3 Position Analysis

Problem 3.3: R-RTR-RRT mechanism

The planar R-RTR-RRT mechanism is considered in Fig. P3.3. The driver is the rigid link 1 (the element AB) and makes an angle $\phi=\phi_1=\pi/6$ with the horizontal. The length of the links are AB=0.02 m, BC=0.03 m, and CD=0.06 m. The following dimensions are given: AE=0.05 m and L_a=0.02 m. Find the positions of the joints and the angles of the links with the horizontal axis.

Solution

Position of joint A:

A cartesian reference frame $xOyz$ with the unit vectors $[\mathbf{\imath}, \mathbf{\jmath}, \mathbf{k}]$ is selected. Since joint A is in the origin of the reference system $A \equiv O$ then

$$x_A = y_A = 0.$$

Position of joint E:

The the coordinates of joint E are

$$x_E = -AE = -0.05 \text{ m} \quad \text{and} \quad y_E = 0.$$

Position of joint B:

Because the joint A is fixed and the angle ϕ is known, the coordinates of joint B are computed with

$$x_B = AB \cos\phi = 0.02 \cos\pi/6 = 0.017 \,\text{m},$$
$$y_B = AB \sin\phi = 0.02 \sin\pi/6 = 0.010 \,\text{m}.$$

Position of joint C:

Joints E, B, and C are located on the same straight line, EBC. The slope of this straight line is

$$m = \frac{y_B - y_E}{x_B - x_E} = \frac{y_C - y_E}{x_C - x_E} \quad \text{or} \quad \frac{0.010}{0.017 - (-0.05)} = \frac{y_C}{x_C - (-0.05)}. \qquad (3.1)$$

The length of the link BC is constant and a quadratic equation can be written as:

$$(x_C - x_B)^2 + (y_C - y_B)^2 = BC^2 \quad \text{or}$$
$$(x_C - 0.017)^2 + (y_C - 0.01)^2 = 0.03^2. \qquad (3.2)$$

Solving Eq. (3.1) and Eq. (3.2) two sets of solutions are found for the position of joint C. These solutions are

$$x_{C_1} = -0.012\,\text{m}, \quad y_{C_1} = 0.005\,\text{m},$$
$$x_{C_2} = 0.046\,\text{m}, \quad y_{C_2} = 0.014\,\text{m}.$$

The points C_1 and C_2 are the intersections of the circle of radius BC (with its center at B), with the straight line EC. To determine the position of joint C for this position of the mechanism ($\phi = \pi/6$), an additional constraint condition is needed: $x_C > x_B$. With this constraint the coordinates of joint C have the following numerical values:

$$x_C = x_{C_2} = 0.046\,\text{m} \quad \text{and} \quad y_C = y_{C_2} = 0.014\,\text{m}.$$

Position of joint D:

The $x-$coordinate of D is $x_D = L_a = 0.02$ m. The length of the link CD is constant and a quadratic equation can be written:

$$(x_D - x_C)^2 + (y_D - y_C)^2 = CD^2 \quad \text{or}$$
$$(0.02 - 0.046)^2 + (y_D - 0.014)^2 = 0.06^2. \tag{3.3}$$

Solving Eq. (3.3), two sets of solutions are found for the position of the joint D. These solutions are

$$y_{D_1} = -0.039\,\text{m} \quad \text{and} \quad y_{D_2} = 0.067\,\text{m}.$$

The points D_1 and D_2 are the intersections of the circle of radius CD (with its center at C) with the vertical line $x = L_a$. To determine the correct position of joint D for the angle $\phi = \pi/6$, an additional constraint condition is needed: $y_D < y_C$. With this constraint the coordinates of joint D are

$$x_D = 0.02\,\text{m} \quad \text{and} \quad y_D = y_{D_1} = -0.039\,\text{m}.$$

Angle ϕ_2:

The angle of link 2 (or link 3) with the horizontal axis is calculated from the slope of the straight line EB:

$$\phi_2 = \phi_3 = \arctan \frac{y_B - y_E}{x_B - x_E} = \arctan \frac{0.010}{0.017 - (-0.050)} = 0.147 \text{ rad} = 8.449°.$$

Angle ϕ_4:

The angle of link 4 with the horizontal axis is obtained from the slope of the straight line CD:

$$\phi_4 = \arctan\frac{y_C - y_D}{x_C - x_D} = \arctan\frac{0.014 + 0.039}{0.046 - 0.020} = 1.104 \text{ rad} = 63.261^\circ.$$

Problem 3.4: R-TRR-RRT mechanism

The mechanism is shown in Fig. P3.4. The following data are given: AC=0.100 m, BC=0.300 m, BD=0.900 m, and L_a=0.100 m. If the angle of link 1 with the horizontal axis is ϕ=45°, find the position of joint D.

Solution

Position of joint A:

A cartesian reference frame with the origin at A is selected. The coordinates of the joint A are

$$x_A = y_A = 0.$$

Position of joint C:

The coordinates of joint C are

$$x_C = AC = 0.100 \text{ m} \quad \text{and} \quad y_C = 0.$$

Position of joint B:

The slope of the line AB is

$$\tan\phi = \frac{y_B}{x_B} \quad \text{or} \quad \tan 45^\circ = \frac{y_B}{x_B}. \tag{3.4}$$

The length of the link BC is constant and the following equation can be written:

$$(x_B - x_C)^2 + (y_B - y_C)^2 = BC^2 \quad \text{or} \quad (x_B - 0.1)^2 + y_B^2 = 0.3^2. \tag{3.5}$$

Equations (3.4) and (3.5) form a system of two equations with the unknowns x_B and y_B. The following numerical results are obtained:

$$x_{B_1} = -0.156\,\text{m}, \quad y_{B_1} = -0.156\,\text{m},$$
$$x_{B_2} = 0.256\,\text{m}, \quad y_{B_2} = 0.256\,\text{m}.$$

To determine the correct position of the joint B for the angle $\phi = 45^\circ$, an additional constraint condition is needed: $x_B > x_C$. With this constraint the coordinates of joint B are

$$x_B = x_{B_2} = 0.256\,\text{m} \quad \text{and} \quad y_B = y_{B_2} = 0.256\,\text{m}.$$

Position of joint D:

The slider 5 has a translational motion in the horizontal direction and $y_D = L_a$. There is only one unknown, x_D, for joint D. The following expression can be written:

$$(x_B - x_D)^2 + (y_B - y_D)^2 = BD^2 \quad \text{or}$$
$$(0.256 - x_D)^2 + (0.256 - 0.1)^2 = 0.9^2 \tag{3.6}$$

Solving Eq. (3.6), two numerical values are obtained:

$$x_{D_1} = -0.630\,\text{m}, \quad x_{D_2} = 1.142\,\text{m}. \tag{3.7}$$

For continuous motion of the mechanism, a geometric constraint $x_D > x_B$ has to be selected. Using this relation the coordinates of joint D are

$$x_D = 1.142\,\text{m} \quad \text{and} \quad y_D = 0.100\,\text{m}.$$