Slider-Crank Mechanism

Velocity and Acceleration Analysis

The R-RRT (slider-crank) mechanism shown in the figure has the dimensions: \( AB = 1 \text{ m} \) and \( BC = 1 \text{ m} \). The driver link 1 makes an angle \( \phi = \phi_1 = \pi/4 \text{ rad} \) with the horizontal axis and rotates with a constant speed of \( n = 30/\pi \text{ rpm} \). The point \( A \) is selected as the origin of the \( xyz \) reference frame. The position vectors of the joints \( B \) and \( C \) are:

\[ r_B = x_B \hat{i} + y_B \hat{j} = \frac{\sqrt{2}}{2} \hat{i} + \frac{\sqrt{2}}{2} \hat{j} \text{ [m]} \quad \text{and} \quad r_C = x_C \hat{i} + y_C \hat{j} = \sqrt{2} \hat{i} + 0 \hat{j} \text{ [m]}. \]

The angular velocity of link 1 is

\[ \omega_1 = \omega = \frac{\pi n}{30} \text{ k} = \frac{\pi (30/\pi)}{30} \text{ k} = 1 \text{ k \ rad/s}. \]

Velocity of joint \( B \)

The velocity of the point \( B = B_1 \) on the link 1 is

\[ \mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_A + \mathbf{\omega}_1 \times \mathbf{r}_{AB} = \mathbf{\omega}_1 \times \mathbf{r}_B, \]

where \( \mathbf{v}_A \equiv \mathbf{0} \) is the velocity of the origin \( A \equiv O \).

The velocity of point \( B_2 \) on the link 2 is \( \mathbf{v}_{B_2} = \mathbf{v}_{B_1} \) because the links 1 and 2 are connected at a rotational joint. The velocity of \( B_1 = B_2 \) is

\[ \mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \begin{bmatrix} 1 & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_1 \\ x_B & y_B & 0 \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \end{bmatrix} = -\frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j} \text{ m/s}. \]

Velocity of joint \( C \)

The points \( B_2 \) and \( C_2 \) are on the link 2 and

\[ \mathbf{v}_C = \mathbf{v}_{C_2} = \mathbf{v}_{B_2} + \mathbf{\omega}_2 \times \mathbf{r}_{BC} = \mathbf{v}_B + \mathbf{\omega}_2 \times (\mathbf{r}_C - \mathbf{r}_B), \]

where the angular velocity of link 2 is \( \mathbf{\omega}_2 = \omega_2 \mathbf{k} \) (\( \omega_2 \) is unknown).
On the other hand the velocity of $C$ is along the vertical axis ($x$-axis) because slider 2 translates along $x$-axis

$$v_C = v_{C_3} = v_C \mathbf{1}. \quad (2)$$

Equations (1) and (2) give

$$v_B + \omega_2 \times (r_C - r_B) = v_C \mathbf{1},$$

or

$$v_B + \begin{vmatrix} 1 & J & k \\ 0 & 0 & \omega_2 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} = v_C \mathbf{1} \quad (3)$$

Equation (3) represents a vectorial equation with two scalar components on $x$-axis and $y$-axis and with two unknowns $\omega_2$ and $v_C$

$$v_{Bx} - \omega_2(y_C - y_B) = v_C,$$

$$v_{Bx} + \omega_2(x_C - x_B) = 0,$$

or

$$\frac{\sqrt{2}}{2} - \omega_2(0 - \frac{\sqrt{2}}{2}) = v_C,$$

$$\frac{\sqrt{2}}{2} + \omega_2(\sqrt{2} - \frac{\sqrt{2}}{2}) = 0.$$

It results

$$\omega_2 = -1 \text{ rad/s} \text{ and } v_C = -\sqrt{2} \text{ m/s}.$$

**Acceleration of joint $B$**

The acceleration of the point $B = B_1$ on the link 1 is

$$a_B = a_{B_1} = a_{B_2} = a_A + \alpha_1 \times r_B + \omega_1 \times (\omega_1 \times r_B) = \alpha_1 \times r_B - \omega^2_1 r_B$$

$$=-\omega^2_1 r_B = -1^2(\frac{\sqrt{2}}{2} \mathbf{1} + \frac{\sqrt{2}}{2} \mathbf{j}) = -\frac{\sqrt{2}}{2} \mathbf{1} - \frac{\sqrt{2}}{2} \mathbf{j} \text{ m/s}^2.$$

The angular acceleration of link 1 is $\alpha_1 = \dot{\omega}_1 = 0$. 
Acceleration of joint $C$

The points $C_2$ and $B_2$ are on the link 2 and

\[ \mathbf{a}_C = \mathbf{a}_{C_2} = \mathbf{a}_{B_2} + \alpha_2 \mathbf{r}_{BC} - \omega_2^2 \mathbf{r}_{BC} = \mathbf{a}_B + \alpha_2 \times (\mathbf{r}_C - \mathbf{r}_B) - \omega_2^2 (\mathbf{r}_C - \mathbf{r}_B), \] (4)

where the angular acceleration of link 2 is $\alpha_2 = \alpha_2 \mathbf{k}$ ($\alpha_2$ is unknown).

The slider $C$ has a translational motion along $x$-axis and

\[ \mathbf{a}_C = \mathbf{a}_{C_3} = a_C \mathbf{i}. \] (5)

Equations (4) and (5) give

\[ \mathbf{a}_B + \alpha_2 \times (\mathbf{r}_C - \mathbf{r}_B) - \omega_2^2 (\mathbf{r}_C - \mathbf{r}_B) = a_C \mathbf{i}, \]

or

\[
\mathbf{a}_B + \begin{bmatrix} 1 & \mathbf{j} & \mathbf{k} \end{bmatrix} \begin{bmatrix} 0 & 0 & \alpha_2 \\ x_C - x_B & y_C - y_B & 0 \end{bmatrix} - \omega_2^2 [(x_C - x_B) \mathbf{i} + (y_C - y_B) \mathbf{j}] = a_C \mathbf{i}. \] (6)

Equation (6) represents a vectorial equation with two scalar components on $x$-axis and $y$-axis and with two unknowns $\alpha_2$ and $\alpha_3$

\[ a_{Bx} - \alpha_2 (y_C - y_B) - \omega_2^2 (x_C - x_B) = a_C, \]
\[ a_{By} + \alpha_2 (x_C - x_B) - \omega_2^2 (y_C - y_B) = 0, \]

or

\[ -\frac{\sqrt{2}}{2} - \alpha_2 \left( 0 - \frac{\sqrt{2}}{2} \right) - (-1)^2 \left( \sqrt{2} - \frac{\sqrt{2}}{2} \right) = a_C, \]
\[ -\frac{\sqrt{2}}{2} + \alpha_2 \left( \sqrt{2} - \frac{\sqrt{2}}{2} \right) - (-1)^2 \left( 0 - \frac{\sqrt{2}}{2} \right) = 0. \]

It results

\[ \alpha_2 = 0 \text{ rad/s}^2 \] and \[ a_C = -\sqrt{2} \text{ m/s}^2. \]