure P6.2: Mechanism P6.2

For ϕ=30° the kinematics of the mechanism are given by:
x_B=0.129904 m, y_B=0.075 m,
x_C=-0.0689445 m, y_C=0.422073 m,
x_E=-0.298288 m, y_E=0.404712 m,
x_F=-0.37 m, y_F=0.186177 m,
ϕ_2=-1.05052 rad, ϕ_3=0.0755515 rad, ϕ_4=1.25372 rad.
$\dot{x}_B$=-0.471239 m/s, $\dot{y}_B$=0.81621 m/s,
$\ddot{x}_B$=-5.1284 m/s^2, $\ddot{y}_B$=-2.96088 m/s^2,
$\dot{x}_C$=-0.0788027 m/s, $\dot{y}_C$=1.04105 m/s,

$\ddot{x}_C$=2.87595 m/s^2, $\ddot{y}_C$=1.03567 m/s^2,
$\dot{x}_E$=-0.127788 m/s, $\dot{y}_E$=1.68819 m/s,
$\ddot{x}_E$=4.66371 m/s^2, $\ddot{y}_E$=1.67947 m/s^2,
$\dot{x}_F$=0 m/s, $\dot{y}_F$=1.64625 m/s,
$\ddot{x}_F$=0 m/s^2, $\ddot{y}_F$=3.29262 m/s^2,
$\omega_2=\dot{\phi}_2$=-1.1307 rad/s, $\alpha_2=\ddot{\phi}_2$=-22.33 rad/s^2,
$\omega_3=\dot{\phi}_3$=-2.82169 rad/s, $\alpha_3=\ddot{\phi}_3$=-2.20443 rad/s^2.
$\omega_4=\dot{\phi}_3$=0.58475 rad/s, $\alpha_4=\ddot{\phi}_3$=-21.453 rad/s^2.

Problem 6.3

The mechanism in Fig. P6.3 has the dimensions: AB=200 mm, AC=600 mm, BD=1000 mm, L_a=150 mm, L_b=250 mm, and EF=600 mm. The driver link 1 rotates with a constant angular speed of n=60 rpm.

The links are homogeneous rectangular prisms made of steel with the width $h = 0.01$ m and the depth $d = 0.001$ m. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. The steel sliders 3 and 4 have the width $w_{Slider} = 0.05$ m, the height $h_{Slider} = 0.02$ m, and the depth $d = 0.001$ m.

For ϕ=ϕ_1=120° find:

1. the accelerations of the mass centers of the links $\mathbf{a}_{C_i}$, $i = 1, \dots 5$;

2. the terms $m_i\, \mathbf{a}_{C_i}$ and $I_{C_i}\, \boldsymbol{\alpha}_i$, where m_i is the mass, I_{C_i} is the mass moment of inertia, and $\boldsymbol{\alpha}_i$ is the angular acceleration of the link $i = 1, \dots 5$.

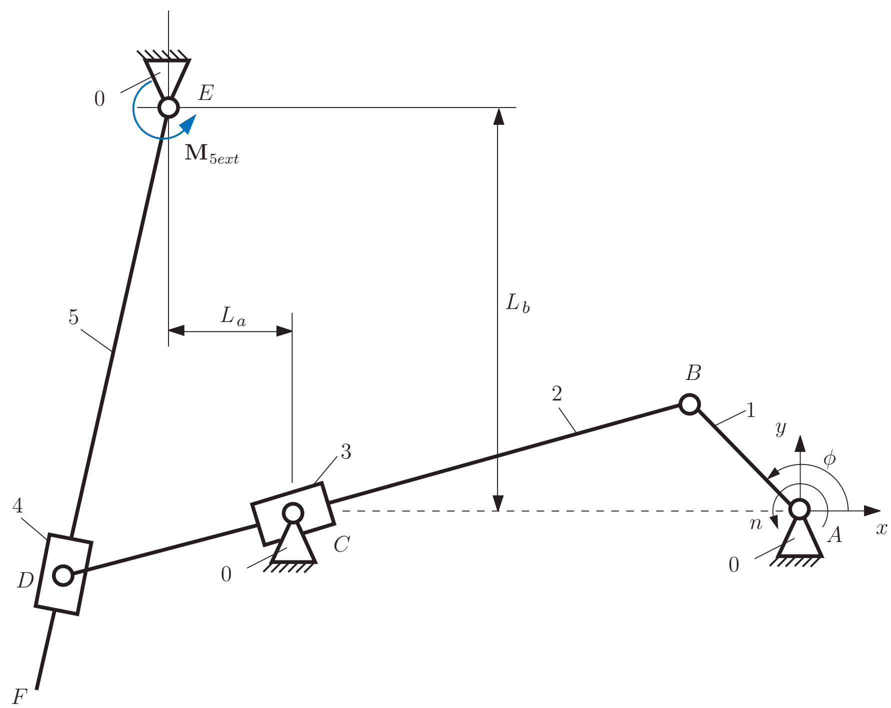

Figure P6.3: Mechanism P6.3

For ϕ=120° the kinematics of the mechanism are given by:
x_B=-0.1 m, y_B=0.173205 m,
x_C=-0.6 m, y_C=0 m,
x_D=-1.04491 m, y_D=-0.154122 m,
ϕ_2=0.333473 rad, ϕ_5=0.940376 rad,
$\dot{x}_{B_1}$=-1.08828 m/s, $\dot{y}_{B_1}$=-0.628319 m/s,
$\ddot{x}_{B_1}$=3.94784 m/s^2, $\ddot{y}_{B_1}$=-6.83786 m/s^2,
$v_{C_2C_3}$=-1.23399 m/s, ω_2=-0.448799 rad/s,
$\dot{x}_{D_2}$=-1.23518 m/s, $\dot{y}_{D_2}$=-0.204243 m/s,
$a^{cor}_{C_2C_3x}$=-0.362557 m/s^2, $a^{cor}_{C_2C_3y}$=1.04661 m/s^2,
$a_{C_2C_3}$=1.59873 m/s^2, α_3=-16.7458 rad/s^2,
$\ddot{x}_{D_2}$=-1.34318 m/s^2, $\ddot{y}_{D_2}$=9.05135 m/s^2,
$v_{D_5D_4}$=0.893105 m/s, ω_5=-1.75371 rad/s,
$a^{cor}_{D_5D_4x}$=2.53037 m/s^2, $a^{cor}_{D_5D_4y}$=-1.84656 m/s^2,
$a_{D_5D_4}$=-4.98108m/s^2, α_5=-6.57248 rad/s^2.