

## 4 Homework 6: Velocity and Acceleration Analysis

### Problem 4.5

The mechanism in Fig. P4.5 has the dimensions:  $AB=200$  mm,  $AC=600$  mm,  $BD=1000$  mm,  $L_a=150$  mm, and  $L_b=250$  mm. The driver link 1 rotates with a constant angular speed of  $n=60$  rpm. Find the velocities and the accelerations of the mechanism for  $\phi=\phi_1=120^\circ$ . For  $\phi=120^\circ$  the position of the mechanism is given by:  $x_B=-0.1$  m,  $y_B=0.173205$  m,  $x_C=-0.6$  m,  $y_C=0$  m,  $x_D=-1.04491$  m,  $y_D=-0.154122$  m,  $\phi_2=0.333473$  rad,  $\phi_5=0.940376$  rad.

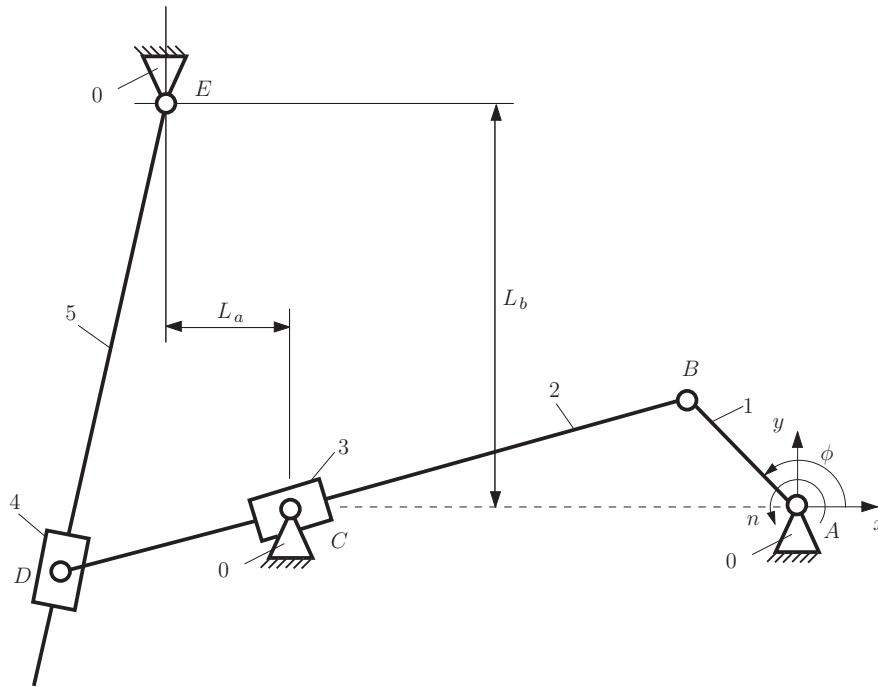


Figure P4.5: Mechanism P4.5

## Results

$\dot{x}_{B_1} = -1.08828$  m/s,  $\dot{y}_{B_1} = -0.628319$  m/s,  $\ddot{x}_{B_1} = 3.94784$  m/s<sup>2</sup>,  $\ddot{y}_{B_1} = -6.83786$  m/s<sup>2</sup>,  
 $v_{C_2C_3} = -1.23399$  m/s,  $\omega_2 = -0.448799$  rad/s,  $\dot{x}_{D_2} = -1.23518$  m/s,  $\dot{y}_{D_2} = -0.204243$  m/s,  
 $a_{C_2C_3x}^{cor} = -0.362557$  m/s<sup>2</sup>,  $a_{C_2C_3y}^{cor} = 1.04661$  m/s<sup>2</sup>,  $a_{C_2C_3} = 1.59873$  m/s<sup>2</sup>,  $\alpha_3 = -$   
 $16.7458$  rad/s<sup>2</sup>,  $\ddot{x}_{D_2} = -1.34318$  m/s<sup>2</sup>,  $\ddot{y}_{D_2} = 9.05135$  m/s<sup>2</sup>,  $v_{D_5D_4} = 0.893105$  m/s,  
 $\omega_5 = -1.75371$  rad/s,  $a_{D_5D_4x}^{cor} = 2.53037$  m/s<sup>2</sup>,  $a_{D_5D_4y}^{cor} = -1.84656$  m/s<sup>2</sup>,  $a_{D_5D_4} = -$   
 $4.98108$  m/s<sup>2</sup>,  $\alpha_5 = -6.57248$  rad/s<sup>2</sup>.

rA = [0, 0, 0] (m)

rB = [ -0.1, 0.173, 0] (m)

rC = [ -0.6, 0, 0] (m)

rD = [ -1.04, -0.154, 0] (m)

rE = [ -0.75, 0.25, 0] (m)

phi2 = 120 (degrees)

phi2 = phi3 = 19.1 (degrees)

phi4 = phi5 = 53.9 (degrees)

omega1 = [0, 0, 6.28] (rad/s)

alpha1 = [0, 0, 0] (rad/s<sup>2</sup>)

vB = vB1 = vB2 = [ -1.09, -0.628, 0] (m/s)

aB = aB1 = aB2 = [ 3.95, -6.84, 0] (m/s<sup>2</sup>)

omega2z = -0.449 (rad/s)

vC23 = -1.23 (m/s)

omega3 = omega2 = [0, 0, -0.449] (rad/s)

vC2C3 = [ -1.17, -0.404, 0] (m/s)

alpha2z = -16.7 (rad/s)

aC23 = 1.6 (m/s<sup>2</sup>)

alpha2 = alpha3 = [0, 0, -16.7] (rad/s<sup>2</sup>)

aC2C3 = [ 1.51, 0.523, 0] (m/s<sup>2</sup>)

vD2 = vD4 = [ -1.24, -0.204, 0] (m/s)

$$\omega_{5z} = -1.75 \text{ (rad/s)}$$

$$v_{D54} = 0.893 \text{ (m/s)}$$

$$\omega_4 = \omega_5 = [0, 0, -1.75] \text{ (rad/s)}$$

$$v_{D5D4} = [0.526, 0.721, 0] \text{ (m/s)}$$

$$a_{D54cor} = [2.53, -1.85, 0] \text{ (m/s}^2\text{)}$$

$$\alpha_{5z} = -6.57 \text{ (rad/s}^2\text{)}$$

$$a_{D54} = -4.98 \text{ (m/s}^2\text{)}$$

$$\alpha_4 = \alpha_5 = [0, 0, -6.57] \text{ (rad/s}^2\text{)}$$

$$a_{D5D4} = [-2.94, -4.02, -0] \text{ (m/s}^2\text{)}$$

**Problem 4.6**

The mechanism in Fig. P4.6 has the dimensions:  $AB=150$  mm,  $AC=350$  mm,  $BD=530$  mm,  $DE=300$  mm,  $EF=200$  mm,  $L_a=55$  mm, and  $L_b=125$  mm. The constant angular speed of the driver link 1 is  $n=30$  rpm. Find the velocities and the accelerations of the mechanism for  $\phi=\phi_1=120^\circ$ . For  $\phi=120^\circ$  the position of the mechanism is given by:  $x_B=-0.075$  m,  $y_B=0.129904$  m,  $x_D=-0.554223$  m,  $y_D=-0.0964704$  m,  $x_E=-0.482421$  m,  $y_E=0.19481$  m,  $\phi_2=0.441306$  rad,  $\phi_4=1.32911$  rad,  $\phi_5=-0.356559$  rad.

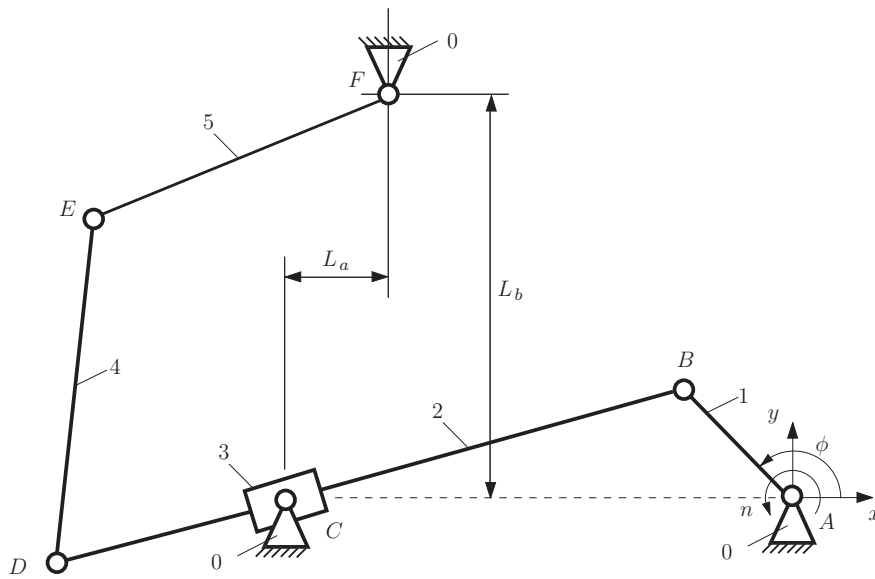


Figure P4.6: Mechanism P4.6

## Results

$\dot{x}_B = -0.408105 \text{ m/s}$ ,  $\dot{y}_B = -0.235619 \text{ m/s}$ ,  $\ddot{x}_B = 0.74022 \text{ m/s}^2$ ,  $\ddot{y}_B = -1.2821 \text{ m/s}^2$ ,  
 $v_{C_3C_2x} = 0.42465 \text{ m/s}$ ,  $v_{C_3C_2y} = 0.200595 \text{ m/s}$ ,  $\omega_2 = -0.127362 \text{ rad/s}$ ,  $\omega_4 = -1.16943 \text{ rad/s}$ ,  
 $\omega_5 = 1.37953 \text{ rad/s}$ ,  $\dot{x}_E = -0.0963053 \text{ m/s}$ ,  $\dot{y}_E = -0.258552 \text{ m/s}$ ,  $a_{C_3C_2x}^{cor} = 0.0510963 \text{ m/s}^2$ ,  
 $a_{C_3C_2y}^{cor} = -0.108168 \text{ m/s}^2$ ,  $\alpha_2 = -5.24453 \text{ rad/s}^2$ ,  $a_{C_3C_2x} = -0.114494 \text{ m/s}^2$ ,  $a_{C_3C_2y} = -$   
 $0.0540842 \text{ m/s}^2$ ,  $\ddot{x}_D = -0.439231 \text{ m/s}^2$ ,  $\ddot{y}_D = 1.23487 \text{ m/s}^2$ ,  $\alpha_4 = -3.94679 \text{ rad/s}^2$ ,  
 $\alpha_5 = -3.66019 \text{ rad/s}^2$ ,  $\ddot{x}_E = 0.612199 \text{ m/s}^2$ ,  $\ddot{y}_E = 0.553139 \text{ m/s}^2$ .

$rA = [ 0.000, 0.000, 0 ] \text{ (m)}$   
 $rD = [-0.554, -0.096, 0] \text{ (m)}$   
 $rB = [-0.075, 0.130, 0] \text{ (m)}$   
 $rC = [-0.350, 0.000, 0] \text{ (m)}$   
 $rE = [-0.482, 0.195, 0] \text{ (m)}$   
 $rF = [-0.295, 0.125, 0] \text{ (m)}$   
 $\text{phi2} = 25.285 \text{ (degrees)}$   
 $\text{phi3} = 25.285 \text{ (degrees)}$   
 $\text{phi4} = 40.509 \text{ (degrees)}$   
 $\text{phi5} = -20.429 \text{ (degrees)}$

## Velocity and acceleration analysis

$\omega_1 = [0, 0, 3.142] \text{ (rad/s)}$   
 $\alpha_1 = [0, 0, 0] \text{ (rad/s}^2\text{)}$

$vB = [-0.408, -0.236, 0] \text{ (m/s)}$   
 $aB = [ 0.740, -1.282, 0] \text{ (m/s}^2\text{)}$

$vC2 = vB + \omega_2 \times rBC = vC3 + vC23 \Rightarrow$   
 $x\text{-axis: } 0.13 \cdot \omega_{2z} - 0.904 \cdot v_{23} - 0.408 = 0$   
 $y\text{-axis: } -0.275 \cdot \omega_{2z} - 0.427 \cdot v_{23} - 0.236 = 0$   
 $\Rightarrow$

$\omega_{2z} = -0.127 \text{ (rad/s)}$   
 $v_{C23} = -0.470 \text{ (m/s)}$

$\omega_3 = \omega_2 = [0, 0, -0.127] \text{ (rad/s)}$   
 $v_{C2C3} = [-0.425, -0.201, -0] \text{ (m/s)}$

$$a_{C23cor} = [-0.051, 0.108, 0] \text{ (m/s}^2\text{)}$$

$$a_{C2} = a_B + \omega_{2x} r_{BC} - (\omega_2 \cdot \omega_2) r_{BC} = a_{C3} + a_{C23} + a_{C23cor} \Rightarrow$$

$$\text{x-axis: } 0.13 \cdot \alpha_{2z} - 0.904 \cdot a_{23} + 0.796 = 0$$

$$\text{y-axis: } -0.427 \cdot a_{23} - 0.275 \cdot \alpha_{2z} - 1.39 = 0$$

$\Rightarrow$

$$\alpha_{2z} = -5.245 \text{ (rad/s)}$$

$$a_{C23} = 0.127 \text{ (m/s)}$$

$$\alpha_{32} = \alpha_{23} = [0, 0, -5.245] \text{ (rad/s}^2\text{)}$$

$$a_{C3C2} = [0.114, 0.054, 0] \text{ (m/s}^2\text{)}$$

$$v_D = [-0.437, -0.175, 0] \text{ (m/s)}$$

$$a_D = [-0.439, 1.235, 0] \text{ (m/s}^2\text{)}$$

$$v_E = v_D + \omega_4 \times r_{DE} = v_F + \omega_5 \times r_{FE} \Rightarrow$$

$$\text{x-axis: } 0.0698 \cdot \omega_{5z} - 0.291 \cdot \omega_{4z} - 0.437 = 0$$

$$\text{y-axis: } 0.0718 \cdot \omega_{4z} + 0.187 \cdot \omega_{5z} - 0.175 = 0$$

$\Rightarrow$

$$\omega_{4z} = -1.169 \text{ (rad/s)}$$

$$\omega_{5z} = 1.380 \text{ (rad/s)}$$

$$\omega_4 = [0, 0, -1.169] \text{ (rad/s)}$$

$$\omega_5 = [0, 0, 1.380] \text{ (rad/s)}$$

$$v_E = [-0.096, -0.259, 0] \text{ (m/s)}$$

$$a_E = a_D + \alpha_4 \times r_{DE} - (\omega_4 \cdot \omega_4) r_{DE}$$

$$a_E = a_F + \alpha_5 \times r_{FE} - (\omega_5 \cdot \omega_5) r_{FE}$$

$$\text{x-axis: } 0.0698 \cdot \alpha_{5z} - 0.291 \cdot \alpha_{4z} - 0.894 = 0$$

$$\text{y-axis: } 0.0718 \cdot \alpha_{4z} + 0.187 \cdot \alpha_{5z} + 0.969 = 0$$

$\Rightarrow$

$$\alpha_{4z} = -3.947 \text{ (rad/s}^2\text{)}$$

$$\alpha_{5z} = -3.660 \text{ (rad/s}^2\text{)}$$

$$\alpha_4 = [0, 0, -3.947] \text{ (rad/s}^2\text{)}$$

$$\alpha_5 = [0, 0, -3.660] \text{ (rad/s}^2\text{)}$$

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$$\mathbf{aE} = [ 0.612, 0.553, 0 ] \text{ (m/s}^2\text{)}$$