4 Homework: Velocity and Acceleration Analysis

Motion of a point \( A \) that moves relative to a rigid body

A point \( A \) is not assumed to be a point of the rigid body, \( A \notin (RB) \).

Show the mathematical proof that the acceleration of the point \( A \) relative to the primary reference frame \((x_0y_0z_0)\) is

\[
a_A = a_O + a_{rel}^{A(xyz)} + 2 \omega \times v_{rel}^{A(xyz)} + \alpha \times r + \omega \times (\omega \times r),
\]

where

\((xyz)\) is a body fixed (mobile or rotating) reference frame with its origin at a point \( O \) of the rigid body \((O \in (RB))\), and is a moving reference frame relative to the primary reference;

\( a_O \) is the acceleration of \( O \) relative to the primary reference;

\( r = r_{OA} = x\hat{i} + y\hat{j} + z\hat{k} \) is the position vector of \( A \) relative to the origin \( O \), of the body fixed reference frame, and \( x, y, \) and \( z \) are the coordinates of \( A \) in terms of the body fixed reference frame.

\[
v_{rel}^{A(xyz)} = \frac{(xyz)d}{dt}r = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}, \text{ is the velocity of } A \text{ relative to the body fixed reference frame or relative to the rigid body;}
\]

\[
a_{rel}^{A(xyz)} = \frac{(xyz)d^2}{dt^2}r = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}, \text{ is the acceleration of } A \text{ relative to the body fixed reference frame or relative to the rigid body;}
\]

\( \omega \) is the angular velocity vector of the rigid body;

\( \alpha \) is the angular acceleration vector of the rigid body;

\[
a_{cor}^{A(xyz)} = 2 \omega \times v_{rel}^{A(xyz)} \text{ is the Coriolis acceleration.}
\]