Slider-Crank Mechanism

Velocity and Acceleration Analysis

The R-RRT (slider-crank) mechanism shown in the figure has the dimensions: $AB = 1$ m and $BC = 1$ m. The driver link 1 makes an angle $\phi = \phi_1 = \pi/4$ rad with the horizontal axis and rotates with a constant speed of $n = 30/\pi$ rpm. The point A is selected as the origin of the xyz reference frame. The position vectors of the joints B and C are:

$$\mathbf{r}_B = x_B\mathbf{\imath} + y_B\mathbf{\jmath} = \frac{\sqrt{2}}{2}\mathbf{\imath} + \frac{\sqrt{2}}{2}\mathbf{\jmath}\ [\text{m}] \quad \text{and} \quad \mathbf{r}_C = x_C\mathbf{\imath} + y_C\mathbf{\jmath} = \sqrt{2}\mathbf{\imath} + 0\mathbf{\jmath}\ [\text{m}].$$

The angular velocity of link 1 is

$$\boldsymbol{\omega}_1 = \boldsymbol{\omega} = \omega_1\ \mathbf{k} = \frac{\pi n}{30}\ \mathbf{k} = \frac{\pi(30/\pi)}{30}\ \mathbf{k} = 1\ \mathbf{k}\ \ \text{rad/s}.$$

Velocity of joint B

The velocity of the point $B = B_1$ on the link 1 is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_A + \boldsymbol{\omega}_1 \times \mathbf{r}_{AB} = \boldsymbol{\omega}_1 \times \mathbf{r}_B,$$

where $\mathbf{v}_A \equiv \mathbf{0}$ is the velocity of the origin $A \equiv O$.
The velocity of point B_2 on the link 2 is $\mathbf{v}_{B_2} = \mathbf{v}_{B_1}$ because the links 1 and 2 are connected at a rotational joint. The velocity of $B_1 = B_2$ is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_1 \\ x_B & y_B & 0 \end{vmatrix} = \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = -\frac{\sqrt{2}}{2}\mathbf{\imath} + \frac{\sqrt{2}}{2}\mathbf{\jmath}\ \ \text{m/s}.$$

Velocity of joint C

The points B_2 and C_2 are on the link 2 and

$$\mathbf{v}_C = \mathbf{v}_{C_2} = \mathbf{v}_{B_2} + \boldsymbol{\omega}_2 \times \mathbf{r}_{BC} = \mathbf{v}_B + \boldsymbol{\omega}_2 \times (\mathbf{r}_C - \mathbf{r}_B), \tag{1}$$

where the angular velocity of link 2 is $\boldsymbol{\omega}_2 = \omega_2\mathbf{k}$ (ω_2 is unknown).

On the other hand the velocity of C is along the vertical axis (x-axis) because slider 2 translates along x-axis

$$\mathbf{v}_C = \mathbf{v}_{C_3} = v_C \boldsymbol{\imath}. \tag{2}$$

Equations (1) and (2) give

$$\mathbf{v}_B + \boldsymbol{\omega}_2 \times (\mathbf{r}_C - \mathbf{r}_B) = v_C \boldsymbol{\imath},$$

or

$$\mathbf{v}_B + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_2 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} = v_C \boldsymbol{\imath} \tag{3}$$

Equation (3) represents a vectorial equation with two scalar components on x-axis and y-axis and with two unknowns ω_2 and v_C

$$v_{Bx} - \omega_2(y_C - y_B) = v_C,$$
$$v_{Bx} + \omega_2(x_C - x_B) = 0,$$

or

$$-\frac{\sqrt{2}}{2} - \omega_2(0 - \frac{\sqrt{2}}{2}) = v_C,$$
$$\frac{\sqrt{2}}{2} + \omega_2(\sqrt{2} - \frac{\sqrt{2}}{2}) = 0.$$

It results

$$\omega_2 = -1 \text{ rad/s} \quad \text{and} \quad v_C = -\sqrt{2} \text{ m/s}.$$

Acceleration of joint B

The acceleration of the point $B = B_1$ on the link 1 is

$$\begin{aligned} \mathbf{a}_B &= \mathbf{a}_{B_1} = \mathbf{a}_{B_2} = \mathbf{a}_A + \boldsymbol{\alpha}_1 \times \mathbf{r}_B + \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_B) = \boldsymbol{\alpha}_1 \times \mathbf{r}_B - \boldsymbol{\omega}_1^2 \mathbf{r}_B \\ &= -\boldsymbol{\omega}_1^2 \mathbf{r}_B = -1^2(\frac{\sqrt{2}}{2}\boldsymbol{\imath} + \frac{\sqrt{2}}{2}\boldsymbol{\jmath}) = -\frac{\sqrt{2}}{2}\boldsymbol{\imath} - \frac{\sqrt{2}}{2}\boldsymbol{\jmath} \ \text{ m/s}^2. \end{aligned}$$

The angular acceleration of link 1 is $\boldsymbol{\alpha}_1 = \dot{\boldsymbol{\omega}}_1 = \mathbf{0}$.

Acceleration of joint C

The points C_2 and B_2 are on the link 2 and

$$\mathbf{a}_C = \mathbf{a}_{C_2} = \mathbf{a}_{B_2} + \boldsymbol{\alpha}_2 \times \mathbf{r}_{BC} - \omega_2^2\ \mathbf{r}_{BC} = \mathbf{a}_B + \boldsymbol{\alpha}_2 \times (\mathbf{r}_C - \mathbf{r}_B) - \omega_2^2\ (\mathbf{r}_C - \mathbf{r}_B), \quad (4)$$

where the angular acceleration of link 2 is $\boldsymbol{\alpha}_2 = \alpha_2 \mathbf{k}$ (α_2 is unknown).

The slider C has a translational motion along x-axis and

$$\mathbf{a}_C = \mathbf{a}_{C_3} = a_C \boldsymbol{\imath}. \quad (5)$$

Equations (4) and (5) give

$$\mathbf{a}_B + \boldsymbol{\alpha}_2 \times (\mathbf{r}_C - \mathbf{r}_B) - \omega_2^2\ (\mathbf{r}_C - \mathbf{r}_B) = a_C \boldsymbol{\imath},$$

or

$$\mathbf{a}_B + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \alpha_2 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} - \omega_2^2\, [(x_C - x_B)\boldsymbol{\imath} + (y_C - y_B)\boldsymbol{\jmath}] = a_C \boldsymbol{\imath}. \quad (6)$$

Equation (6) represents a vectorial equation with two scalar components on x-axis and y-axis and with two unknowns α_2 and α_3

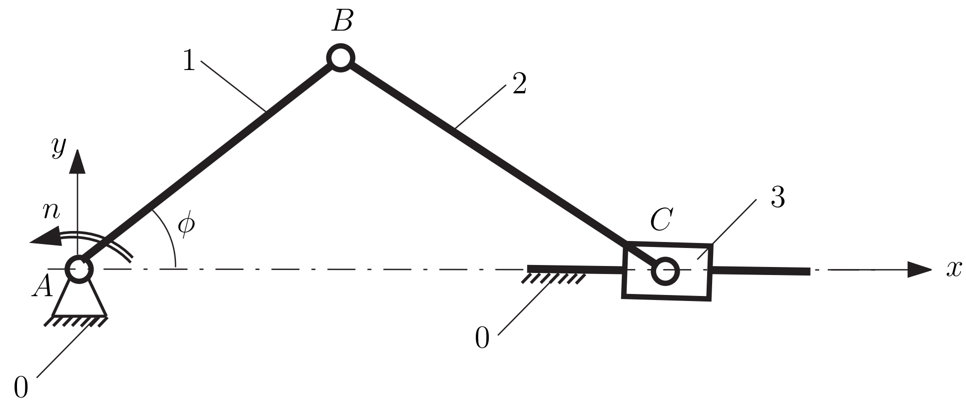
$$a_{Bx} - \alpha_2(y_C - y_B) - \omega_2^2(x_C - x_B) = a_C,$$
$$a_{By} + \alpha_2(x_C - x_B) - \omega_2^2(y_C - y_B) = 0,$$

or

$$-\frac{\sqrt{2}}{2} - \alpha_2(0 - \frac{\sqrt{2}}{2}) - (-1)^2(\sqrt{2} - \frac{\sqrt{2}}{2}) = a_C,$$
$$-\frac{\sqrt{2}}{2} + \alpha_2(\sqrt{2} - \frac{\sqrt{2}}{2}) - (-1)^2(0 - \frac{\sqrt{2}}{2}) = 0.$$

It results

$$\alpha_2 = 0 \text{ rad/s}^2 \text{ and } a_C = -\sqrt{2} \text{ m/s}^2.$$



Figure