Contents

3 Position Analysis

3.1 Slider-Crank (R-RRT) Mechanism

The R-RRT (slider-crank) mechanism has the dimensions: $AB = 0.5$ m and $BC = 1$ m. The driver link 1 makes an angle $\phi = \phi_1 = 45°$ with the horizontal axis. Find the positions of the joints and the angles of the links with the horizontal axis.

Solution

The MATLAB program starts with the statements

```
clear % clears all variables from the workspace
clc % clears the command window and homes the cursor
close all  % closes all the open figure windows
```

MATLAB commands for $AB = 0.1$ m and $BC = 1$ m.

```
AB=0.5;  BC=1.;
```

MATLAB command for the angle of the driver link 1 with the horizontal axis $\phi = 45°$.

```
phi=pi/4;
```

where `pi` has a numerical value approximately equal to 3.14159.

Position of joint A

A Cartesian reference frame xOy is selected. The joint A is in the origin of the reference frame, that is, $A \equiv O$.

MATLAB commands for $x_A = 0,\ y_A = 0$

```
xA=0; yA=0;
```

Position of joint B

The unknowns are the coordinates of the joint B, x_B and y_B. Because the joint A is fixed and the angle ϕ is known, the coordinates of the joint B are computed from the following expressions

$$
\begin{aligned}
x_B &= AB\cos\phi = (0.5)\ \cos 45^\circ = 0.353553\,\text{m},\\
y_B &= AB\sin\phi = (0.5)\ \sin 45^\circ = 0.353553\,\text{m}.
\end{aligned} \tag{3.1}
$$

The MATLAB commands for Eq. (3.1)

```
xB=AB*cos(phi);
yB=AB*Sin(phi);
```

where `phi` is the angle ϕ in radians.

Position of joint C

The unknowns are the coordinates of the joint C, x_C and y_C.
The joint C is located on the horizontal axis.
MATLAB command for $y_C = 0$

```
yC=0;
```

The length of the segment BC is constant

$$(x_B - x_C)^2 + (y_B - y_C)^2 = BC^2, \tag{3.2}$$

or

$$(0.353553 - x_C)^2 + (0.353553 - 0)^2 = 1^2.$$

MATLAB command for Eq. (3.2)

```
eqnC='(xB-xCsol)^2+(yB-yC)^2=BC^2';
```

where `xCsol` is the unknown.

To solve the equation, a specific MATLAB command will be used.
The command

```
solve('eqn1','eqn2',...,'eqnN','var1','var2',...'varN')
```

attempts to solve an equation or set of equations `'eqn1','eqn2',...,'eqnN'` for the variables `'eqnN','var1','var2',...'varN'`. The set of equations are symbolic expressions or strings specifying equations.

MATLAB command to find the solution `xCsol` from the equation
`eqnC='(xB-xCsol)^2+(yB-yC)^2=BC^2'`

```
solC=solve(eqnC,'xCsol');
```

Because it is a quadratic equation two solutions are found for the position of C. The two solutions are given in a vector form: `solC` is a vector with two components `solC(1)` and `solC(2)`. To obtain the numerical solutions the `eval` command has to be used

```
xC1=eval(solC(1));
xC2=eval(solC(2));
```

The command `eval(s)`, where `s` is a string, executes the string as an expression or statement.

These two solutions for x_C are located at the intersection of the horizontal axis $0x$ with the circle centered in B and radius CB, as shown in Fig. 3.2(b), and they have the following numerical values:

$$x_{C_1} = 1.2890 \text{ m} \quad \text{and} \quad x_{C_2} = -0.5819 \text{ m}.$$

To determine the correct position of the joint C for the mechanism, an additional condition is needed. For the first quadrant, $0 \leq \phi \leq 90°$, the condition is $x_C > xB$.

This MATLAB condition for `xC` located in the first quadrant is

```
if xC1 > xB xC = xC1; else xC = xC2; end
```

The general form of the `if` statement is
`if` *expression statements* `else` *statements* `end`

The x-coordinate of the joint C is $x_C = x_{C_1} = 1.2890$ m.

The angle of the link 2 (link BC) with the horizontal is

$$\phi_2 = \arctan \frac{y_B - y_C}{x_B - x_C},$$

or in MATLAB

```
phi2 = atan((yB-yC)/(xB-xC));
```

The statement `atan(s)` is the arctangent of the elements of `s`.

The numerical solutions for B, C, and ϕ_2 are printed using the statements

```
fprintf('xB = %g (m) \n', xB);
fprintf('yB = %g (m) \n', yB);
fprintf('xC = %g (m) \n', xC);
fprintf('yC = %g (m) \n', yC);
fprintf('phi2 = %g (degrees) \n', phi2*180/pi);
```

The statement `fprintf(f,format,s)` writes data in the real part of array `s` to the file `f`. The data is formated under control of the specified `format` string.

The results of the program are displayed as

```
xB = 0.353553 (m)
yB = 0.353553 (m)
xC = 1.06821 (m)
yC = 1.28897 (m)
phi2 = -20.7048 (degrees)
>>
```

The mechanism is plotted with the help of the command `plotf`. The statement `plotf(x,y,c)` plots vector `y` versus vector `x`, and `c` is a character string.
For the R-RRT mechanism two straight lines AB and BC are plotted

```
plot( [xA,xB],[yA,yB],'r-o', [xB,xC],[yB,yC],'b-o' ),...
```

The line AB is a red (`r` red), solid line (`-` solid), with a circle (`o` circle) at each data point and the line BC is a blue (`b` blue), solid line with a circle at each data point and the.

The x-axis and y-axis are labeled using the commands

```
xlabel('x (m)'),...
ylabel('y (m)'),...
```

and a title is added

```
title('positions for \phi = 45 (deg)'),...
```

On the figure the joints A, B, and C are identified with the statements

```
text(xA,yA,' A'),...
text(xB,yB,' B'),...
text(xC,yC,' C'),...
axis([-0.2 1.4 -0.2 1.4]),...
grid
```

The commas (`,`) and ellipses (`...`) after the command are used to execute the commands together. Otherwise, the data will be plotted, then the labels will be added and the data replotted, and so on.

The statement `axis([xMIN xMAX yMIN yMAX])` sets scaling for the x and y axes on the current plot. To improve the graph a background grid was added with the command `grid`.

The MATLAB program for the positions is given in Program 1.

3.2 R-RRR-RRT Mechanism

The considered planar R-RRR-RRT mechanism is shown in Fig. 3.3 (see lecture notes). The driver link is the rigid link 1 (the element AB). The following data are given: AB=0.150 m, BC=0.400 m, CD=0.370 m, CE=0.230 m, EF=CE, L_a=0.300 m, L_b=0.450 m, and L_c=CD. The angle of the driver link 1 with the horizontal axis is $\phi = \phi_1 = 45°$.
Find the positions of the joints and the angles of the links with the horizontal axis.

Solution
Position of joint A
A cartesian reference frame $xOyz$ with the versors $[\mathbf{\imath}, \mathbf{\jmath}, \mathbf{k}]$ is selected, Fig. 3.4. Since the joint A is in the origin of the reference system $A \equiv O$ the coordinates of A are $x_A = 0$, $y_A = 0$ and the position vector of A is $\mathbf{r}_A = x_A\,\mathbf{\imath} + y_A\,\mathbf{\jmath}$. The vector $\mathbf{r}_A$ is introduced in MATLAB as

```
rA = [xA yA 0];
```

Position of joint D
The coordinates of the joint D are $x_D = L_a$, $y_D = L_b$ and the position vector of D is $\mathbf{r}_D = x_D\,\mathbf{\imath} + y_D\,\mathbf{\jmath}$.
Position of joint B
The unknowns are the coordinates of the joint B, x_B and y_B. Because the joint A is fixed and the angle ϕ is known, the coordinates of the joint B are computed from the following expressions
$x_B = AB\cos\phi = 0.106\,\text{m}, \quad y_B = AB\sin\phi = 0.106\,\text{m}.$
The position vector of B is $\mathbf{r}_B = x_B\,\mathbf{\imath} + y_B\,\mathbf{\jmath}$.
The MATLAB program for this part is

```
AB=0.15; BC=0.40; CD=0.37; CE=0.23; EF=CE;
La=0.30; Lb=0.45; Lc=CD;
phi = pi/6 ;
xA = 0; yA = 0; rA = [xA yA 0];
xD = La ; yD = Lb ; rD = [xD yD 0];
xB = AB*cos(phi); yB = AB*sin(phi); rB = [xB yB 0];
```

Position of joint C

The unknowns are the coordinates of the joint C, x_C and y_C. Knowing the positions of the joints B and D, the position of the joint C can be computed using the fact that the lengths of the links BC and CD are constants

$$(x_C - x_B)^2 + (y_C - y_B)^2 = BC^2,$$
$$(x_C - x_D)^2 + (y_C - y_D)^2 = CD^2, \tag{3.3}$$

or

$$(x_C - 0.106)^2 + (y_C - 0.106)^2 = 0.400^2,$$
$$(x_C - 0.300)^2 + (y_C - 0.450)^2 = 0.370^2, \tag{3.4}$$

Equations (3.4) consist of two quadratic equations. Solving this system of equations, two sets of solutions are found for the position of the joint C. These solutions are

$$x_{C_1} = -0.069\,\text{m}, \quad y_{C_1} = 0.465\,\text{m},$$
$$x_{C_2} = 0.504\,\text{m}, \quad y_{C_2} = 0.141\,\text{m}. \tag{3.5}$$

The MATLAB program for calculating the coordinates of C_1 and C_2 is

```
eqnC1 = '( xCsol - xB )^2 + ( yCsol - yB )^2 = BC^2';
eqnC2 = '( xCsol - xD )^2 + ( yCsol - yD )^2 = CD^2';
solC = solve(eqnC1, eqnC2, 'xCsol, yCsol');
xCpositions = eval(solC.xCsol);
yCpositions = eval(solC.yCsol);
xC1 = xCpositions(1); % first component of the vector xCpositions
xC2 = xCpositions(2); % second component of the vector xCpositions
yC1 = yCpositions(1); % first component of the vector yCpositions
yC2 = yCpositions(2); % second component of the vector yCpositions
```

The points C_1 and C_2 are the intersections of the circle of radius BC (with its center at B) with the circle of radius CD (with its center at D) (see lecture notes Fig. 3.4).

To determine the position of the joint C for this mechanism, an additional constraint condition is needed: $x_C < x_D$. Because $x_D = 0.300$ m, the coordinates of joint C have the following numerical values

$$x_C = x_{C_1} = -0.069\,\text{m}, \qquad y_C = y_{C_1} = 0.465\,\text{m}. \tag{3.6}$$

The MATLAB program for selecting the correct position of C is

```
if xC1 < xD
   xC = xC1; yC=yC1;
else
   xC = xC2; yC=yC2;
end
rC = [xC yC 0]; % Position vector of C
```

Position of joint E

The unknowns are the coordinates of the joint E, x_E and y_E. The position of the joint E is determined from the equation

$$(x_E - x_C)^2 + (y_E - y_C)^2 = CE^2, \tag{3.7}$$

or

$$(x_E + 0.069)^2 + (y_E - 0.465)^2 = 0.230^2.$$

The joints D, C and E are located on the same straight element DE. For these joints, the following equation can be written

$$\frac{y_D - y_C}{x_D - x_C} = \frac{y_E - y_C}{x_E - x_C}, \tag{3.8}$$

or

$$\frac{0.450 - 0.465}{0.300 + 0.069} = \frac{y_E - 0.465}{x_E + 0.069}.$$

Equations (3.7) and (3.8) form a system from which the coordinates of the joint E can be computed. Two solutions are obtained, Fig. 3.5, and the numerical values are

$$x_{E_1} = -0.299\text{ m}, \quad y_{E_1} = 0.474\text{ m},$$
$$x_{E_2} = 0.160\text{ m}, \quad y_{E_2} = 0.455\text{ m}. \tag{3.9}$$

The MATLAB program for calculating the coordinates of E_1 and E_2 is

```
eqnE1 = '( xEsol - xC )^2 + ( yEsol - yC )^2 = CE^2 ';
eqnE2 = '(yD-yC)/(xD-xC)=(yEsol-yC )/(xEsol-xC)';
solE = solve(eqnE1, eqnE2, 'xEsol, yEsol');
xEpositions=eval(solE.xEsol); yEpositions=eval(solE.yEsol);
xE1 = xEpositions(1); xE2 = xEpositions(2);
yE1 = yEpositions(1); yE2 = yEpositions(2);
```

For continuous motion of the mechanism, a constraint condition is needed, $x_E < x_C$. Using this condition, the coordinates of the joint E are

$$x_E = x_{E_1} = -0.300\,\text{m},\;\; y_E = y_{E_1} = 0.475\,\text{m}.$$

The MATLAB program for selecting the correct position of E is

```
if xE1 < xC
   xE = xE1; yE=yE1;
else
   xE = xE2; yE=yE2;
end
rE = [xE yE 0]; % Position vector of E
```

Position of joint F

The joint F is restricted to move in a vertical direction, i.e. $x_F = -L_c = 0.370$ m. The coordinate y_F of the joint F can be calculated from the following quadratic equation

$$(x_F - x_E)^2 + (y_F - y_E)^2 = EF^2, \tag{3.10}$$

or

$$(0.370 + 0.300)^2 + (y_F - 0.475)^2 = 0.230^2,$$

The solutions of Eq. (3.10) are

$$y_{F_1} = 0.256\,\text{m}, \qquad y_{F_2} = 0.693\,\text{m}. \tag{3.11}$$

The points F_1 and F_2 are the intersections between the circle of radius EF (centered at E) and the vertical line with $x = x_F$, Fig. 3.7. For the mechanism depicted in Fig. 3.4, with $\theta = \pi/4$ the y coordinate of the joint F should be smaller that the y coordinate of the joint E, $y_F < y_E$. The y coordinate of the joint F is

$$y_F = y_{F_1} = 0.256\,\mathrm{m}. \tag{3.12}$$

The MATLAB program for the position of F is

```
xF = - Lc ;
eqnF = '( xF - xE )^2 + ( yFsol - yE )^2 = EF^2 ';
solF = solve(eqnF,'yFsol');
yFpositions=eval(solF);
yF1 = yFpositions(1); yF2 = yFpositions(2);
if yF1 < yE
   yF=yF1;
else
   yF=yF2;
end
rF = [xF yF 0];
```

The angles of the links 2, 3, and 4 with the horizontal are

$$\phi_2 = \arctan\frac{y_B - y_C}{x_B - x_C}, \quad \phi_3 = \arctan\frac{y_D - y_C}{x_D - x_C}, \quad \phi_4 = \arctan\frac{y_F - y_E}{x_F - x_E},$$

and in MATLAB

```
phi2 = atan((yB-yC)/(xB-xC));
phi3 = atan((yD-yC)/(xD-xC));
phi4 = atan((yF-yE)/(xF-xE));
```

The results are printed using the statements

```
fprintf('rA = [ %g, %g, %g ] (m) \n', rA);
fprintf('rD = [ %g, %g, %g ] (m) \n', rD);
fprintf('rB = [ %g, %g, %g ] (m) \n', rB);
fprintf('rC = [ %g, %g, %g ] (m) \n', rC);
fprintf('rE = [ %g, %g, %g ] (m) \n', rE);
fprintf('rF = [ %g, %g, %g ] (m) \n', rF);
fprintf('phi2 = %g (degrees) \n', phi2*180/pi);
fprintf('phi3 = %g (degrees) \n', phi3*180/pi);
```

```
fprintf('phi4 = %g (degrees) \n', phi4*180/pi);
```

The graph of the mechanism using MATLAB for $\phi = \pi/4$ is given by

```
plot([xA,xB],[yA,yB],'r-o','LineWidth',1.5)
hold on % holds the current plot
plot([xB,xC],[yB,yC],'b-o','LineWidth',1.5)
hold on
plot([xD,xE],[yD,yE],'g-o','LineWidth',1.5)
hold on
plot([xE,xF],[yE,yF],'b-o','LineWidth',1.5)
grid on,...  % adds major grid lines to the current axes
xlabel('x (m)'), ylabel('y (m)'),...
title('positions for \phi = 45 (deg)'),...
text(xA,yA,'\leftarrow A = ground','HorizontalAlignment','left'),...
text(xB,yB,' B'),...
text(xC,yC,'\leftarrow C = ground','HorizontalAlignment','left'),...
text(xD,yD,'\leftarrow D = ground','HorizontalAlignment','left'),...
text(xE,yE,' E'), text(xF,yF,' F'), axis([-0.4 0.45 -0.1 0.6])
```

The MATLAB program for the positions and the results is given in Program 2.