

4 Homework 7: Velocity and Acceleration Analysis

Problem 4.7

The dimensions of the mechanism shown in Fig. P4.7 are: $AB=200$ mm, $AC=300$ mm, $CD=500$ mm, $DE=250$ mm, and $L_a=400$ mm. The constant angular speed of the driver link 1 is $n=40$ rpm. Find the velocities and the accelerations of the mechanism when the angle of the driver link 1 with the horizontal axis is $\phi=60^\circ$. For $\phi=60^\circ$ the position of the mechanism is given by: $x_B=0.1$ m, $y_B=0.173205$ m, $x_D=-0.758831$ m, $y_D=-0.19868$ m, $x_E=-0.7$ m, $y_E=0.0442993$ m, $\phi_2=0.408638$ rad, $\phi_4=1.33324$ rad.

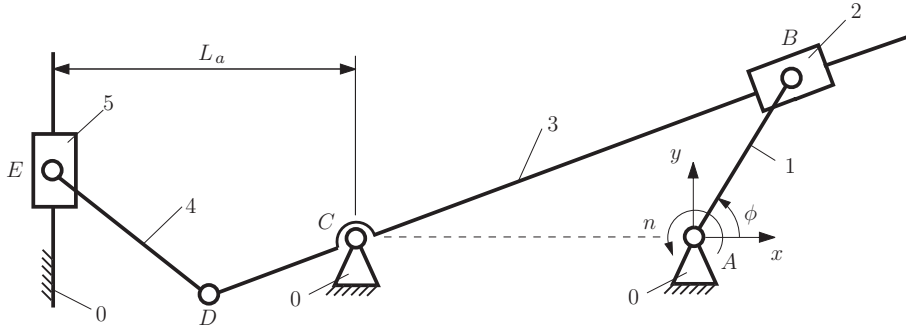


Figure P4.7: Mechanism P4.7

Results

$$\begin{aligned} \dot{x}_{B_1} &= -0.72552 \text{ m/s}, \dot{y}_{B_1} = 0.418879 \text{ m/s}, \ddot{x}_{B_1} = -1.7546 \text{ m/s}^2, \ddot{y}_{B_1} = -3.03905 \text{ m/s}^2, \\ \dot{x}_D &= 0.30661 \text{ m/s}, \dot{y}_D = -0.708086 \text{ m/s}, \ddot{x}_D = 0.841861 \text{ m/s}^2, \ddot{y}_D = 1.05257 \text{ m/s}^2, \\ \dot{x}_E &= 0 \text{ m/s}, \dot{y}_E = -0.633848 \text{ m/s}, \ddot{x}_E = 0 \text{ m/s}^2, \ddot{y}_E = 0.846817 \text{ m/s}^2, \\ \omega_2 &= \dot{\phi}_2 = 1.54324 \text{ rad/s}, \alpha_2 = \dot{\omega}_2 = -1.26276 \text{ rad/s}^2, \\ \omega_4 &= \dot{\phi}_4 = 1.26188 \text{ rad/s}, \alpha_4 = \dot{\omega}_4 = 3.0792 \text{ rad/s}^2. \end{aligned}$$

Results

$$rA = [0, 0, 0] \text{ (m)}$$

$$rB = [0.1, 0.173, 0] \text{ (m)}$$

$$rC = [-0.3, 0, 0] \text{ (m)}$$

$$rD = [-0.759, -0.199, 0] \text{ (m)}$$

$$rE = [-0.7, 0.0443, 0] \text{ (m)}$$

$$\text{phi3} = 23.4 \text{ (degrees)}$$

$$\text{phi4} = 76.4 \text{ (degrees)}$$

Velocity and acceleration analysis

$$\text{omega1} = [0, 0, 4.19] \text{ (rad/s)}$$

$$\text{alpha1} = [0, 0, 0] \text{ (rad/s}^2\text{)}$$

$$vB = vB1 = vB2 = [-0.726, 0.419, 0] \text{ (m/s)}$$

$$aB = aB1 = aB2 = [-1.75, -3.04, 0] \text{ (m/s}^2\text{)}$$

$$vB3 = vC + \text{omega3} \times rCB = vB2 - vB3B2 \Rightarrow$$

$$\text{x-axis: } 0.726 - 0.918 \cdot vB32 - 0.173 \cdot \text{omega3z} = 0$$

$$\text{y-axis: } 0.4 \cdot \text{omega3z} - 0.397 \cdot vB32 - 0.419 = 0$$

\Rightarrow

$$\text{omega3z} = 1.54 \text{ (rad/s)}$$

$$vB32 = 0.499 \text{ (m/s)}$$

$$\text{omega2} = \text{omega3} = [0, 0, 1.54] \text{ (rad/s)}$$

$$vB3B2 = [0.458, 0.198, 0] \text{ (m/s)}$$

$$aB32\text{cor} = [-0.612, 1.41, 0] \text{ (m/s}^2\text{)}$$

$$aB3 = aC + \text{omega3} \times rCB - (\text{omega3} \cdot \text{omega3}) rCB = aB2 + aB3B2 + aB3B2\text{cor} \Rightarrow$$

$$\text{x-axis: } 1.41 - 0.173 \cdot \text{alpha3z} - 0.918 \cdot aB32 = 0$$

$$\text{y-axis: } 0.4 \cdot \text{alpha3z} - 0.397 \cdot aB32 + 1.21 = 0$$

\Rightarrow

$$\text{alpha3z} = -1.26 \text{ (rad/s}^2\text{)}$$

$$aB32 = 1.78 \text{ (m/s}^2\text{)}$$

$$\text{alpha2} = \text{alpha3} = [0, 0, -1.26] \text{ (rad/s}^2\text{)}$$

$$a_{B3B2} = [1.63, 0.707, 0] \text{ (m/s}^2\text{)}$$

$$v_D = [0.307, -0.708, 0] \text{ (m/s)}$$

$$a_D = [0.842, 1.05, 0] \text{ (m/s}^2\text{)}$$

$$v_{E5} = v_{E4} = v_D + \text{cross}(\omega_4, r_E - r_D) \Rightarrow$$

$$\text{x-axis: } 0.243 \cdot \omega_4 - 0.307 = 0$$

$$\text{y-axis: } v_{Ey} - 0.0588 \cdot \omega_4 + 0.708 = 0$$

\Rightarrow

$$\omega_4 = 1.26 \text{ (rad/s)}$$

$$v_{Ey} = -0.634 \text{ (m/s)}$$

$$\omega_4 = [0, 0, 1.26] \text{ (rad/s)}$$

$$v_E = [0, -0.634, 0] \text{ (m/s)}$$

$$a_{E5} = a_{E4} = a_D + \omega_4 \times r_{DE} - (\omega_4 \cdot \omega_4) r_{DE} \Rightarrow$$

$$\text{x-axis: } 0.748 - 0.243 \cdot \alpha_4 = 0$$

$$\text{y-axis: } 0.0588 \cdot \alpha_4 - 1.0 \cdot a_{Ey} + 0.666 = 0$$

\Rightarrow

$$\alpha_4 = 3.08 \text{ (rad/s}^2\text{)}$$

$$a_E = 0.847 \text{ (m/s}^2\text{)}$$

$$\alpha_4 = [0, 0, 3.08] \text{ (rad/s}^2\text{)}$$

$$a_E = [0, 0.847, 0] \text{ (m/s}^2\text{)}$$

Problem 4.8

The dimensions of the mechanism shown in Fig. P4.8 are: $AB=180$ mm, $AC=90$ mm, and $CD=200$ mm. The constant angular speed of the driver link 1 is $n=180$ rpm. Find the velocities and the accelerations of the mechanism for $\phi=\phi_1=60^\circ$. For $\phi=60^\circ$ the position of the mechanism is given by: $x_B=0.09$ m, $y_B=0.155885$ m, $x_D=-0.16138$ m, $y_D=-0.0281381$ m, $\phi_2=0.631914$ rad= 36.206° , $\phi_4=0.172624$ rad= 9.89064° .

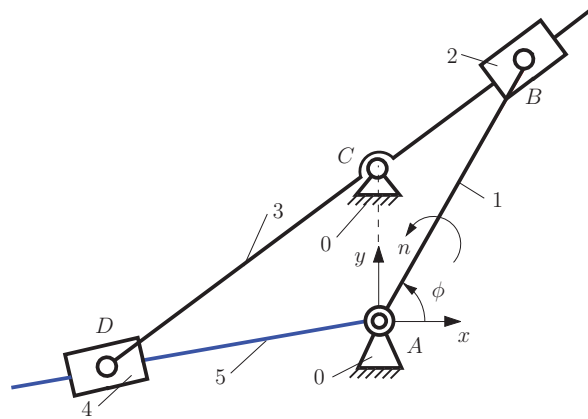


Figure P4.8: Mechanism P4.8

Results

$$\begin{aligned} \dot{x}_{B_1} &= -2.93835 \text{ m/s}, \quad \dot{y}_{B_1} = 1.69646 \text{ m/s}, \quad \ddot{x}_{B_1} = -31.9775 \text{ m/s}^2, \quad \ddot{y}_{B_1} = -55.3867 \text{ m/s}^2, \\ \dot{x}_{D_3} &= 3.28823 \text{ m/s}, \quad \dot{y}_{D_3} = -4.4918 \text{ m/s}, \quad \ddot{x}_{D_3} = 178.405 \text{ m/s}^2, \quad \ddot{y}_{D_3} = 18.6037 \text{ m/s}^2, \\ \omega_2 &= \dot{\phi}_2 = 27.8338 \text{ rad/s}, \quad \alpha_2 = \ddot{\phi}_2 = 451.854 \text{ rad/s}^2, \\ \omega_4 &= \dot{\phi}_4 = 30.4604 \text{ rad/s}, \quad \alpha_4 = \ddot{\phi}_4 = 992.942 \text{ rad/s}^2. \end{aligned}$$