Contents

4 Dynamic Force Analysis ................................................. 1
  4.1 Equation of Motion for General Planar Motion ................. 1
  4.2 D’Alembert’s Principle ....................................... 8
  4.3 Free-Body Diagrams ........................................... 10
  4.4 Force Analysis Using Dyads ................................. 11
  4.5 Force Analysis Using Contour Method ..................... 14
  4.6 R-RRT (slider-crank) Mechanism ............................ 15
    4.6.1 Newton-Euler Equations of Motion .................. 20
    4.6.2 D’Alembert’s Principle .............................. 25
    4.6.3 Dyad Method ........................................ 30
    4.6.4 Contour Method ..................................... 32
  4.7 R-RTR-RTR Mechanism ....................................... 38
    4.7.1 Newton-Euler Equations of Motion .................. 46
    4.7.2 Dyad Method ........................................ 54
    4.7.3 Contour Method ..................................... 60
Dynamic Force Analysis

For a kinematic chain it is important to know how forces and moments are transmitted from the input to the output, so that the links can be properly designated. The friction effects are assumed to be negligible in the force analysis presented here.

4.1 Equation of Motion for General Planar Motion

Consider a system of $N$ particles. A particle is an object whose shape and geometrical dimensions are not significant to the investigation of its motion. An arbitrary collection of matter with total mass $m$ can be divided into $N$ particles, the $i$th particle having mass, $m_i$ [Fig. 4.1(a)] $m = \sum_{i=1}^{N} m_i$.

A rigid body can be considered as a collection of particles in which the number of particles approaches infinity and in which the distance between any two points remains constant. As $N$ approaches infinity, each particle is treated as a differential mass element, $m_i \to dm$, and the summation is replaced by integration over the body $m = \int_{\text{body}} dm$.

The position of the mass center of a collection of particles is defined by

$$r_C = \frac{1}{m} \sum_{i=1}^{N} m_i r_i,$$

where $r_i = r_{OP_i} = r_{P_i}$ is the position vector from the origin $O$ to the $i$th particle. The particle $i$ is located at the point $P_i$.

As $N \to \infty$, the summation is replaced by integration over the body

$$r_C = \frac{1}{m} \int_{\text{body}} r \, dm,$$

where $r$ is the vector from the origin $O$ to differential element $dm$.

The time derivative of Eq. (4.1) gives

$$\sum_{i=1}^{N} m_i \frac{d^2 r_i}{dt^2} = m \frac{d^2 r_C}{dt^2} = m a_C,$$

where $a_C$ is the acceleration of the mass center. The acceleration of the mass center can be related to the external forces acting on the system. This
Dynamic Force Analysis with MATLAB

relationship is obtained by applying Newton’s laws to each of the individual particles in the system. Any such particle is acted on by two types of forces. One type is exerted by other particles that are also part of the system. Such forces are called internal forces (internal to the system). Additionally, a particle can be acted on by a force that is exerted by a particle or object not included in the system. Such a force is known as an external force (external to the system). Let \( f_{ij} \) be the internal force exerted on the \( j \)th particle by the \( i \)th particle. Newton’s third law (action and reaction) states that the \( j \)th particle exerts a force on the \( i \)th particle of equal magnitude, and opposite direction, and collinear with the force exerted by the \( i \)th particle on the \( j \)th particle [Fig. 4.1(a)] \( f_{ji} = -f_{ij}, \ j \neq i \). Newton’s second law for the \( i \)th particle must include all of the internal forces exerted by all of the other particles in the system on the \( i \)th particle, plus the sum of any external forces exerted by particles, objects outside of the system on the \( i \)th particle

\[
\sum_j f_{ji} + F^\text{ext}_i = m_i \frac{d^2 r_i}{dt^2}, \quad j \neq i, \quad (4.4)
\]

where \( F^\text{ext}_i \) is the external force on the \( i \)th particle. Equation (4.4) is written for each particle in the collection of particles. Summing the resulting equations over all of the particles from \( i = 1 \) to \( N \) the following relation is obtained

\[
\sum_i \sum_j f_{ji} + \sum_i F^\text{ext}_i = m a_C, \quad j \neq i. \quad (4.5)
\]

The sum of the internal forces includes pairs of equal and opposite forces. The sum of any such pair must be zero. The sum of all of the internal forces on the collection of particles is zero (Newton’s third law) \( \sum_i \sum_j f_{ji} = 0, \ j \neq i \).

The term \( \sum_i F^\text{ext}_i \) is the sum of the external forces on the collection of particles \( \sum_i F^\text{ext}_i = F \). The sum of the external forces acting on a closed system equals the product of the mass and the acceleration of the mass center

\[
m a_C = F. \quad (4.6)
\]

Considering Fig. 4.2(b) for a rigid body and introducing the distance \( q \) in Eq. (4.2) gives

\[
r_C = \frac{1}{m} \int \text{body} r \, dm = \frac{1}{m} \int \text{body} (r_C + q) \, dm = r_C + \frac{1}{m} \int \text{body} q \, dm. \quad (4.7)
\]
It results
\[
\frac{1}{m} \int_{\text{body}} q \, dm = 0, \quad (4.8)
\]
that is the weighed average of the displacement vector about the mass center is zero. The equation of motion for the differential element \( dm \) is
\[
a \, dm = dF,
\]
where \( dF \) is the total force acting on the differential element. For the entire body
\[
\int_{\text{body}} a \, dm = \int_{\text{body}} dF = F, \quad (4.9)
\]
where \( F \) is the resultant of all forces. This resultant contains contributions only from the external forces, as the internal forces cancel each other. Introducing Eq. (4.7) into Eq. (4.9), the Newton’s second law for a rigid body is obtained
\[
m \, a_C = F
\]
The derivation of the equations of motion is valid for the general motion of a rigid body. These equations are equally applicable to planar and three-dimensional motions.

Resolving the sum of the external forces into cartesian rectangular components
\[
F = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k},
\]
and the position vector of the mass center
\[
r_C = x_C(t) \mathbf{i} + y_C(t) \mathbf{j} + z_C(t) \mathbf{k},
\]
Newton’s second law for the rigid body is
\[
m \ddot{r}_C = F, \quad (4.10)
\]
or
\[
m \ddot{x}_C = F_x, \quad m \ddot{y}_C = F_y, \quad m \ddot{z}_C = F_z. \quad (4.11)
\]
Angular Momentum Principle for a System of Particles

The total angular momentum of the system of $N$ particles [Fig. 4.1(a)] about its mass center $C$, is the sum of the angular momenta of the particles about $C$

\[
H_C = \sum_{i=1}^{N} r_{CP_i} \times m_i v_i,
\]

(4.12)

where $v_i = \frac{dr_i}{dt}$ is the velocity of the particle $P_i$.

The total angular momentum of the system about $O$ is the sum of the angular momenta of the particles

\[
H_O = \sum_{i=1}^{N} r_i \times m_i v_i = \sum_{i=1}^{N} (r_i + r_{CP_i}) \times m_i v_i = \sum_{i=1}^{N} (r_C \times m_C + H_C) \times m_C + H_C,
\]

(4.13)

or the total angular momentum about $O$ is the sum of the angular momentum about $O$ due to the velocity $v_C$ of the mass center of the system and the total angular momentum about the mass center.

The rate of change of the angular momentum about $O$ equals the sum of the moments about $O$ due to external forces and couples

\[
\frac{dH_O}{dt} = \sum M_O.
\]

(4.14)

Using Eqs. (4.13) and (4.14), the following result is obtained

\[
\sum M_O = \frac{d}{dt} (r_C \times m_C + H_C) = r_C \times ma_C + \frac{dH_C}{dt},
\]

(4.15)

where $a_C$ is the acceleration of the mass center.

With the relation

\[
\sum M_O = \sum M_C + r_C \times F = \sum M_C + r_C \times ma_C,
\]

Eq. (4.15) becomes

\[
\frac{dH_C}{dt} = \sum M_C.
\]

(4.16)

The rate of change of the angular momentum about the mass center equals the sum of the moments about the mass center.
Equations of Motion

An arbitrary rigid body with the mass \( m \) can be divided into \( N \) particles \( P_i, i = 1, 2, ..., N \). The mass of the \( P_i \) particle is \( m_i \). Figure 4.2(a) represents the rigid body moving with general planar motion in the \((x, y)\) plane. The origin of the cartesian reference frame is \( O \). The mass center \( C \) of the rigid body is located in the plane of the motion, \( C \in (x,y) \). Let \( d_O = Oz \) be the axis through the fixed origin point \( O \) that is perpendicular to the plane of motion of the rigid body. Let \( d_C = Czz \) be the parallel axis through the mass center \( C \). The rigid body has a general planar motion and the angular velocity vector is \( \omega = \omega k \). The unit vector of the \( d_C = Czz \) axis is \( k \).

The sum of the moments about \( O \) due to external forces and couples is

\[
\sum M_O = \frac{dH_O}{dt} = \frac{d}{dt}[(r_C \times m v_C) + H_C].
\] (4.17)

The magnitude of the angular momentum about \( d_C \) is

\[
H_C = \sum m_i r_i^2 \omega.
\] (4.18)

The summation \( \sum m_i r_i^2 \) or the integration over the body \( \int r^2 dm \) is defined as mass moment of inertia \( I_{Czz} \) of the body about the \( z \)-axis through \( C \)

\[
I_{Czz} = \sum m_i r_i^2.
\]

The term \( r_i \) is the perpendicular distance from \( d_C \) to the \( P_i \) particle.

The mass moment of inertia \( I_{Czz} \) is a constant property of the body and is a measure of the rotational inertia or resistance to change in angular velocity due to the radial distribution of the rigid body mass around \( z \)-axis through \( C \).

The angular momentum of the rigid body about \( d_C \) (\( z \)-axis through \( C \)) is

\[
H_C = I_{Czz} \omega \quad \text{or} \quad \mathbf{H}_C = I_{Czz} \omega \mathbf{k} = I_{Czz} \mathbf{\omega}.
\]

Substituting this expression into Eq. (4.17) gives

\[
\sum M_O = \frac{d}{dt}[(r_C \times m v_C) + I_{Czz} \omega] = (r_C \times ma_C) + I_{Czz} \alpha.
\] (4.19)

The rotational equation of motion for the rigid body is

\[
I_{Czz} \alpha = \sum M_C \quad \text{or} \quad I_{Czz} \mathbf{\alpha} = \sum M_C \mathbf{k}.
\] (4.20)
For general planar motion the angular acceleration is \( \mathbf{a} = \ddot{\omega} = \ddot{\theta} \mathbf{k} \), where the angle \( \theta \) describes the position, or orientation, of the rigid body about a fixed axis. If the rigid body is a plate moving in the plane of motion \((X,Y)\), the mass moment of inertia of the rigid body about \(z\)-axis through \(C\) becomes the polar mass moment of inertia of the rigid body about \(C\), \( I_{Czz} = I_C \). For this case the Eq. (4.20) gives

\[
I_C \mathbf{a} = \sum \mathbf{M}_C.
\]  

(4.21)

A special application of Eq. (4.21) is for rotation about a fixed point. Consider the special case when the rigid body rotates about the fixed point \(O\) as shown in Fig. 4.2(b). It follows that the acceleration of the mass center is expressed as

\[
\mathbf{a}_C = \mathbf{a} \times \mathbf{r}_C - \omega^2 \mathbf{r}_C.
\]  

(4.22)

The relation between the sum of the moments of the external forces about the fixed point \(O\) and the product \(I_{Czz} \mathbf{a}\) is given by Eq. (4.19)

\[
\sum \mathbf{M}_O = \mathbf{r}_C \times m \mathbf{a}_C + I_{Czz} \mathbf{a}.
\]  

(4.23)

Equations (4.22) and (4.23) give

\[
\sum \mathbf{M}_O = \mathbf{r}_C \times m (\mathbf{a} \times \mathbf{r}_C - \omega^2 \mathbf{r}_C) + I_{Czz} \mathbf{a} = \\
m \mathbf{r}_C \times (\mathbf{a} \times \mathbf{r}_C) + I_{Czz} \mathbf{a} = \\
m [(\mathbf{r}_C \cdot \mathbf{r}_C) \mathbf{a} - (\mathbf{r}_C \cdot \mathbf{r}_C) \mathbf{r}_C] + I_{Czz} \mathbf{a} = \\
m \mathbf{r}_C^2 \mathbf{a} + I_{Czz} \mathbf{a} = (m \mathbf{r}_C^2 + I_{Czz}) \mathbf{a}.
\]  

(4.24)

According to parallel-axis theorem

\[
I_{Ozz} = I_{Czz} + m \mathbf{r}_C^2,
\]

where \(I_{Ozz}\) denotes the mass moment of inertia of the rigid body about \(z\)-axis through \(O\). For the special case of rotation about a fixed point \(O\) one can use the formula

\[
I_{Ozz} \mathbf{a} = \sum \mathbf{M}_O.
\]  

(4.25)

The general equations of motion for a rigid body in plane motion are (Fig. 4.3)

\[
\mathbf{F} = m \mathbf{a}_C \quad \text{or} \quad \mathbf{F} = m \mathbf{\ddot{r}}_C,
\]  

(4.26)

\[
\sum \mathbf{M}_C = I_{Czz} \mathbf{a},
\]  

(4.27)
or using the cartesian components
\[ m\ddot{x}_C = \sum F_x, \quad m\ddot{y}_C = \sum F_y, \quad I_{Czz}\ddot{\theta} = \sum M_C. \]  
(4.28)

Equations (4.26) and (4.27) are interpreted in two ways

1. The forces and moments are known and the equations are solved for the motion of the rigid body (direct dynamics).

2. The motion of the RB is known and the equations are solved for the force and moments (inverse dynamics).

The dynamic force analysis in this chapter is based on the known motion of the mechanism.
4.2 D’Alembert’s Principle

Newton’s second law can be written as

\[ \mathbf{F} + (-m \mathbf{a}_C) = 0, \quad \text{or} \quad \mathbf{F} + \mathbf{F}_{\text{in}} = 0, \quad (4.29) \]

where the term \( \mathbf{F}_{\text{in}} = -m \mathbf{a}_C \) is the \textit{inertia force}. Newton’s second law can be regarded as an “equilibrium” equation.

Equation (4.23) relates the total moment about a fixed point \( O \) to the acceleration of the mass center and the angular acceleration

\[ \sum \mathbf{M}_O = (\mathbf{r}_C \times m \mathbf{a}_C) + I_{Czz} \alpha, \]

or

\[ \sum \mathbf{M}_O + [\mathbf{r}_C \times (-m \mathbf{a}_C)] + (-I_{Czz} \alpha) = 0. \quad (4.30) \]

The term \( \mathbf{M}_{\text{in}} = -I_{Czz} \alpha \) is the \textit{inertia moment}. The sum of the moments about any point, including the moment due to the inertial force \(-m \mathbf{a}\) acting at mass center and the inertial moment, equals zero.

The equations of motion for a rigid body are analogous to the equations for static equilibrium:

The sum of the forces equals zero and the sum of the moments about any point equals zero when the inertial forces and moments are taken into account. This is called \textit{D’Alembert’s principle}.

The dynamic force analysis is expressed in a form similar to static force analysis

\[ \sum \mathbf{R} = \sum \mathbf{F} + \mathbf{F}_{\text{in}} = 0, \quad (4.31) \]
\[ \sum \mathbf{T}_C = \sum \mathbf{M}_C + \mathbf{M}_{\text{in}} = 0, \quad (4.32) \]

where \( \sum \mathbf{F} \) is the vector sum of all external forces (resultant of external force), and \( \sum \mathbf{M}_C \) is the sum of all external moments about the center of mass \( C \) (resultant external moment).

For a rigid body in plane motion in the \( xy \) plane,

\[ \mathbf{a}_C = \ddot{x}_C \mathbf{i} + \ddot{y}_C \mathbf{j}, \quad \alpha = \alpha \mathbf{k}, \]

with all external forces in that plane, Eqs. (4.31) and (4.32) become

\[ \sum R_x = \sum F_x + F_{\text{in}x} = \sum F_x + (-m \ddot{x}_C) = 0, \quad (4.33) \]
\[ \sum R_y = \sum F_y + F_{\text{in}y} = \sum F_y + (-m \ddot{y}_C) = 0, \quad (4.34) \]
\[ \sum T_C = \sum M_C + M_{\text{in}} = \sum M_C + (-I_C \alpha) = 0. \quad (4.35) \]
With d’Alembert’s principle the moment summation can be about any arbitrary point \( P \)

\[
\sum T_P = \sum M_P + M_{in} + r_{PC} \times F_{in} = 0, \tag{4.36}
\]

where
- \( \sum M_P \) is the sum of all external moments about \( P \),
- \( M_{in} \) is the inertia moment,
- \( F_{in} \) is the inertia force, and
- \( r_{PC} \) is a vector from \( P \) to \( C \).

The dynamic analysis problem is reduced to a static force and moment balance problem where the inertia forces and moments are treated in the same way as external forces and moments.
4.3 Free-Body Diagrams

A free-body diagram is a drawing of a part of a complete system, isolated in order to determine the forces acting on that rigid body.

The following force convention is defined: $F_{ij}$ represents the force exerted by link $i$ on link $j$.

Figure 4.4 shows various free-body diagrams that are considered in the analysis of a slider-crank mechanism Fig. 4.4(a).

In Fig. 4.4(b), the free body consists of the three moving links isolated from the frame $0$. The forces acting on the system include an external driven force $F$, and the forces transmitted from the frame at joint $A$, $F_{01}$, and at joint $C$, $F_{03}$. Figure 4.4(c) is a free-body diagram of the two links 1 and 2 and Fig. 4.4(d) is a free-body diagram of the two links 0 and 1. Figure 4.4(e) is a free-body diagram of crank 1 and Fig. 4.4(f) is a free-body diagram of slider 3.

The force analysis can be accomplished by examining individual links or a subsystem of links. In this way the joint forces between links as well as the required input force or moment for a given output load are computed.
4.4 Force Analysis Using Dyads

The inertia force \( \mathbf{F}_{inj} = F_{injx} \mathbf{i} + F_{injy} \mathbf{j} \) and the gravitational force \( \mathbf{G}_j = -m_j g \mathbf{j} \) act on link \( j \) at the center of mass \( C_j \), \( j = 2, 3 \). The inertia moment on link \( j \) is \( \mathbf{M}_{inj} = M_{injz} \mathbf{k} \).

**RRR dyad**

Figure 4.5(a) shows an RRR dyad with two links 2 and 3, and three pin joints, \( B, C, \) and \( D \). First, the exterior unknown joint reaction forces are considered:

\[
\mathbf{F}_{12} = F_{12x} \mathbf{i} + F_{12y} \mathbf{j} \quad \text{and} \quad \mathbf{F}_{43} = F_{43x} \mathbf{i} + F_{43y} \mathbf{j}.
\]

To determine \( \mathbf{F}_{12} \) and \( \mathbf{F}_{43} \), the following equations are written:

- sum of all forces on links 2 and 3 is zero

\[
\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_{in2} + \mathbf{G}_2 + \mathbf{F}_{in3} + \mathbf{G}_3 + \mathbf{F}_{43} = \mathbf{0},
\]

or

\[
\sum \mathbf{F}^{(2\&3)} \cdot \mathbf{i} = F_{12x} + F_{in2x} + F_{in3x} + F_{43x} = 0, \quad (4.37)
\]

\[
\sum \mathbf{F}^{(2\&3)} \cdot \mathbf{j} = F_{12y} + F_{in2y} - m_2 g + F_{in3y} - m_3 g + F_{43y} = 0. \quad (4.38)
\]

- sum of moments of all forces and moments on link 2 about \( C \) is zero

\[
\sum \mathbf{M}^{(2)}_C = (\mathbf{r}_B - \mathbf{r}_C) \times \mathbf{F}_{12} + (\mathbf{r}_C - \mathbf{r}_C) \times (\mathbf{F}_{in2} + \mathbf{G}_2) + \mathbf{M}_{in2} = \mathbf{0}. \quad (4.39)
\]

- sum of moments of all forces and moments on link 3 about \( C \) is zero

\[
\sum \mathbf{M}^{(3)}_C = (\mathbf{r}_D - \mathbf{r}_C) \times \mathbf{F}_{43} + (\mathbf{r}_C - \mathbf{r}_C) \times (\mathbf{F}_{in3} + \mathbf{G}_3) + \mathbf{M}_{in3} = \mathbf{0}. \quad (4.40)
\]

The components \( F_{12x}, F_{12y}, F_{43x}, \) and \( F_{43y} \) are calculated from Eqs. (4.37), (4.38), (4.39), and (4.40).

The reaction force \( \mathbf{F}_{32} = -\mathbf{F}_{23} \) is computed from the sum of all forces on link 2

\[
\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_{in2} + \mathbf{G}_2 + \mathbf{F}_{32} = \mathbf{0} \quad \text{or} \quad \mathbf{F}_{32} = -\mathbf{F}_{12} - \mathbf{F}_{in2} - \mathbf{G}_2.
\]
RRT dyad

Figure 4.5(b) shows an RRT dyad with the unknown joint reaction forces \( F_{12}, F_{43}, \) and \( F_{23} = -F_{32} \). The joint reaction force \( F_{43} \) is perpendicular to the sliding direction \( F_{43} \perp \Delta \) or

\[
F_{43} \cdot \Delta = (F_{43x} \hat{\textbf{i}} + F_{43y} \hat{\textbf{j}}) \cdot (\cos \theta \hat{\textbf{i}} + \sin \theta \hat{\textbf{j}}) = 0. 
\] (4.41)

In order to determine \( F_{12} \) and \( F_{43} \) the following equations are written

- sum of all the forces on links 2 and 3 is zero

\[
\sum F^{(2&3)} = F_{12} + F_{in\,2} + G_2 + F_{in\,3} + G_3 + F_{43} = 0,
\]

or

\[
\sum F^{(2&3)} \cdot \hat{\textbf{i}} = F_{12x} + F_{in\,2x} + F_{in\,3x} + F_{43x} = 0, \quad (4.42)
\]

\[
\sum F^{(2&3)} \cdot \hat{\textbf{j}} = F_{12y} + F_{in\,2y} - m_2 g + F_{in\,3y} - m_3 g + F_{43y} = 0. \quad (4.43)
\]

- sum of moments of all the forces and the moments on link 2 about \( C \) is zero

\[
\sum M^{(2)}_C = (r_B - r_C) \times F_{12} + (r_{C_2} - r_C) \times (F_{in\,2} + G_2) + M_{in\,2} = 0. \quad (4.44)
\]

The components \( F_{12x}, F_{12y}, F_{43x}, \) and \( F_{43y} \) are calculated from Eqs. (4.41), (4.42), (4.43), and (4.44).

The reaction force components \( F_{32x} \) and \( F_{32y} \) are computed from the sum of all the forces on link 2

\[
\sum F^{(2)} = F_{12} + F_{in\,2} + G_2 + F_{32} = 0 \quad \text{or} \quad F_{32} = -F_{12} - F_{in\,2} - G_2.
\]

RTR dyad

The unknown joint reaction forces \( F_{12} \) and \( F_{43} \) are calculated from the relations [Fig. 4.5(c)]

- sum of all the forces on links 2 and 3 is zero

\[
\sum F^{(2&3)} = F_{12} + F_{in\,2} + G_2 + F_{in\,3} + G_3 + F_{43} = 0,
\]

or

\[
\sum F^{(2&3)} \cdot \hat{\textbf{i}} = F_{12x} + F_{in\,2x} + F_{in\,3x} + F_{43x} = 0, \quad (4.45)
\]

\[
\sum F^{(2&3)} \cdot \hat{\textbf{j}} = F_{12y} + F_{in\,2y} - m_2 g + F_{in\,3y} - m_3 g + F_{43y} = 0. \quad (4.46)
\]
• sum of the moments of all the forces and moments on links 2 and 3 about $B$ is zero

$$
\sum M_{B}^{(2k3)} = (r_D - r_B) \times F_{43} + (r_{C3} - r_B) \times (F_{in3} + G_3) + M_{in3}
+ (r_{C2} - r_B) \times (F_{in2} + G_2) + M_{in2} = 0.
$$

(4.47)

• sum of all the forces on link 2 projected onto the sliding direction $\Delta = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}$ is zero

$$
\sum F^{(2)} \cdot \Delta = (F_{12} + F_2) \cdot (\cos \theta \mathbf{i} + \sin \theta \mathbf{j}) = 0.
$$

(4.48)

The components $F_{12x}, F_{12y}, F_{43x}, \text{and } F_{43y}$ are calculated from Eqs. (4.45), (4.46), (4.47), and (4.48).

The force components $F_{32x}$ and $F_{32y}$ are computed from the sum of all the forces on link 2

$$
\sum F^{(2)} = F_{12} + F_{in2} + G_2 + F_{32} = 0 \text{ or } F_{32} = -(F_{12} + F_{in2} + G_2).
$$
4.5 Force Analysis Using Contour Method

An analytical method to compute joint forces that can be applied for both planar and spatial mechanisms will be presented. The method is based on the decoupling of a closed kinematic chain and writing the dynamic equilibrium equations. The kinematic links are loaded with external forces and inertia forces and moments.

A general monocontour closed kinematic chain is considered in Fig. 4.6. The joint force between the links \( i - 1 \) and \( i \) (joint \( A_i \)) will be determined. When these two links \( i - 1 \) and \( i \) are separated the joint forces \( F_{i-1,i} \) and \( F_{i,i-1} \) are introduced and

\[
F_{i-1,i} + F_{i,i-1} = 0. \tag{4.49}
\]

Table 4.1 shows the joint forces for one degree of freedom joints. It is helpful to “mentally disconnect” the two links \((i - 1)\) and \(i\), which create joint \( A_i \), from the rest of the mechanism. The joint at \( A_i \) will be replaced by the joint forces \( F_{i-1,i} \) and \( F_{i,i-1} \). The closed kinematic chain has been transformed into two open kinematic chains, and two paths \( I \) and \( II \) are associated. The two paths start from \( A_i \).

For the path \( I \) (counterclockwise), starting at \( A_i \) and following \( I \) the first joint encountered is \( A_{i-1} \). For the link \( i - 1 \) left behind, dynamic equilibrium equations are written according to the type of the joint at \( A_{i-1} \). Following the same path \( I \), the next joint encountered is \( A_{i-2} \). For the subsystem \((i - 1 \) and \( i - 2)\) equilibrium conditions corresponding to the type of joint at \( A_{i-2} \) can be specified, and so on. A similar analysis is performed for the path \( II \) of the open kinematic chain. The number of equilibrium equations written is equal to the number of unknown scalars introduced by joint \( A_i \) (joint forces at this joint). For a joint, the number of equilibrium conditions is equal to the number of relative mobilities of the joint.
4.6 R-RRT (slider-crank) Mechanism

Figure 4.7(a) is a schematic diagram of a R-RRT (slider-crank) mechanism comprised of a crank 1, a connecting rod 2, and a slider 3. The mechanism shown in the figure has the dimensions: $AB = 1$ m and $BC = 1$ m. The driver link 1 makes an angle $\phi = \phi_1 = \pi/4$ rad with the horizontal axis and rotates with a constant speed of $n = 30/\pi$ rpm. The point $A$ is selected as the origin of the $xyz$ reference frame. The position vectors of the joints $B$ and $C$ are

$$\mathbf{r}_B = x_B \mathbf{i} + y_B \mathbf{j} = \frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j} \text{ m and } \mathbf{r}_C = x_C \mathbf{i} + y_C \mathbf{j} = \sqrt{2} \mathbf{i} + 0 \mathbf{j} \text{ m.}$$

The angular velocities of links 1 and 2 are

$$\omega_1 = \omega_1 \mathbf{k} = 1 \mathbf{k} \text{ rad/s and } \omega_2 = \omega_2 \mathbf{k} = -1 \mathbf{k} \text{ rad/s.}$$

The angular accelerations of link 1 and 2 are $\alpha_1$ and $\alpha_2$. For this particular configuration of the mechanism $\alpha_1 = \alpha_2 = 0$.

The velocity and acceleration of $B$ are

$$\mathbf{v}_B = -\frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j} \text{ m/s and } \mathbf{a}_B = -\frac{\sqrt{2}}{2} \mathbf{i} - \frac{\sqrt{2}}{2} \mathbf{j} \text{ m/s}^2.$$

The velocity and acceleration of $C$ are

$$\mathbf{v}_C = -\sqrt{2} \mathbf{i} \text{ m/s and } \mathbf{a}_C = -\sqrt{2} \mathbf{i} \text{ m/s}^2.$$

The center of mass of link 1 is $C_1$, the center of mass of link 2 is $C_2$, and the center of mass of slider 3 is $C$. The position vectors of the $C_i$, $i = 1, 2, 3$ are

$$\mathbf{r}_{C_1} = \mathbf{r}_B / 2 = x_{C_1} \mathbf{i} + y_{C_1} \mathbf{j} = \frac{\sqrt{2}}{4} \mathbf{i} + \frac{\sqrt{2}}{4} \mathbf{j} \text{ m},$$

$$\mathbf{r}_{C_2} = (\mathbf{r}_B + \mathbf{r}_C) / 2 = x_{C_2} \mathbf{i} + y_{C_2} \mathbf{j} = \frac{3\sqrt{2}}{4} \mathbf{i} + \frac{\sqrt{2}}{4} \mathbf{j} \text{ m},$$

$$\mathbf{r}_{C_3} = \mathbf{r}_C = x_{C_3} \mathbf{i} + y_{C_3} \mathbf{j} = \sqrt{2} \mathbf{i} \text{ m.}$$

The acceleration vectors of the $C_i$, $i = 1, 2, 3$ are

$$\mathbf{a}_{C_1} = \mathbf{a}_B / 2 = a_{C_{1x}} \mathbf{i} + a_{C_{1y}} \mathbf{j} = -\frac{\sqrt{2}}{4} \mathbf{i} - \frac{\sqrt{2}}{4} \mathbf{j} \text{ m/s}^2,$$

$$\mathbf{a}_{C_2} = (\mathbf{a}_B + \mathbf{a}_C) / 2 = a_{C_{2x}} \mathbf{i} + a_{C_{2y}} \mathbf{j} = -\frac{3\sqrt{2}}{4} \mathbf{i} - \frac{\sqrt{2}}{4} \mathbf{j} \text{ m/s}^2,$$

$$\mathbf{a}_{C_3} = \mathbf{a}_C = a_{C_{3x}} \mathbf{i} + a_{C_{3y}} \mathbf{j} = -\sqrt{2} \mathbf{i} \text{ m/s}^2.$$
The MATLAB commands for the kinematics of the mechanism (positions, velocities, and accelerations) are

\[
\begin{align*}
AB &= 1; \ BC = 1; \ \phi = 45^\circ; \\
xA &= 0; \ yA = 0; \ rA = [xA \ yA \ 0]; \\
xB &= AB \cos(\phi); \ yB = AB \sin(\phi); \ rB = [xB \ yB \ 0]; \\
yC &= 0; \ xC = xB + \sqrt{BC^2 - (yC - yB)^2}; \ rC = [xC \ yC \ 0]; \\
n &= 30/\pi; \ \omega_{1} = [0 \ 0 \ pi*n/30]; \ \alpha_{1} = [0 \ 0 \ 0]; \\
vA &= [0 \ 0 \ 0]; \ aA = [0 \ 0 \ 0]; \\
vB1 &= vA + \cross(\omega_1, rB); \ vB2 = vB1; \\
aB1 &= aA + \cross(\alpha_1, rB) - \dot{\cross}(\omega_{1}, \omega_{1}) \cdot rB; \ aB2 = aB1; \\
\omega_{2z} &= \sym(\omega_{2z}, 'real'); \ vCx = \sym(vCx, 'real'); \\
\omega_{2} &= [0 \ 0 \ \omega_{2z}]; \ vC = [vCx \ 0 \ 0]; \\
eqvC &= vC - (vB2 + \cross(\omega_{2}, rC - rB)); \\
eqvCx &= \equivvC(1); \ eqvCy = \equivvC(2); \\
solvC &= \solve(eqvCx, eqvCy); \ \omega_{2zs} = \eval(solvC.omega2z); \\
vCx = \eval(solvC.vCx); \ \Omega_{2} = [0 \ 0 \ \omega_{2zs}]; \\
vCs &= [vCx \ 0 \ 0]; \\
alpha_{2z} &= \sym(\alpha_{2z}, 'real'); \ aCx = \sym(aCx, 'real'); \\
alpha_{2} &= [0 \ 0 \ \alpha_{2z}]; \ aC = [aCx \ 0 \ 0]; \\
eqaC &= aC - (aB1 + \cross(\alpha_{2}, rC - rB) - \dot{\cross}(\Omega_{2}, \Omega_{2}) \cdot (rC - rB)); \\
eqaCx &= \eqaC(1); \ eqaCy = \eqaC(2); \\
solaC &= \solve(eqaCx, eqaCy); \\
alpha_{20z} &= \eval(solaC.alpha2z); \ aCx = \eval(solaC.aCx); \\
alpha_{20} &= [0 \ 0 \ \alpha_{20z}]; \ aCs = [aCx \ 0 \ 0]; \ \alpha_{30} = [0 \ 0 \ 0]; \\
rC1 &= (rA + rB)/2; \ \text{fprintf}('rC1 = [ \%g, \%g, \%g ] (m) \n', rC1); \\
rC2 &= (rB + rC)/2; \ \text{fprintf}('rC2 = [ \%g, \%g, \%g ] (m) \n', rC2); \\
rC3 &= rC; \ \text{fprintf}('rC3 = [ \%g, \%g, \%g ] (m) \n', rC3); \\
aC1 &= aB1/2; \ \text{fprintf}('aC1 = [ \%g, \%g, \%g ] (m/s^2) \n', aC1); \\
aC2 &= (aB1 + aCs)/2; \ \text{fprintf}('aC2 = [ \%g, \%g, \%g ] (m/s^2) \n', aC2); \\
aC3 &= aCs; \ \text{fprintf}('aC3 = [ \%g, \%g, \%g ] (m/s^2) \n', aC3); \\
\end{align*}
\]

The external driven force \( F_{ext} \) applied on link 3 is opposed to the motion of the link (opposed to \( v_C \)). Because \( v_C = -\sqrt{2} \text{ m/s} \), the external force vector will be

\[
F_{ext} = [-\text{Sign}(v_C)] 100 \text{ i } 100 \text{ i N}.
\]
The MATLAB commands for the external force on link 3 are

\[
\begin{align*}
fe &= 100; \\
Fe &= -\text{sign}(vCs(1)) \times [fe \ 0 \ 0];
\end{align*}
\]

The signum function in MATLAB is \( \text{sign}(x) \). If \( x \) is greater than zero \( \text{sign}(x) \) returns 1, if \( x \) is zero \( \text{sign}(x) \) returns zero, and if if \( x \) is less than zero \( \text{sign}(x) \) returns -1.

The height of the links 1 and 2 is \( h = 0.01 \) m. The width of the links 3 is \( w_{\text{Slider}} = 0.01 \) m and the height is \( h_{\text{Slider}} = 0.01 \) m [Fig. 4.7(b)]. All three moving links are rectangular prisms with the depth \( d = 0.001 \) m. The acceleration of gravity is \( g = 10 \) m/s\(^2\).

\[
\begin{align*}
h &= 0.01; \ d &= 0.001; \ h_{\text{Slider}} &= 0.01; \ w_{\text{Slider}} &= 0.01; \ g &= 10.;
\end{align*}
\]

**Inertia forces and moments**

*Link 1*

The mass of the crank 1 is

\[
m_1 = \rho_1 \ AB \ h \ d,
\]

where the density of the material is \( \rho_1 \). For the simplicity of calculations \( m_1 = 1 \) kg.

The inertia force on link 1 at \( C_1 \) is

\[
F_{in1} = -m_1 a_{C_1} = \frac{\sqrt{2}}{4} \mathbf{i} + \frac{\sqrt{2}}{4} \mathbf{j} \ N.
\]

The gravitational force on crank 1 at \( C_1 \) is

\[
G_1 = -m_1 \ g \ \mathbf{j} = -10 \ \mathbf{j} \ N.
\]

The total force on link 1 at the mass center \( C_1 \) is

\[
F_1 = F_{in1} + G_1 = \frac{\sqrt{2}}{4} \mathbf{i} + (\frac{\sqrt{2}}{4} - 10) \mathbf{j} \ N.
\]

The mass moment of inertia of the link 1 is

\[
I_{C_1} = m_1 (AB^2 + h^2)/12 = 0.0833417 \ kg \cdot m^2.
\]
The moment of inertia on link 1 is

\[ M_{in1} = -I_{C1} \alpha_1 = 0. \]

**Link 2**

The mass of connecting rod 2 is

\[ m_2 = \rho_2 BC \ h \ d, \]

where the density of the material of link 2 is \( \rho_2 \). For the simplicity of calculations \( m_2 = 1 \) kg.

The inertia force on link 2 at \( C_2 \) is

\[ F_{in2} = -m_2 a_{C2} = \frac{3\sqrt{2}}{4} \mathbf{i} + \frac{\sqrt{2}}{4} \mathbf{j} \text{ N}. \]

The gravitational force on link 2 at \( C_2 \) is

\[ G_2 = -m_2 g \mathbf{j} = -10 \mathbf{j} \text{ N}. \]

The total force on link 2 at the mass center \( C_2 \) is

\[ F_2 = F_{in2} + G_2 = \frac{3\sqrt{2}}{4} \mathbf{i} + (\frac{\sqrt{2}}{4} - 10) \mathbf{j} \text{ N}. \]

The mass moment of inertia of link 2 is

\[ I_{C2} = m_2 (BC^2 + h^2)/12 = 0.0833417 \text{ kg} \cdot \text{m}^2. \]

The moment of inertia on link 2 is

\[ M_{in2} = -I_{C2} \alpha_2 = 0. \]

**Link 3**

The mass of the link 3 is

\[ m_3 = \rho_3 \ h_{Slider} \ w_{Slider} \ d, \]

where the density of the material of link 3 is \( \rho_3 \). For the simplicity of calculations \( m_3 = 1 \) kg.

The inertia force on link 3 at \( C_3 = C \) is

\[ F_{in3} = -m_3 a_{C3} = \sqrt{2} \mathbf{i} \text{ N}. \]
The gravitational force on link 3 at $C_3 = C$ is

$$G_3 = -m_3 \ g \ \mathbf{j} = -10 \ \mathbf{j} \ N.$$  

The total force on link 3 at the mass center $C$ is

$$F_3 = F_{in3} + G_3 = \sqrt{2} \ \mathbf{1} - 10 \ \mathbf{j} \ N.$$  

The mass moment of inertia of slider 3 is

$$I_{C_3} = m_3 \ (h_{Slider}^2 + w_{Slider}^2) / 12 = 0.000166667 \ \text{kg} \cdot \text{m}^2.$$  

The moment of inertia on slider 3 is

$$M_{in3} = -I_{C_3} \ \mathbf{a}_3 = 0.$$  

The MATLAB commands for the forces and moments of inertia are

```matlab
m1 = 1;
IC1 = m1*(AB^2+h^2)/12;
G1 = [ 0 -m1*g 0 ];
Fin1 = - m1*aC1;
Min1 = - IC1*alpha1;
m2 = 1;
IC2 = m2*(BC^2+h^2)/12;
G2 = [ 0 -m2*g 0 ];
Fin2 = - m2*aC2;
Min2 = - IC2*alpha20;
m3 = 1;
IC3 = m3*(hSlider^2+wSlider^2)/12;
G3 = [ 0 -m3*g 0 ];
Fin3 = - m3*aC3;
Min3 = - IC3*alpha30;
```

For a given value of the crank angle $\phi$ ($\phi = \pi/4$) and a known driven force $F_{ext}$ find the joint reactions and the drive moment $M$ on the crank.
Joint forces and drive moment

4.6.1 Newton-Euler Equations of Motion

Figure 4.7(b) shows the free-body diagrams of the crank 1, the connecting rod 2, and the slider 3. For each moving link the dynamic equilibrium equations are applied (Newton-Euler equations of motion)

\[ m \mathbf{a}_C = \sum \mathbf{F} \quad \text{and} \quad I_C \mathbf{\alpha} = \sum \mathbf{M}_C, \]

where \( C \) is the center of mass of the link.

The force analysis starts with the link 3 because the external driven force \( \mathbf{F}_{ext} \) on the slider is given.

The reaction joint force of the ground 0 on the slider 3, \( \mathbf{F}_{03} \), is perpendicular on the sliding direction, \( x \)-axis: \( \mathbf{F}_{03} \perp \mathbf{i} \) (Figure 4.8). The application point \( Q \) of the reaction force \( \mathbf{F}_{03} \) is determined using Euler moment equation

\[ I_{C_3} \mathbf{\alpha}_3 = \mathbf{r}_{CQ} \times \mathbf{F}_{03} \quad \text{or} \quad \mathbf{0} = \mathbf{r}_{CQ} \times \mathbf{F}_{03} \quad \implies \quad \mathbf{r}_{CQ} = \mathbf{0} \quad \text{or} \quad C = Q. \]

It results the reaction force \( \mathbf{F}_{03} \) acts at \( C \).

For the slider 3 the vector sum of the net forces (external forces \( \mathbf{F}_{ext} \), gravitational force \( \mathbf{g}_3 \), joint forces \( \mathbf{F}_{23}, \mathbf{F}_{03} \)) is equal to \( m_3 \mathbf{a}_{C_3} \) (Fig. 4.8)

\[ m_3 \mathbf{a}_{C_3} = \mathbf{F}_{23} + \mathbf{g}_3 + \mathbf{F}_{ext} + \mathbf{F}_{03}, \]

where \( \mathbf{F}_{23} = F_{23x}\mathbf{i} + F_{23y}\mathbf{j} \) and \( \mathbf{F}_{03} = F_{03x}\mathbf{i} + F_{03y}\mathbf{j} \).

Projecting this vectorial equation onto \( x \) and \( y \) axes gives

\[ m_3 a_{C_3x} = F_{23x} + F_{ext}, \]

\[ m_3 a_{C_3y} = F_{23y} - m_3 g + F_{03y}, \]

or numerically

\[ (1)(-\sqrt{2}) = F_{23x} + 100, \quad (4.50) \]

\[ 0 = F_{23y} - (1)(10) + F_{03y}, \quad (4.51) \]

There are two equations Eqs. (4.50) and (4.51) and three unknowns \( F_{03y}, F_{23x} \) and \( F_{23y} \) and that is why the analysis will continue with link 2.

The MATLAB commands for Newton-Euler equations for slider 3 are
Dynamic Force Analysis with Matlab

\[ F_{03} = \begin{bmatrix} 0 & \text{sym('F03y','real')} & 0 \end{bmatrix} \; ; \]
\[ F_{23} = \begin{bmatrix} \text{sym('F23x','real')} & \text{sym('F23y','real')} & 0 \end{bmatrix} \; ; \]
\[ \text{eqF3} = F_{03} + F_{23} + F_e + G_3 - m_3 a_{C3} ; \]
\[ \text{eqF3x} = \text{eqF3}(1) ; \]
\[ \text{eqF3y} = \text{eqF3}(2) ; \]

For the connecting rod 2 (Fig. 4.9), Newton equation gives

\[ m_2 a_{C2} = F_{32} + G_2 + F_{12} . \]

The previous equation can be projected on \( x \) and \( y \) axes

\[ m_2 a_{C2x} = F_{32x} + F_{12x} , \]
\[ m_2 a_{C2y} = F_{32y} - m_2 g + F_{12y} , \]

or numerically

\[ (1)(-\frac{3\sqrt{2}}{4}) = F_{32x} + F_{12x} , \]  \hspace{1cm} (4.52)
\[ (1)(-\frac{\sqrt{2}}{4}) = F_{32y} - 1(10) + F_{12y} . \]  \hspace{1cm} (4.53)

For the link 2 a moment equation can be written with respect to \( C_2 \)

\[ I_{C2} \alpha_2 = r_{C2C} \times F_{32} + r_{C2B} \times F_{12} , \]

or

\[ I_{C2} \alpha_2 \mathbf{k} = \begin{vmatrix} 1 & \mathbf{J} & \mathbf{k} \\ x_C - x_{C2} & y_C - y_{C2} & 0 \\ F_{32x} & F_{32y} & 0 \end{vmatrix} + \begin{vmatrix} 1 & \mathbf{J} & \mathbf{k} \\ x_B - x_{C2} & y_B - y_{C2} & 0 \\ F_{12x} & F_{12y} & 0 \end{vmatrix} \]

or

\[ I_{C2} \alpha_2 = (x_C - x_{C2})F_{32y} - (y_C - y_{C2})F_{32x} + (x_B - x_{C2})F_{12y} - (y_B - y_{C2})F_{12x} , \]

or numerically

\[ 0 = (\sqrt{2} - \frac{3\sqrt{2}}{4})F_{32y} - (-\frac{\sqrt{2}}{4})F_{32x} + (\frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{4})F_{12y} - (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4})F_{12x} . \]  \hspace{1cm} (4.54)

The Matlab commands for Newton-Euler equations for link 2 are
\[ F_{32} = -F_{23}; \]
\[ F_{12} = [ \text{sym}(F_{12x}, 'real') \text{sym}(F_{12y}, 'real') \ 0 ]; \]
\[ eqF2 = F_{32} + F_{12} + G_{2} - m_{2} \alpha_{20}; \]
\[ eqF2x = eqF2(1); \]
\[ eqF2y = eqF2(2); \]
\[ eqM2 = \text{cross}(r_{B} - r_{C2}, F_{12}) + \text{cross}(r_{C} - r_{C2}, F_{32}) - IC_{2} \alpha_{20}; \]
\[ eqM2z = eqM2(3); \]

Equations (4.50), (4.51), (4.52), (4.53), and (4.54) form a system of 5 equations with five scalar unknowns. The system can be solved using the \texttt{solve} statement.

\[
\text{sol32} = \text{solve}(eqF3x, eqF3y, eqF2x, eqF2y, eqM2z); \\
F_{03y} = \text{eval(sol32.F03y);} \\
F_{23x} = \text{eval(sol32.F23x);} \\
F_{23y} = \text{eval(sol32.F23y);} \\
F_{12x} = \text{eval(sol32.F12x);} \\
F_{12y} = \text{eval(sol32.F12y);} \\
F_{03} = [ \ 0, F_{03y}, \ 0 ]; \\
F_{23} = [ F_{23x}, F_{23y}, \ 0 ]; \\
F_{12} = [ F_{12x}, F_{12y}, \ 0 ];
\]

The numerical values for the joint forces for the links 3 and 2 are

\[
F_{03y} = -85 - \frac{3\sqrt{2}}{2} \text{ N}, \\
F_{23x} = -100 - \sqrt{2} \text{ N}, \quad F_{23y} = 95 + \frac{3\sqrt{2}}{2} \text{ N}, \\
F_{12x} = -\frac{1}{4}(400 + 7\sqrt{2}) \text{ N}, \quad F_{12y} = \frac{5}{4}(84 + \sqrt{2}) \text{ N},
\]

or

\[
F_{03} = |F_{03}| = 85 + \frac{3\sqrt{2}}{2} \text{ N}, \]
\[
F_{23} = |F_{23}| = \sqrt{F_{23x}^2 + F_{23y}^2} = \sqrt{\frac{38063}{2} + 485\sqrt{2}} \text{ N}, \]
\[
F_{12} = |F_{12}| = \sqrt{F_{12x}^2 + F_{12y}^2} = \frac{1}{2}\sqrt{84137 + 2450\sqrt{2}} \text{ N}.
\]
For the crank 1 [Fig. 4.4], there are two vectorial equations

\[ m_1 a_{C_1} = F_{21} + G_1 + F_{01}, \]

\[ I_{C_1} \alpha_1 = r_{C_1B} \times F_{21} + r_{C_1A} \times F_{01} + M, \]

where \( M \) is the input (motor) moment on the crank, \( F_{21} = -F_{12} \), and \( F_{01} = F_{01x}I + F_{01y}J \).

The above vectorial equations give three scalar equations on \( x, y, \) and \( z \)

\[ m_1 a_{C_1x} = F_{21x} + F_{01x}, \]

\[ m_1 a_{C_1y} = F_{21y} - m_1 g + F_{01y}, \]

\[ I_{C_1} \alpha_1 k = \left[ \begin{array}{ccc} 1 & J & k \\ x_B - x_{C_1} & y_B - y_{C_1} & 0 \\ F_{21x} & F_{21y} & 0 \end{array} \right] + \left[ \begin{array}{ccc} 1 & J & k \\ x_A - x_{C_1} & y_A - y_{C_1} & 0 \\ F_{01x} & F_{01y} & 0 \end{array} \right] + M \]

\[ + M k = 0, \]

or

\[ m_1 a_{C_1x} = F_{21x} + F_{01x}, \]

\[ m_1 a_{C_1y} = F_{21y} - m_1 g + F_{01y}, \]

\[ I_{C_1} \alpha_1 = (x_B - x_{C_1}) F_{21y} - (y_{C_2} - y_{C_1}) F_{21x} + (x_A - x_{C_1}) F_{01y} - (y_A - y_{C_1}) F_{01x} + M; \]

or numerically

\[ 1\left(-\frac{\sqrt{2}}{4}\right) = \frac{1}{4}(400 + 7\sqrt{2}) + F_{01x}, \quad (4.55) \]

\[ 1\left(-\frac{\sqrt{2}}{4}\right) = -\frac{5}{4}(84 + \sqrt{2}) - 1(10) + F_{01y}, \quad (4.56) \]

\[ 0 = \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4}\right) \left[-\frac{5}{4}(84 + \sqrt{2})\right] - \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4}\right) \left[\frac{1}{4}(400 + 7\sqrt{2})\right] \]

\[ -\frac{\sqrt{2}}{4} F_{01y} + \frac{\sqrt{2}}{4} F_{01y} + M = 0. \quad (4.57) \]

The MATLAB commands for Newton-Euler equations for the crank 1 are

\[ F01=\text{m1*aC1-G1+F12s}; \]
\[ \text{Mm}=-\text{cross(rB,-F12s)}-\text{cross(rC1,G1-m1*aC1)}-\text{IC1*alpha1}; \]
Equations (4.55), (4.56) and (4.57) give

\[ F_{01x} = -2(50 + \sqrt{2}) \text{ N}, \quad F_{01y} = 115 + \sqrt{2} \text{ N}, \]
\[ M = 3 + 105\sqrt{2} \text{ N} \cdot \text{m}, \]

or

\[ F_{01} = |F_{01}| = \sqrt{F_{01x}^2 + F_{01y}^2} = \sqrt{23235 + 630\sqrt{2}} \text{ N}, \]
\[ M = |M| = 3 + 105\sqrt{2} \text{ N} \cdot \text{m}. \]

Another way of calculating the moment \( M \) required for dynamic equilibrium is to write the moment equation of motion for link 1 about the fixed point \( A \)

\[ I_A \alpha_1 \mathbf{k} = \mathbf{r}_{AC_1} \times \mathbf{G}_1 + \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{M} \implies \mathbf{M} = \mathbf{r}_B \times \mathbf{F}_{12} - \mathbf{r}_{C_1} \times \mathbf{G}_1, \]

where \( I_A = I_{C_1} + m_1 (AB/2)^2 \). The reaction force \( F_{01} \) does not appear in this moment equation.

The MATLAB program using Newton-Euler equations and the results are given in Program 4.1.
4.6.2 D’Alembert’s Principle

For each moving link the dynamic equilibrium equations are applied (d’Alembert’s principle)

\[ \sum \mathbf{F} + \mathbf{F}_{in} = 0 \quad \text{and} \quad \sum \mathbf{M}_C + \mathbf{M}_{in} = 0, \]

where \( C \) is the center of mass of the link. With d’Alembert’s principle the moment summation can be about any arbitrary point \( P \)

\[ \sum \mathbf{M}_P + \mathbf{M}_{in} + \mathbf{r}_{PC} \times \mathbf{F}_{in} = 0. \]

The force analysis starts with the link 3 because the external driven force \( \mathbf{F}_{ext} \) is given. For the slider 3 the vector sum of all the forces (external forces \( \mathbf{F}_{ext} \), gravitational force \( \mathbf{G}_3 \), inertia forces \( \mathbf{F}_{in3} \), joint forces \( \mathbf{F}_{23}, \mathbf{F}_{03} \)) is zero (Fig. 4.8)

\[ \sum \mathbf{F}(3) = \mathbf{F}_{23} + \mathbf{F}_{in3} + \mathbf{G}_3 + \mathbf{F}_{ext} + \mathbf{F}_{03} = 0, \]

where \( \mathbf{F}_{23} = F_{23x}\mathbf{i} + F_{23y}\mathbf{j} \) and \( \mathbf{F}_{03} = F_{03y}\mathbf{j} \).

Projecting this force onto \( x \) and \( y \) axes gives

\[ \sum \mathbf{F}(3) \cdot \mathbf{i} = F_{23x} + (-m_3 a_{C3x}) + F_{ext} = 0, \]

\[ \sum \mathbf{F}(3) \cdot \mathbf{j} = F_{23y} - m_3 g + F_{03y} = 0, \]

or numerically

\[ F_{23x} + (-1)(-\sqrt{2}) + 100 = 0, \quad (4.58) \]

\[ F_{23y} - (1)(10) + F_{03y} = 0. \quad (4.59) \]

The MATLAB commands for slider 3 are

\[ \begin{align*}
F03 &= [0 \text{ sym}('F03y','\text{real}') \ 0]; \\
F23 &= [\text{sym}('F23x','\text{real}') \ \text{sym}('F23y','\text{real}') \ 0]; \\
eq_{F3} &= F03+F23+Fe+G3+Fin3;
\end{align*} \]

For the connecting rod 2 (Fig. 4.9), the sum of the forces is equal to zero

\[ \sum \mathbf{F}^{(2)} = \mathbf{F}_{32} + \mathbf{F}_{in2} + \mathbf{G}_2 + \mathbf{F}_{12} = 0, \]
The previous equation can be projected on $x$ and $y$ axes

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{i} = F_{32x} + (-m_2 a_{C2x}) + F_{12x} = 0,$$

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{j} = F_{32y} + (-m_2 a_{C2y}) - m_2 g + F_{12y} = 0,$$

or numerically

$$F_{32x} + (-1)(-\frac{3\sqrt{2}}{4}) + F_{12x} = 0, \quad (4.60)$$

$$F_{32y} + (-1)(-\frac{\sqrt{2}}{4}) - 1(10) + F_{12y} = 0. \quad (4.61)$$

For the link 2 a moment equation can be written with respect to $C_2$

$$\sum \mathbf{M}^{(2)}_{C_2} = r_{C2C} \times \mathbf{F}_{32} + r_{C2B} \times \mathbf{F}_{12} + \mathbf{M}_{in2} = 0.$$

Instead of the previous one can use the sum of the moments with respect to $B$

$$\sum \mathbf{M}^{(2)}_B = r_{BC} \times \mathbf{F}_{32} + r_{BC_2} \times (\mathbf{F}_{in2} + \mathbf{G}_2) + \mathbf{M}_{in2} = 0,$$

or

$$\begin{vmatrix} 1 & \mathbf{J} & \mathbf{k} \\ x_C - x_B & y_C - y_B & 0 \\ F_{32x} & F_{32y} & 0 \end{vmatrix} + \begin{vmatrix} 1 & \mathbf{J} & \mathbf{k} \\ x_{C_2} - x_B & y_{C_2} - y_B & 0 \\ -m_2 a_{C2x} & -m_2 a_{C2y} - m_2 g & 0 \end{vmatrix}$$

$$-I_{C_2} a_2 \mathbf{k} = 0.$$

or

$$(x_C - x_B)F_{32y} - (y_C - y_B)F_{32x} + (x_{C_2} - x_B)(-m_2 a_{C2y} - m_2 g)$$

$$- (y_{C_2} - y_B)(-m_2 a_{C2x}) - I_{C_2} a_2 = 0,$$

or numerically

$$\begin{align*}
(\sqrt{2} - \frac{\sqrt{2}}{2})F_{32y} - (-\frac{\sqrt{2}}{2})F_{32x} + (3\sqrt{2} - \frac{\sqrt{2}}{2}) \left[ -1(-\frac{\sqrt{2}}{4}) - 1(10) \right] \\
- (\frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{2}) \left[ -1(-\frac{3\sqrt{2}}{4}) \right] - 0 = 0.
\end{align*} \quad (4.62)$$
For the connecting rod 2 the MATLAB commands are

\[
\begin{align*}
F32 &= -F23; \\
F12 &= \text{[ sym('F12x','real') sym('F12y','real') 0 ]}; \\
eq F2 &= F32+F12+G2+\text{Fin2}; \\
eq F2x &= eqF2(1); \\
eq F2y &= eqF2(2); \\
eq M2 &= \text{cross(rB-rC2,F12)+cross(rC-rC2,F32)+Min2}; \\
eq M2z &= eqM2(3);
\end{align*}
\]

For the crank 1 (Fig. 4.10), there are two vectorial equations

\[
\sum F^{(1)} = F_{21} + F_{in1} + G_1 + F_{01} = 0, \\
\sum M^{(1)}_A = r_B \times F_{21} + r_C \times (F_{in1} + G_1) + M_{in1} + M = 0,
\]

where \( M = |M| \) is the magnitude of the input moment on the crank, \( F_{21} = -F_{12} \), and \( F_{01} = F_{01x} + F_{01y} j \).

The above vectorial equations give three scalar equations on \( x, y, \) and \( z \)

\[
\sum F^{(1)} \cdot i = F_{21x} + (-m_1 a_{C_{1x}}) + F_{01x} = 0, \\
\sum F^{(1)} \cdot j = F_{21y} + (-m_1 a_{C_{1y}}) - m_1 g + F_{01y} = 0, \\
\begin{vmatrix}
1 & j & k \\
x_B & y_B & 0 \\
F_{21x} & F_{21y} & 0
\end{vmatrix}
\begin{vmatrix}
1 & j & k \\
x_C & y_C & 0 \\
-m_1 a_{C_{1x}} & -m_1 a_{C_{1y}} - m_1 g & 0
\end{vmatrix}
\]

\[-I_{C_1} a_1 k + M k = 0,
\]

or

\[
F_{21x} + (-m_1 a_{C_{1x}}) + F_{01x} = 0, \\
F_{21y} + (-m_1 a_{C_{1y}}) - m_1 g + F_{01y} = 0, \\
x_B F_{21y} - y_B F_{21x} + x_C (-m_1 a_{C_{1y}} - m_1 g) - y_C (-m_1 a_{C_{1x}}) - I_{C_1} a_1 + M = 0,
\]

or numerically

\[
\frac{1}{4}(400 + 7\sqrt{2}) + \left[ -\frac{\sqrt{2}}{4} \right] + F_{01x} = 0, \quad (4.63) \\
-\frac{5}{4}(84 + \sqrt{2}) + \left[ -\frac{\sqrt{2}}{4} \right] - 1(10) + F_{01y} = 0, \quad (4.64)
\]
\[\frac{\sqrt{2}}{2} \left[ -\frac{5}{4}(84 + \sqrt{2}) \right] - \frac{\sqrt{2}}{2} \left[ \frac{1}{4}(400 + 7\sqrt{2}) \right] + \frac{\sqrt{2}}{4} \left[ -1(-\frac{\sqrt{2}}{4}) - 1(10) \right] - \frac{\sqrt{2}}{4} \left[ -1(-\frac{\sqrt{2}}{4}) \right] - 0 + M = 0. \]  

(4.65)

For the crank 1 the MATLAB commands are

\[
\begin{align*}
F01 &= \text{[ sym('F01x','real') sym('F01y','real') 0 ]}; \\
Mm &= \text{[ 0 0 sym('Mmz','real') ]}; \\
eqxF1 &= F01+Fin1+G1-F12; \\
eqxF1x &= eqF1(1); \\
eqxF1y &= eqF1(2); \\
eqXM1 &= \text{cross(rB-rC1,-F12)+cross(rA-rC1,F01)+Min1+Mm}; \\
eqXM1z &= eqM1(3); \\
\end{align*}
\]

Equations (4.58), (4.59), (4.60), (4.61), (4.62), (4.63), (4.64) and (4.65) form a system of 8 equations with eight scalar unknowns.

The MATLAB commands for solving the system of equations are

\[
\begin{align*}
\text{sol321=}\text{solve(eqF3x,eqF3y,eqF2x,eqF2y,eqM2z,eqF1x,eqF1y,eqM1z)}; \\
F03ys &= \text{eval(sol321.F03y)}; \\
F23xs &= \text{eval(sol321.F23x)}; \\
F23ys &= \text{eval(sol321.F23y)}; \\
F12xs &= \text{eval(sol321.F12x)}; \\
F12ys &= \text{eval(sol321.F12y)}; \\
F01xs &= \text{eval(sol321.F01x)}; \\
F01ys &= \text{eval(sol321.F01y)}; \\
Mmzs &= \text{eval(sol321.Mmz)}; \\
\end{align*}
\]

The following numerical solutions are obtained

\[
\begin{align*}
F_{03y} &= -85 - \frac{3\sqrt{2}}{2} \text{ N}, \\
F_{23x} &= -100 - \sqrt{2} \text{ N}, \\
F_{23y} &= 95 + \frac{3\sqrt{2}}{2} \text{ N}, \\
F_{12x} &= -\frac{1}{4}(400 + 7\sqrt{2}) \text{ N}, \\
F_{12y} &= \frac{5}{4}(84 + \sqrt{2}) \text{ N},
\end{align*}
\]
\[ F_{01x} = -2(50 + \sqrt{2}) \text{ N}, \quad F_{01y} = 115 + \sqrt{2} \text{ N}, \]
\[ M = 3 + 105\sqrt{2} \text{ N} \cdot \text{m}. \]

The MATLAB program using D’Alembert principle and the results are given in Program 4.2.
4.6.3 Dyad Method

$B_R C_R C_T$ dyad

Figure 4.11 shows the dyad $B_R C_R C_T$ with the unknown joint reactions $F_{12}$ and $F_{03}$. The joint reaction $F_{03}$ is perpendicular to the sliding direction $\Delta = \mathbf{i}$ or $F_{03} = F_{03y}$.

The following equations are written to determine $F_{12}$ and $F_{03}$

- sum of all the forces on links 2 and 3 is zero

$$\sum \mathbf{F}^{(2,3)} = F_{12} + F_{in2} + \mathbf{G}_2 + F_{in3} + \mathbf{G}_3 + F_{ext} + F_{03} = 0,$$

or

$$\sum \mathbf{F}^{(2,3)} \cdot \mathbf{i} = F_{12x} + F_{in2x} + F_{in3x} + F_{ext} = 0, \quad (4.66)$$

$$\sum \mathbf{F}^{(2,3)} \cdot \mathbf{j} = F_{12y} + F_{in2y} - m_2 g + F_{in3y} - m_3 g + F_{03y} = 0. \quad (4.67)$$

- sum of moments of all the forces and moments on link 2 about $C_R$ is zero

$$\sum \mathbf{M}^{(2)}_C = (r_B - r_C) \times F_{12} + (r_{C_2} - r_C) \times (F_{in2} + \mathbf{G}_2) + \mathbf{M}_{in2} = 0. \quad (4.68)$$

The MATLAB commands for the Eqs. (4.66), (4.67), and (4.68) and for finding the unknowns are

```matlab
F03 = [ 0 sym('F03y','real') 0 ];
F12 = [ sym('F12x','real') sym('F12y','real') 0 ];
eqF32 = F03+Fe+G3+Fin3+Fin2+G2+F12;
eqF32x = eqF32(1);
eqF32y = eqF32(2);
eqM2C = cross(rB-rC,F12)+cross(rC2-rC,Fin2+G2)+Min2;
eqM2Cz = eqM2C(3);
fprintf('%s = 0 (1)\n', char(vpa(eqF32x,6)));
fprintf('%s = 0 (2)\n', char(vpa(eqF32y,6)));
fprintf('%s = 0 (3)\n', char(vpa(eqM2Cz,6)));
fprintf('Eqs(1)-(3) => F03y, F12x, F12y \n');
sol32=solve(eqF32x,eqF32y,eqM2Cz);
F03ys=eval(sol32.F03y);
F12xs=eval(sol32.F12x);
F12ys=eval(sol32.F12y);
```
\[ F_{03s} = [0, F_{03ys}, 0]; \]
\[ F_{12s} = [F_{12xs}, F_{12ys}, 0]; \]

The numerical values of the joint forces are

\[ F_{12x} = -102.475 \text{ N}, \quad F_{12y} = 106.768 \text{ N}, \quad \text{and} \quad F_{03y} = -87.1213 \text{ N}. \]

For the crank 1 (Fig. 4.11), there are two vectorial equations

\[
\sum F^{(1)} = F_{21} + F_{\text{in}1} + G_{1} + F_{01} = 0,
\]
\[
\sum M_{A}^{(1)} = r_{B} \times F_{21} + r_{C_{1}} \times (F_{\text{in}1} + G_{1}) + M_{\text{in}1} + M = 0,
\]

and the MATLAB statements for finding \( F_{01} \) and \( M \) are

\[
F_{01} = -F_{\text{in}1} - G_{1} + F_{12s};
M_{m} = -\text{cross}(r_{B}, -F_{12s}) - \text{cross}(r_{C_{1}}, G_{1} + F_{\text{in}1}) - M_{\text{in}1};
\]

The MATLAB program using the dyad method and the results are given in Program 4.3.
4.6.4 Contour Method

The diagram representing the mechanism is shown in Fig. 4.12 and has one contour, 0-1-2-3-0.

The dynamic force analysis can start with any joint.

Reaction $F_{03}$

The reaction force $F_{03}$ is perpendicular to the sliding direction of joint $C_T$ ($C_{Translation}$) as shown in Fig. 4.13.

$$F_{03} = F_{03y} \mathbf{j}.$$ 

The application point of the unknown reaction force $F_{03}$ is computed from a moment equation about $C_R$ ($C_{Rotation}$) for link 3 (path $I$) (Fig. 4.13)

$$\sum M_C^{(3)} = r_{CP} \times F_{03} = (r_P - r_C) \times F_{03} = 0,$$

or

$$xF_{05y} = 0 \Rightarrow x = 0.$$ 

The application point of the reaction force $F_{03}$ is at $C$ ($P \equiv C$).

The magnitude of the reaction force $F_{03y}$ is obtained from a moment equation about $B_R$ for the links 3 and 2 (path $I$)

$$\sum M_B^{(3k2)} = r_{BC} \times (F_{03} + F_{in3} + G_3 + F_{ext}) + r_{BC2} \times (F_{in2} + G_2) + M_{in2} = 0,$$

or

$$\begin{vmatrix} 1 & x_C - x_B & y_C - y_B & 0 \\ x_C - x_B & y_C - y_B & 0 \\ 1 & x_C - x_B & y_C - y_B & 0 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} + \begin{vmatrix} F_{in3x} + F_{ext} & F_{03y} + F_{in3y} - m_3 g & 0 \\ F_{in3x} + F_{ext} & F_{03y} + F_{in3y} - m_3 g & 0 \end{vmatrix} + M_{in2} \mathbf{k} = 0,$$

or numerically

$$\frac{3}{2} - \frac{5}{\sqrt{2}} + 45\sqrt{2} + \frac{F_{03y}}{\sqrt{2}} = 0.$$ 

The reaction $F_{03y}$ is

$$F_{03y} = -85 - \frac{3\sqrt{2}}{2} \text{ N.}$$
The MATLAB statements for finding \( \mathbf{F}_{03} \) are

```matlab
% Joint C,T
F03 = [ 0 sym('F03y','real') 0 ];
eqM32B = cross(rC-rB,F03+G3+Fin3+Fe)+cross(rC2-rB,Fin2+G2)+Min2;
eqM32Bz = eqM32B(3);
fprintf('%s = 0 (1)

\text{Eq}(1) \Rightarrow F_{03y}

solF03=solve(eqM32Bz);
F03ys=eval(solF03);
F03s=[ 0, F03ys, 0 ];
fprintf('F03 = [ %g, %g, %g ] (N)
```

Reaction \( \mathbf{F}_{23} \)

The pin joint at \( C_R \), between 2 and 3, is replaced with the reaction force (Fig. 4.14)

\[
\mathbf{F}_{23} = -\mathbf{F}_{32} = \mathbf{F}_{23x}\hat{\mathbf{i}} + \mathbf{F}_{23y}\hat{\mathbf{j}}.
\]

For the path I, an equation for the forces projected onto the sliding direction of the joint \( C_T \) is written for link 3

\[
\sum \mathbf{F}^{(3)} \cdot \mathbf{i} = (\mathbf{F}_{23} + \mathbf{F}_{in3} + \mathbf{G}_3 + \mathbf{F}_{ext}) \cdot \mathbf{i} = \mathbf{F}_{23x} + \mathbf{F}_{in3x} + \mathbf{F}_{ext} = \mathbf{F}_{23x} + 100 + \sqrt{2} = 0. \quad (4.69)
\]

For the path II, shown Fig. 4.7, a moment equation about \( B_R \) is written for link 2

\[
\sum \mathbf{M}^{(2)}_{B} = \mathbf{r}_{BC} \times ( -\mathbf{F}_{23} ) + \mathbf{r}_{BC2} \times ( \mathbf{F}_{in2} + \mathbf{G}_2 ) + \mathbf{M}_{in2} = \mathbf{0},
\]

or numerically

\[
\frac{1}{2} - \frac{5\sqrt{2}}{2} - \frac{F_{23x}\sqrt{2}}{2} - \frac{F_{23y}\sqrt{2}}{2} = 0. \quad (4.70)
\]

The joint force \( \mathbf{F}_{23} \) is obtained from the system of Eqs. (4.69) and (4.70)

\[
F_{23x} = -100 - \sqrt{2} \text{ N and } F_{23y} = 95 + \frac{3\sqrt{2}}{2} \text{ N.}
\]
The MATLAB statements for finding $\mathbf{F}_{23}$ are

```matlab
% Joint C_R
F23 = [ sym('F23x','real') sym('F23y','real') 0 ];
eqF3 = F23+Fe+G3+Fin3;
eqF3x = eqF3(1);
eqM2B = cross(rC-rB,-F23)+cross(rC2-rB,Fin2+G2)+Min2;
eqM2Bz = eqM2B(3);
fprintf('%s = 0 (2)
', char(vpa(eqF3x,6)));
fprintf('%s = 0 (3)
', char(vpa(eqM2Bz,6)));
fprintf('Eqs(2)-(3) => F23x, F23y
');
solF23=solve(eqF3x,eqM2Bz);
F23xs=eval(solF23.F23x);
F23ys=eval(solF23.F23y);
F23s = [ F23xs, F23ys, 0 ];
fprintf('F23 = [ %g, %g, %g ] (N)
', F23s );
```

Reaction $\mathbf{F}_{12}$

The pin joint at $B_R$, between 1 and 2, is replaced with the reaction force (Fig. 4.15)

$$ \mathbf{F}_{12} = -\mathbf{F}_{21} = F_{12x} \hat{i} + F_{12y} \hat{j}. $$

For the path $I$, shown Fig. 4.8, a moment equation about $C_R$ is written for link 2

$$ \sum M_C^{(2)} = \mathbf{r}_{CB} \times \mathbf{F}_{12} + \mathbf{r}_{CC2} \times (\mathbf{F}_{in2} + \mathbf{G}_2) + \mathbf{M}_{in2} = 0, $$

or numerically

$$ -\frac{1}{2} + \frac{5\sqrt{2}}{2} - \frac{F_{12x}\sqrt{2}}{2} - \frac{F_{12y}\sqrt{2}}{2} = 0. \quad (4.71) $$

Continuing on path $I$, an equation for the forces projected onto the sliding direction of the joint $C_T$ is written for links 2 and 3

$$ \sum \mathbf{F}^{(2\&3)} \cdot \mathbf{i} = (\mathbf{F}_{12} + \mathbf{F}_{in2} + \mathbf{G}_2 + \mathbf{F}_{in3} + \mathbf{G}_3 + \mathbf{F}_{ext}) \cdot \mathbf{i} = 
F_{12x} + F_{in2x} + F_{in3x} + F_{ext} = F_{23x} + 100 + \sqrt{2} + \frac{3\sqrt{2}}{2} = 0. \quad (4.72) $$
The joint force $F_{12}$ is obtained from the system of Eqs. (4.72) and (4.71)

$$F_{12x} = -\frac{1}{4}(400 + 7\sqrt{2}) \text{ N} \quad \text{and} \quad F_{12y} = \frac{5}{4}(84 + \sqrt{2}) \text{ N}.$$ 

The MATLAB statements for finding $F_{12}$ are

% Joint B_R
F12 = [ sym('F12x','real') sym('F12y','real') 0 ];
eqM2C = cross(rB-rC,F12)+cross(rC2-rC,Fin2+G2)+Min2;
eqM2Cz = eqM2C(3);
eqF23 = (F12+Fin2+G2+G3+Fin3+Fe);
eqF23x = eqF23(1);
fprintf('%s = 0 (4)\n', char(vpa(eqM2Cz,6)));
fprintf('%s = 0 (5)\n', char(vpa(eqF23x,6)));
fprintf('Eqs(4)-(5) => F12x, F12y\n');
solF12=solve(eqM2Cz,eqF23x);
F12xs=eval(solF12.F12x);
F12ys=eval(solF12.F12y);
F12s = [ F12xs, F12ys, 0 ];
fprintf('F12 = [ %g, %g, %g ] (N)\n', F12s );

Reaction $F_{01}$ and equilibrium moment $M$

The pin joint $A_R$, between 0 and 1, is replaced with the unknown reaction (Fig. 4.16)

$$F_{01} = F_{01x} \mathbf{i} + F_{01y} \mathbf{j}.$$ 

The unknown equilibrium moment is $M = M \mathbf{k}$. If the path $I$ is followed [Fig. 4.9] for the pin joint $B_R$, a moment equation is written for link 1

$$\sum M_{B}^{(1)} = r_{BA} \times F_{01} + r_{BC_1} \times (F_{in1} + G_1) + M_{in1} + M = 0,$$

or numerically

$$\frac{5\sqrt{2}}{2} + \frac{F_{01x}\sqrt{2}}{2} + \frac{F_{01y}\sqrt{2}}{2} + M = 0. \quad (4.73)$$
Continuing on path I the next joint encountered is the pin joint \( C_R \), and a moment equation is written for links 1 and 2

\[
\sum \mathbf{M}^{(1\&2)}_C = \mathbf{r}_{CA} \times \mathbf{F}_{01} + \mathbf{r}_{CC_1} \times (\mathbf{F}_{in1} + \mathbf{G}_1) + \mathbf{M}_{in1} + \mathbf{M} + \mathbf{r}_{CC_2} \times (\mathbf{F}_{in2} + \mathbf{G}_2) + \mathbf{M}_{in2} = 0,
\]

or numerically

\[
-\sqrt{2} F_{01y} + M - 1 + 10\sqrt{2} = 0. \tag{4.74}
\]

Continuing on path I the next joint encountered is the slider joint \( C_T \), and a force equation is written for links 1, 2, and 3

\[
\sum \mathbf{F}^{(1\&2\&3)} \cdot \mathbf{i} = (\mathbf{F}_{01} + \mathbf{F}_{in1} + \mathbf{G}_1 + \mathbf{F}_{in2} + \mathbf{G}_2 + \mathbf{F}_{in3} + \mathbf{G}_3 + \mathbf{F}_{ext}) \cdot \mathbf{i} =
\]

\[
F_{01x} + F_{in1x} + F_{in2x} + F_{in3x} + F_{ext} = F_{12x} + 100 + \sqrt{2} + \frac{3\sqrt{2}}{2} = 0. \tag{4.75}
\]

From Eqs. (4.73), (4.74), and (4.75) the components \( F_{01x}, F_{01y} \) and \( M \) are computed

\[
F_{01x} = -2(50 + \sqrt{2}) \text{ N} \quad \text{and} \quad F_{01y} = 115 + \sqrt{2} \text{ N},
\]

\[
M = 3 + 105\sqrt{2} \text{ N} \cdot \text{m}.
\]

The MATLAB statements for finding \( \mathbf{F}_{01} \) and \( \mathbf{M} \) are

\begin{verbatim}
% Joint A_R
F01 = [ sym('F01x','real') sym('F01y','real') 0 ];
Mm = [ 0 0 sym('Mmz','real') ];
eqM1B = cross(-rB,F01)+cross(rC1-rB,Fin1+G1)+Min1+Mm;
eqM1Bz = eqM1B(3);
eqM12C=cross(-rC,F01)+cross(rC1-rC,Fin1+G1)+Min1+Mm+cross(rC2-rC,Fin2+G2)+Min2;
eqM12Cz = eqM12C(3);
\end{verbatim}
eqF123 = (F01+Fin1+G1+Fin2+G2+Fin3+G3+Fe);
eqF123x = eqF123(1);
fprintf('%s = 0 (6\n', char(vpa(eqM1Bz,6)));
fprintf('%s = 0 (7)\n', char(vpa(eqM12Cz,6)));
fprintf('%s = 0 (8)\n', char(vpa(eqF123x,6)));
fprintf('Eqs(6)-(8) => F01x, F01y, Mmz \n');
solF01=solve(eqM1Bz,eqM12Cz,eqF123x);
F01xs=eval(solF01.F01x);
F01ys=eval(solF01.F01y);
Mmzs=eval(solF01.Mmz);
F01s = [ F01xs, F01ys, 0 ];
Mms = [ 0, 0, Mmzs ];
fprintf('F01 = [ %g, %g, %g ] (N)\n', F01s);
fprintf('Mm = [ %g, %g, %g ] (N m)\n', Mms);

The MATLAB program using contour method and the results are given in Program 4.4.
4.7 R-RTR-RTR Mechanism

The planar R-RTR-RTR mechanism considered is shown in Fig. 4.17. The following numerical data are given: $AC = 0.060$ m, $AE = 0.250$ m and $CD = 0.150$ m. The widths of the links 1, 3, and 5 are $AB = 0.140$ m, $DF = 0.400$ m, and respectively, $EG = 0.500$ m. The height of the links 1, 3, and 5 is $h = 0.010$ m. The width of the links 2 and 4 is $w_{Slider} = 0.050$ m, and the height is $h_{Slider} = 0.020$ m. All five moving links are rectangular prisms with the depth $d = 0.001$ m. The angular velocity of the driver link 1 is $n = 50$ rpm. The density of the material is $\rho_{Steel} = \rho = 8000$ kg/m$^3$. The gravitational acceleration is $g = 9.807$ m/s$^2$. The center of mass locations of the links $i = 1, 2, ..., 5$ are designated by $C_i(x_{Ci}, y_{Ci}, 0)$.

The external moment applied on link 5 is opposed to the motion of the link: $M_{5, ext} = -\text{Sign}(\omega_5) |M_{ext}| k$ where $|M_{ext}| = 100$ N·m and $\omega_5$ is the angular velocity of link 5.

Find the motor moment $M_m$ required for the dynamic equilibrium and the joint reaction forces when the driver link 1 makes an angle $\phi = \frac{\pi}{6}$ rad with the horizontal axis.

Solution

The position vectors (in meters) of the joints were calculated at subsection 2.4

position of joint $A$: $r_A = 0$;
position of joint $B$: $r_B = x_B \hat{i} + y_B \hat{j} = 0.121 \hat{i} + 0.070 \hat{j}$;
position of joint $C$: $r_C = y_C \hat{j} = 0.060 \hat{j}$;
position of joint $D$: $r_D = x_D \hat{i} + y_D \hat{j} = -0.149 \hat{i} + 0.047 \hat{j}$;
position of joint $E$: $r_E = y_E \hat{j} = -0.250 \hat{j}$;
position of $F$: $r_F = x_F \hat{i} + y_F \hat{j} = 0.249 \hat{i} + 0.080 \hat{j}$; and
position of $G$: $r_G = x_G \hat{i} + y_G \hat{j} = -0.224 \hat{i} + 0.196 \hat{j}$.

The angles of the links with the horizontal are $\phi_2 = \phi_3 = 4.715^\circ$ and $\phi_4 = \phi_5 = 16.666^\circ$.

The position vector of the center of mass of link 1 is

$$r_{C_1} = x_{C_1} \hat{i} + y_{C_1} \hat{j} = \frac{x_B}{2} \hat{i} + \frac{y_B}{2} \hat{j} = 0.060 \hat{i} + 0.035 \hat{j} \text{ m.}$$

The position vector of the center of mass of slider 2 is

$$r_{C_2} = x_{C_2} \hat{i} + y_{C_2} \hat{j} = r_B.$$
The position vector of the center of mass of link 3 is
\[ \mathbf{r}_{C3} = x_{C3} \hat{i} + y_{C3} \hat{j} = \frac{x_D + x_F}{2} \hat{i} + \frac{y_D + y_F}{2} \hat{j} = 0.049 \hat{i} + 0.064 \hat{j} \text{ m.} \]

The position vector of the center of mass of slider 4 is
\[ \mathbf{r}_{C4} = x_{C4} \hat{i} + y_{C4} \hat{j} = \mathbf{r}_D. \]

The position vector of the center of mass of link 5 is
\[ \mathbf{r}_{C5} = x_{C5} \hat{i} + y_{C5} \hat{j} = \frac{x_E + x_G}{2} \hat{i} + \frac{y_E + y_G}{2} \hat{j} = -0.112 \hat{i} - 0.026 \hat{j} \text{ m.} \]

The velocity and acceleration analysis was carried out at subsection 3.8:
- acceleration of joint B: \( \mathbf{a}_{B1} = \mathbf{a}_{B2} = -3.323 \hat{i} - 1.919 \hat{j} \text{ m/s}^2; \)
- acceleration of joint D: \( \mathbf{a}_{D3} = \mathbf{a}_{D4} = 4.617 \hat{i} - 1.811 \hat{j} \text{ m/s}^2; \)
- acceleration of joint F: \( \mathbf{a}_F = -7.695 \hat{i} + 3.019 \hat{j} \text{ m/s}^2; \)
- acceleration of joint G: \( \mathbf{a}_G = 2.767 \hat{i} + 0.919 \hat{j} \text{ m/s}^2; \)
- angular velocity of link 5: \( \omega_5 = 0.917 \text{ rad/s}; \)
- angular acceleration of link 1: \( \alpha_1 = 0 \text{ rad/s}^2; \)
- angular acceleration of links 2 and 3: \( \alpha_2 = \alpha_3 = 14.568 \text{ rad/s}^2; \)
- angular acceleration of links 4 and 5: \( \alpha_4 = \alpha_5 = -5.771 \text{ rad/s}^2. \)

The acceleration vector of the center of mass of link 1 is
\[ \mathbf{a}_{C1} = \frac{\mathbf{a}_{B1}}{2} = -1.661 \hat{i} - 0.959 \hat{j} \text{ m/s}^2. \]

The acceleration vector of the center of mass of slider 2 is
\[ \mathbf{r}_{C2} = \mathbf{a}_{B2} = -3.323 \hat{i} - 1.919 \hat{j} \text{ m/s}^2. \]

The acceleration vector of the center of mass of link 3 is
\[ \mathbf{a}_{C3} = \frac{\mathbf{a}_{D3} + \mathbf{a}_F}{2} = -1.539 \hat{i} + 0.603 \hat{j} \text{ m/s}^2. \]

The acceleration vector of the center of mass of slider 4 is
\[ \mathbf{a}_{C4} = \mathbf{a}_{D4} = 4.617 \hat{i} - 1.811 \hat{j} \text{ m/s}^2. \]

The acceleration vector of the center of mass of link 5 is
\[ \mathbf{a}_{C5} = \frac{\mathbf{a}_E + \mathbf{a}_G}{2} = 1.383 \hat{i} + 0.459 \hat{j} \text{ m/s}^2. \]
Dynamic Force Analysis with MATLAB

The MATLAB program for positions, velocities, and accelerations is

```matlab
AB = 0.14; AC = 0.06; AE = 0.25; CD = 0.15; DF=0.4; EG=0.5;
phi = 30*(pi/180);
xA = 0; yA = 0; rA = [xA yA 0];
xC = 0; yC = AC; rC = [xC yC 0];
xE = 0; yE = -AE; rE = [xE yE 0];
xB = AB*cos(phi); yB = AB*sin(phi); rB = [xB yB 0];
eqnD1 = '( xDsol - xC )^2 + ( yDsol - yC )^2 = CD^2 ';
eqnD2 = '(yB - yC)/(xB - xC) = (yDsol - yC)/(xDsol - xC)';
solD = solve(eqnD1, eqnD2, 'xDsol, yDsol');
xDpositions = eval(solD.xDsol); yDpositions = eval(solD.yDsol);
xD1 = xDpositions(1); xD2 = xDpositions(2);
yD1 = yDpositions(1); yD2 = yDpositions(2);
if (phi>=0 && phi<=pi/2)||(phi >= 3*pi/2 && phi<=2*pi)
    if xD1 <= xC xD = xD1; yD=yD1; else xD = xD2; yD=yD2; end
else
    if xD1 >= xC xD = xD1; yD=yD1; else xD = xD2; yD=yD2; end
end
rD = [xD yD 0];
phi2 = atan((yB-yC)/(xB-xC)); phi3 = phi2;
phi4 = atan((yD+yF)/(xD-xE))+pi; phi5 = phi4;
xF = xD + DF*cos(phi3); yF = yD + DF*sin(phi3); rF = [xF yF 0];
xG = xE + EG*cos(phi5); yG = yE + EG*sin(phi5); rG = [xG yG 0];
xC1 = xB/2; yC1 = yB/2; rC1 = [xC1 yC1 0];
rC2 = rB;
xC3 = (xD+xF)/2; yC3 = (yD+yF)/2; rC3 = [xC3 yC3 0];
rC4 = rD;
xC5 = (xE+xG)/2; yC5 = (yE+yG)/2; rC5 = [xC5 yC5 0];
n = 50.;
omega1 = [ 0 0 pi*n/30 ]; alpha1 = [0 0 0 ];
vA = [0 0 0 ]; aA = [0 0 0 ];
vB1 = vA + cross(omega1,rB); vB2 = vB1;
aB1 = aA + cross(alpha1,rB) - dot(omega1,omega1)*rB; aB2 = aB1;
omega3z=sym('omega3z','real'); alpha3z=sym('alpha3z','real');
vB32=sym('vB32','real'); aB32=sym('aB32','real');
omega3 = [ 0 0 omega3z ];
vC = [0 0 0 ];
```

Dynamic Force Analysis with MATLAB

\( v_{B3} = v_{C} + \text{cross}(\omega_{3}, r_{B} - r_{C}) \)
\( v_{B3B2} = v_{B32} \cdot [\cos(\phi_{2}), \sin(\phi_{2}), 0] \)
\( \text{eqvB} = v_{B3} - v_{B2} - v_{B3B2} \)
\( \text{solve} = \text{solve}(\text{eqvBx}, \text{eqvBy}) \)
\( \omega_{3zs} = \text{eval}(\text{solve}.\omega_{3z}) \)
\( v_{B32s} = \text{eval}(\text{solve}.v_{B32}) \)
\( \text{Omega}_3 = [0, 0, \omega_{3zs}] \)
\( \text{Omega}_2 = \text{Omega}_3 \)
\( v_{32} = v_{B32s} \cdot [\cos(\phi_{2}), \sin(\phi_{2}), 0] \)
\( v_{D3} = v_{C} + \text{cross} (\omega_{3}, r_{D} - r_{C}) \)
\( v_{D4} = v_{D3} \)
\( a_{B3B2cor} = 2 \cdot \text{cross} (\omega_{3}, v_{32}) \)
\( \alpha_{3} = [0, 0, \alpha_{3z}] \)
\( a_{C} = [0, 0, 0] \)
\( a_{A} = a_{C} + \text{cross} (\alpha_{3}, r_{A} - r_{C}) - \text{dot} (\omega_{3}, \omega_{3}) \cdot (r_{A} - r_{C}) \)
\( a_{B3B2} = a_{B32} \cdot [\cos(\phi_{2}), \sin(\phi_{2}), 0] \)
\( \text{solve} = \text{solve}(\text{eqvBx}, \text{eqvBy}) \)
\( \alpha_{3zs} = \text{eval}(\text{solve}.\alpha_{3z}) \)
\( a_{B32s} = \text{eval}(\text{solve}.a_{B32}) \)
\( \text{Alpha}_3 = [0, 0, \alpha_{3zs}] \)
\( \text{Alpha}_2 = \text{Alpha}_3 \)
\( a_{D3} = a_{C} + \text{cross} (\alpha_{3}, r_{D} - r_{C}) - \text{dot} (\omega_{3}, \omega_{3}) \cdot (r_{D} - r_{C}) \)
\( a_{D4} = a_{D3} \)
\( \omega_{5z} = \text{sym} ('\omega_{5z}', 'real') \)
\( \alpha_{5z} = \text{sym} ('\alpha_{5z}', 'real') \)
\( v_{D5} = v_{E} + \text{cross} (\omega_{5}, r_{D} - r_{E}) \)
\( v_{D5D4} = v_{D54} \cdot [\cos(\phi_{5}), \sin(\phi_{5}), 0] \)
\( \text{solve} = \text{solve}(\text{eqvDx}, \text{eqvDy}) \)
\( \omega_{5zs} = \text{eval}(\text{solve}.\omega_{5z}) \)
\( v_{D54s} = \text{eval}(\text{solve}.v_{D54}) \)
\( \omega_{5} = [0, 0, \omega_{5z}] \)
\( v_{E} = [0, 0, 0] \)
\( v_{D5} = v_{D54s} \cdot [\cos(\phi_{5}), \sin(\phi_{5}), 0] \)
\( \text{Omega}_4 = \text{Omega}_5 \)
\( a_{D5D4cor} = 2 \cdot \text{cross} (\omega_{5}, v_{54}) \)
\( \alpha_{5} = [0, 0, \alpha_{5z}] \)
\( a_{E} = [0, 0, 0] \)
\( a_{D5} = a_{E} + \text{cross} (\alpha_{5}, r_{D} - r_{E}) - \text{dot} (\omega_{5}, \omega_{5}) \cdot (r_{D} - r_{E}) \)
\( a_{D5D4} = a_{D54} \cdot [\cos(\phi_{5}), \sin(\phi_{5}), 0] \)
\( \text{solve} = \text{solve}(\text{eqvDx}, \text{eqvDy}) \)
\( a_{D5} = a_{D4} + a_{D5D4} - a_{D5D4cor} \)
eqaDx = eqaD(1); eqaDy = eqaD(2);
solaD = solve(eqaDx,eqaDy);
alpha5zs=eval(solaD.alpha5z); aD54s=eval(solaD.aD54);
Alpha5 = [0 0 alpha5zs]; Alpha4 = Alpha5;
aF = aC + cross(Alpha3,rF-rC) - dot(Omega3,Omega3)*(rF-rC);
aG = aE + cross(Alpha5,rG-rE) - dot(Omega5,Omega5)*(rG-rE);
aC1 = aB1/2;
aC2 = aB2;
aC3 = (aD3+aF)/2;
aC4 = aD3;
aC5 = (aE+aG)/2;

The external moment applied on link 5 is opposed to the motion of the link
\[ M_{5_{\text{ext}}} = -\text{Sign}(\omega_5) |M_{\text{ext}}| k = -\text{Sign}(0.917) (100) k = -100k \text{ N m.} \]

**Inertia forces and moments**

*Link 1*

The mass of the link is
\[ m_1 = \rho AB h d = 8000(0.14)(0.01)(0.001) = 0.0112 \text{ kg.} \]

The inertia force of driver 1 at \( C_1 \) is
\[ \mathbf{F}_{\text{in}1} = -m_1 \mathbf{a}_{C_1} = -0.0112(-1.66198i-0.959545j) = 0.0186142i-0.0107469j \text{ N.} \]

The gravitational force on link 1 at \( C_1 \) is
\[ \mathbf{G}_1 = -m_1 g \mathbf{j} = -0.0112(9.807) \mathbf{j} = -0.109838 \mathbf{j} \text{ N.} \]

The mass moment of inertia of link 1 with respect to \( C_1 \) is
\[ I_{C_1} = m_1 (AB^2 + h^2)/12 = 0.0112(0.14^2 + 0.01^2)/12 = 1.83867 \times 10^{-5} \text{ kg m}^2. \]

The moment of inertia of driver 1 is
\[ \mathbf{M}_{\text{in}1} = -I_{C_1} \mathbf{a}_1 = 0. \]
To calculate the inertia force and the moment the following MATLAB commands are used

\[
\begin{align*}
m1 &= \rho AB h d; \\
Fin1 &= -m1 \cdot aC1; \\
G1 &= [0, -m1 \cdot g, 0]; \\
IC1 &= m1 \cdot (AB^2 + h^2)/12; \\
Min1 &= -IC1 \cdot \alpha1;
\end{align*}
\]

**Link 2**
The mass of the slider 2 is

\[
m_2 = \rho h_{\text{Slider}} w_{\text{Slider}} d = 8000(0.02)(0.05)(0.001) = 0.008 \text{ kg.}
\]

The inertia force of slider 2 at \( C_2 \) is

\[
F_{in2} = -m_2 \cdot a_{C2} = -0.008(-3.323\mathbf{i} - 1.919\mathbf{j}) = 0.0265917\mathbf{i} + 0.0153527\mathbf{j} \text{ N.}
\]

The gravitational force of slider 2 at \( C_2 \) is

\[
G_2 = -m_2 \cdot g \cdot \mathbf{j} = -0.008(9.807)\mathbf{j} = -0.109838\mathbf{j} \text{ N.}
\]

The mass moment of inertia of slider 2 with respect to \( C_2 \) is

\[
I_{C2} = m_2 \left( h_{\text{Slider}}^2 + w_{\text{Slider}}^2 \right)/12 = 0.008(0.02^2 + 0.05^2)/12 = 1.93333 \times 10^{-6} \text{ kg m}^2.
\]

The moment of inertia of slider 2 is

\[
M_{in2} = -I_{C2} \cdot \alpha_2 = -1.93333 \times 10^{-6} \cdot (14.568) \mathbf{k} = -2.8165 \times 10^{-5} \mathbf{k} \text{ N m.}
\]

The MATLAB commands to calculate the inertia force and the moment are

\[
\begin{align*}
m2 &= \rho h_{\text{Slider}} w_{\text{Slider}} d; \\
Fin2 &= -m2 \cdot aC2; \\
G2 &= [0, -m2 \cdot g, 0]; \\
IC2 &= m2 \cdot (h_{\text{Slider}}^2 + w_{\text{Slider}}^2)/12; \\
Min2 &= -IC2 \cdot \alpha2;
\end{align*}
\]
**Link 3**
The mass of the link is

\[ m_3 = \rho \, F D \, h \, d = 8000(0.4)(0.01)(0.001) = 0.032 \text{ kg.} \]

The inertia force of link 3 is

\[ \mathbf{F}_{\text{in}3} = -m_3 \, \mathbf{a}_{C3} = -0.032(-1.539\mathbf{i} + 0.603\mathbf{j}) = 0.0492489\mathbf{i} - 0.019326\mathbf{j} \text{ N.} \]

The gravitational force of link 3 is

\[ \mathbf{G}_3 = -m_3 \, g \, \mathbf{j} = -0.032(9.807)\mathbf{j} = -0.313824\mathbf{j} \text{ N.} \]

The mass moment of inertia is

\[ I_{C3} = m_3 \, (F D^2 + h^2)/12 = 0.032(0.4^2 + 0.01^2)/12 = 0.000426933 \text{ kg m}^2. \]

The inertia moment on link 3 is

\[ \mathbf{M}_{\text{in}3} = -I_{C3} \, \alpha_3 = -0.000426933(14.568)\mathbf{k} = -0.00621962\mathbf{k} \text{ N m.} \]

**Link 4**
The mass of the link is

\[ m_4 = \rho \, h_{\text{slider}} \, w_{\text{slider}} \, d = 8000(0.02)(0.05)(0.001) = 0.008 \text{ kg.} \]

The inertia force is

\[ \mathbf{F}_{\text{in}4} = -m_4 \, \mathbf{a}_{C4} = -0.008(4.617\mathbf{i} - 1.811\mathbf{j}) = -0.0369367\mathbf{i} + 0.0144946\mathbf{j} \text{ N.} \]

The gravitational force is

\[ \mathbf{G}_4 = -m_4 \, g \, \mathbf{j} = -0.008(9.807)\mathbf{j} = -0.109838\mathbf{j} \text{ N.} \]

The mass moment of inertia is

\[ I_{C4} = m_4(h_{\text{slider}}^2 + w_{\text{slider}}^2)/12 = 0.008(0.02^2 + 0.05^2)/12 = 1.93333 \times 10^{-6} \text{ kg m}^2. \]

The moment of inertia is

\[ \mathbf{M}_{\text{in}4} = -I_{C4} \, \alpha_4 = -1.93333 \times 10^{-6}(-5.771)\mathbf{k} = 1.11583 \times 10^{-5}\mathbf{k} \text{ N m.} \]

**Link 5**
The mass of the link is

\[ m_5 = \rho \, EG \, h \, d = 8000(0.5)(0.01)(0.001) = 0.04 \text{ kg}. \]

The inertia force is

\[ \mathbf{F}_{in5} = -m_5 \, \mathbf{a}_{C5} = -0.04(1.383 \hat{i} + 0.459 \hat{j}) = -0.0553516 \hat{i} - 0.0183855 \hat{j} \text{ N}. \]

The gravitational force is

\[ \mathbf{G}_5 = -m_5 \, g \, \mathbf{j} = -0.04(9.807) \mathbf{j} = -0.39228 \mathbf{j} \text{ N}. \]

The mass moment of inertia is

\[ I_{C5} = m_5 \left( EG^2 + h^2 \right)/12 = 0.04(0.5^2 + 0.01^2)/12 = 0.000833667 \text{ kg m}^2. \]

The moment of inertia is

\[ \mathbf{M}_{in5} = -I_{C5} \, \alpha_5 = -0.000833667(-5.771) \mathbf{k} = 0.00481155 \mathbf{k} \text{ N m}. \]

The MATLAB commands to calculate the inertia force and the moment for links 3, 4, and 5 are

```matlab
m3 = rho*DF*h*d;
Fin3 = -m3*aC3;
G3 = [0,-m3*g,0];
IC3 = m3*(DF^2+h^2)/12;
Min3 = -IC3*Alpha3;

m4 = rho*hSlider*wSlider*d;
Fin4 = -m4*aC4;
G4 = [0,-m4*g,0];
IC4 = m4*(hSlider^2+wSlider^2)/12;
Min4 = -IC4*Alpha4;

m5 = rho*EG*h*d;
Fin5 = -m5*aC5;
G5 = [0,-m5*g,0];
IC5 = m5*(EG^2+h^2)/12;
Min5 = -IC5*Alpha5;
```
Joint forces and drive moment

4.7.1 Newton-Euler Equations of Motion

The force analysis starts with the link 5 because the external moment $M_{5_{ext}}$ is given. Figure 4.18(a) shows the free body diagram of the link 5. The joint reaction force of the ground 0 on the link 5 at the joint $F$ is $F_{05} = F_{05x}i + F_{05y}j$. The joint reaction force of the link 4 on the link 5 is $F_{45} = F_{45x}i + F_{45y}j$. The application point of the force $F_{45}$ is $P(x_P, y_P)$ and the position vector of $P$ is $r_P = x_Pi + y_Pj$.

The symbolical six unknowns $F_{05x}$, $F_{05y}$, $F_{45x}$, $F_{45y}$, $x_P$, and $y_P$ are introduced in MATLAB using the commands:

```matlab
F05x=sym('F05x','real');
F05y=sym('F05y','real');
F45x=sym('F45x','real');
F45y=sym('F45y','real');
xP=sym('xP','real');
yP=sym('yP','real');
F05=[ F05x, F05y, 0 ];  % unknown joint force of 0 on 5
F45=[ F45x, F45y, 0 ];  % unknown joint force of 4 on 5
rP=[ xP, yP, 0 ];       % unknown application point of force F45
```

The point $P$, the application of the force $F_{45}$, is located on the direction $DE$, that is

$$ (r_D - r_E) \times (r_P - r_E) = 0. \quad (4.76) $$

Equation (4.76) is written in MATLAB as:

```matlab
eqP=cross(rD-rE,rP-rE);
eqPz=eqP(3);
```

The direction of the unknown joint force $F_{45}$ is perpendicular to the sliding direction $r_{DE}$

$$ F_{45} \cdot r_{DE} = 0, \quad (4.77) $$

or in MATLAB
eqF45DE=dot(F45,rD-rE);

For the link 5 the vector sum of the net forces, gravitational force $G_5$, joint forces $F_{05}$, $F_{45}$, is equal to $m_5 \mathbf{a}_{C5}$ [Fig. 4.18(a)]

$$m_5 \mathbf{a}_{C5} = F_{05} + F_{45} + G_5,$$

or using MATLAB commands

$$\text{eqF5}=F05+F45+G5-m5*aC5;$$

Projecting the previous vectorial onto $x$ and $y$ axes gives

$$m_5 a_{C5x} = F_{05x} + F_{45x}, \quad (4.78)$$
$$m_5 a_{C5y} = F_{05y} + F_{45y} - m_5 g, \quad (4.79)$$

or using MATLAB

$$\text{eqF5x}=\text{eqF5}(1); \quad \% \text{ projection on x-axis}$$
$$\text{eqF5y}=\text{eqF5}(2); \quad \% \text{ projection on y-axis}$$

The vector sum of the moments that act on link 5 with respect to the center of mass $C_5$ is equal to $I_{C5} \alpha_5$ [Fig. 4.18(a)]

$$I_{C5} \alpha_5 = r_{C5E} \times F_{05} + r_{C5P} \times F_{45} + M_{5ext}, \quad (4.80)$$

or in MATLAB

$$\text{eqMC5}=\text{cross}(rE-rC5,F05)+\text{cross}(rP-rC5,F45)+Me-IC5*Alpha5;$$
$$\text{eqMC5z}=\text{eqMC5}(3); \quad \% \text{ projection on z-axis}$$

There are five equations Eqs. (4.76)-(4.80) and six unknowns and that is why the analysis will continue with the slider 4. The free body diagram of the slider 4 is shown in Fig. 4.18(b).

The joint reaction force of the link 3 on the slider 4 at $D = C_4$ is $F_{34} = F_{34x1} + F_{34y1}$ and the joint reaction force of the link 5 on the slider 4 is $F_{54} = -F_{45} = -F_{45x1} - F_{45y1}$. The MATLAB commands are

$$F34x=\text{sym}('F34x','real');$$
F34y=sym('F34y','real');
F34=[ F34x, F34y, 0 ]; % unknown joint force of 3 on 4
F54=-F45; % joint force of 5 on 4

For the slider 4, according to Newton's equations of motion, the vector sum of the net forces, gravitational force \( \mathbf{G}_4 \), joint forces \( \mathbf{F}_{34}, \mathbf{F}_{54} \), is equal to \( m_4 \mathbf{a}_4 \)

\[
    m_4 \mathbf{a}_4 = \mathbf{F}_{34} + \mathbf{F}_{54} + \mathbf{G}_4,
\]
or using MATLAB commands

\[
eq \text{eqF4} = \mathbf{F}_{34} - \mathbf{F}_{45} + \mathbf{G}_4 - m_4 \mathbf{a}_4;
\]

Projecting the previous vectorial onto \( x \) and \( y \) axes gives

\[
    m_4 \mathbf{a}_{4x} = F_{34x} + F_{54x}, \quad (4.81)
\]
\[
    m_4 \mathbf{a}_{4y} = F_{34y} + F_{54y} - m_4 g, \quad (4.82)
\]
or using MATLAB

\[
eq \text{eqF4x} = \text{eqF4}(1);
eq \text{eqF4y} = \text{eqF4}(2);
\]

The vector sum of the moments that act on slider 4 with respect to the center of mass \( D = C_4 \) is equal to \( I_{C_4} \mathbf{a}_4 \)

\[
    I_{C_4} \mathbf{a}_4 = \mathbf{r}_{C_4P} \times \mathbf{F}_{54}, \quad (4.83)
\]
or in MATLAB

\[
eq \text{eqMC4} = \text{cross}(\mathbf{r}_P - \mathbf{r}_{C_4}, \mathbf{F}_{54}) - I_{C_4} \mathbf{a}_4;
eq \text{eqMC4z} = \text{eqMC4}(3);
\]

There are eight equations Eqs. (4.76)-(4.83) with eight unknowns \( F_{05x}, F_{05y}, F_{45x}, F_{45y}, x_P, y_P, F_{34x}, \) and \( F_{34y} \). The system is solved using MATLAB

\[
    \text{so145} = \text{solve}([\text{eqF5x}, \text{eqF5y}, \text{eqMC5z}, \text{eqF45DE}, \text{eqPz}, \text{eqF4x}, \text{eqF4y}, \text{eqMC4z}]);
\]
\[
    F05xs = \text{eval}(\text{so145.F05x});
\]
F05s=eval(sol45.F05y);  
F06s=[ F05xs, F05ys, 0 ];  
F45xs=eval(sol45.F45x);  
F45ys=eval(sol45.F45y);  
F45s=[ F45xs, F45ys, 0 ];  
F34xs=eval(sol45.F34x);  
F34ys=eval(sol45.F34y);  
F34s=[ F34xs, F34ys, 0 ];  
yPs=eval(sol45.yP);  
rPs=[xPs, yPs, 0];

The following numerical solution are obtained
\[
\begin{align*}
F_{05} &= 268.165 \hat{i} + 135.057 \hat{j} \text{ N}, \\
F_{45} &= -268.109 \hat{i} - 134.647 \hat{j} \text{ N}, \\
F_{34} &= -268.072 \hat{i} - 134.583 \hat{j} \text{ N}, \quad \text{and} \\
r_p &= -0.149492 \hat{i} + 0.0476701 \hat{j} \text{ m}.
\end{align*}
\]

The force analysis continues with the link 3. Figure 4.19(a) shows the free body diagram of the link 3. The joint reaction force of the link 4 on the link 3 is \( F_{43} = -F_{34} = 268.072 \hat{i} + 134.583 \hat{j} \text{ N}. \) The joint reaction force of the ground 0 on the link 3 at the joint \( C \) is \( F_{03} = F_{03x} \hat{i} + F_{03y} \hat{j}. \) The joint reaction force of the link 2 on the link 3 is \( F_{23} = F_{23x} \hat{i} + F_{23y} \hat{j}. \) The application point of the force \( F_{23} \) is \( Q(x_Q, y_Q) \) and the position vector of \( Q \) is \( r_Q = x_Q \hat{i} + y_Q \hat{j}. \)

The symbolical six unknowns \( F_{03x}, F_{03y}, F_{23x}, F_{23y}, x_Q, \) and \( y_Q \) are introduced in MATLAB using the commands

\[
\begin{align*}
F03x &= \text{sym}(‘F03x’, ‘real’); \\
F03y &= \text{sym}(‘F03y’, ‘real’); \\
F23x &= \text{sym}(‘F23x’, ‘real’); \\
F23y &= \text{sym}(‘F23y’, ‘real’); \\
xQ &= \text{sym}(‘xQ’, ‘real’); \\
yQ &= \text{sym}(‘yQ’, ‘real’); \\
F03 &= [ F03x, F03y, 0 ]; \quad \% \text{unknown joint force of 0 on 3} \\
F23 &= [ F23x, F23y, 0 ]; \quad \% \text{unknown joint force of 2 on 3} \\
rQ &= [xQ, yQ, 0]; \quad \% \text{unknown application point of force F23}
\end{align*}
\]
The point $Q$, the application of the force $F_{23}$, is located on the direction $BC$, that is
\[(r_B - r_C) \times (r_Q - r_C) = 0. \quad (4.84)\]

Equation (4.84) is written in MATLAB as
\[
\text{eqQ} = \text{cross}(r_B - r_C, r_Q - r_C);
\text{eqQz} = \text{eqQ}(3);
\]

The direction of the unknown joint force $F_{23}$ is perpendicular to the sliding direction $r_{BC}$
\[F_{23} \cdot r_{BC} = 0, \quad (4.85)\]
or in MATLAB
\[
\text{eqF23BC} = \text{dot}(F_{23}, r_B - r_C);
\]

For the link 3 the vector sum of the net forces, gravitational force $G_3$, joint forces $F_{43}$, $F_{03}$, $F_{23}$, is equal to $m_3 a_{C_3}$ [Fig. 4.19(a)]
\[m_3 a_{C_3} = F_{43} + F_{03} + F_{23} + G_3, \]
or using MATLAB commands
\[
\text{eqF3} = F_{43} + F_{03} + F_{23} + G_3 - m_3 a_{C_3};
\]

Projecting the previous vectorial onto $x$ and $y$ axes gives
\[
\begin{align*}
m_3 a_{C_{3x}} &= F_{43x} + F_{03x} + F_{23x}, \quad (4.86) \\
m_3 a_{C_{3y}} &= F_{43y} + F_{03y} + F_{23y} - m_3 g, \quad (4.87)
\end{align*}
\]
or using MATLAB
\[
\begin{align*}
\text{eqF3x} &= \text{eqF3}(1); \% \text{projection on x-axis} \\
\text{eqF3y} &= \text{eqF3}(2); \% \text{projection on y-axis}
\end{align*}
\]

The vector sum of the moments that act on link 3 with respect to the center of mass $C_3$ is equal to $I_{C_3} a_3$ [Fig. 4.19(a)]
\[I_{C_3} a_3 = r_{C_3D} \times F_{43} + r_{C_3C} \times F_{03} + r_{C_3Q} \times F_{23}, \quad (4.88)\]
Dynamic Force Analysis with MATLAB

or in MATLAB

\[
\text{eqMC3} = \text{cross(rD-rC3,F43)+cross(rC-rC3,F03)+cross(rQ-rC3,F23)-IC3*Alpha3;}
\]
\[
\text{eqMC3z} = \text{eqMC3}(3); \quad \% \text{projection on z-axis}
\]

There are five equations Eqs. (4.84)-(4.88) and six unknowns and that is why the analysis will continue with the slider 2. The free body diagram of the slider 2 is shown in Fig. 4.19(b).

The joint reaction force of the link 1 on the slider 2 at \(B\) is \(F_{12} = F_{12x}\hat{i} + F_{12y}\hat{j}\) and the joint reaction force of the link 3 on the slider 2 is \(F_{32} = -F_{23} = -F_{23x}\hat{i} - F_{23y}\hat{j}\). The MATLAB commands are

\[
F12x = \text{sym('F12x','real');}
\]
\[
F12y = \text{sym('F12y','real');}
\]
\[
F12 = [F12x, F12y, 0]; \quad \% \text{unknown joint force of 1 on 2}
\]
\[
F32 = -F23; \quad \% \text{joint force of 3 on 2}
\]

For the slider 2 the vector sum of the net forces, gravitational force \(G_2\), joint forces \(F_{32}, F_{12}\), is equal to \(m_2 a_{C_2}\)

\[
m_2 a_{C_2} = F_{32} + F_{12} + G_2,
\]

or using MATLAB commands

\[
\text{eqF2} = F32 + F12 + G2 - m2*aC2;
\]

Projecting the previous vectorial onto \(x\) and \(y\) axes gives

\[
m_2 a_{C_{2x}} = F_{32x} + F_{12x}, \quad (4.89)
\]
\[
m_2 a_{C_{2y}} = F_{32y} + F_{12y} - m_2 g, \quad (4.90)
\]

or using MATLAB

\[
\text{eqF2x} = \text{eqF2}(1);
\]
\[
\text{eqF2y} = \text{eqF2}(2);
\]

The vector sum of the moments that act on slider 2 with respect to the center of mass \(B = C_2\) is equal to \(I_{C_2} \alpha_2\)

\[
I_{C_2} \alpha_2 = r_{C_2Q} \times F_{32}, \quad (4.91)
\]
or in MATLAB

\[
\begin{align*}
\text{eqMC2} &= \text{cross}(rQ-rC2,F32)-IC2*\text{Alpha}2; \\
\text{eqMC2z} &= \text{eqMC2}(3); \quad \% \text{projection on z-axis}
\end{align*}
\]

There are eight equations Eqs. (4.84)-(4.91) with eight unknowns \(F_{03x}, F_{03y}, F_{23x}, F_{23y}, x_Q, y_Q, F_{12x}, \text{and } F_{12y}\). The system is solved using MATLAB

\[
\begin{align*}
\text{sol23} &= \text{solve(eqF3x,eqF3y,eqMC3z,eqF23BC,eqQz,eqF2x,eqF2y,eqMC2z)}; \\
F03xs &= \text{eval(sol23.F03x)}; \\
F03ys &= \text{eval(sol23.F03y)}; \\
F03s &= [F03xs, F03ys, 0]; \\
F23xs &= \text{eval(sol23.F23x)}; \\
F23ys &= \text{eval(sol23.F23y)}; \\
F23s &= [F23xs, F23ys, 0]; \\
F12xs &= \text{eval(sol23.F12x)}; \\
F12ys &= \text{eval(sol23.F12y)}; \\
F12s &= [F12xs, F12ys, 0]; \\
xQs &= \text{eval(sol23.xQ)}; \\
yQs &= \text{eval(sol23.yQ)}; \\
rQs &= [xQs, yQs, 0];
\end{align*}
\]

The following numerical solution are obtained

\[
\begin{align*}
F_{03} &= -256.745 \hat{i} - 272.179 \hat{j} \text{ N}, \\
F_{23} &= -11.3762 \hat{i} + 137.93 \hat{j} \text{ N}, \\
F_{12} &= -11.4028 \hat{i} + 137.993 \hat{j} \text{ N}, \quad \text{and} \\
r_Q &= 0.121243 \hat{i} + 0.07 \hat{j} \text{ m}.
\end{align*}
\]

The force analysis ends with the driver link 1. Figure 4.20 shows the free body diagram of the link 1. The joint reaction force of the link 2 on the link 1 is \(F_{21} = -F_{12} = 11.4028 \hat{i} - 137.993 \hat{j} \text{ N}\). The joint reaction force of the ground 0 on the link 1 at the joint \(A\) is \(F_{01} = F_{01x} \hat{i} + F_{01y} \hat{j}\). For the link 1 the vector sum of the net forces, gravitational force \(G_1\), joint forces \(F_{01}, F_{21}\), is equal to \(m_1 a_{C_1}\) (Fig. 4.20) \(m_1 a_{C_1} = F_{01} - F_{12} + G_1 \quad \Longrightarrow \quad F_{01} = m_1 a_{C_1} + F_{12} - G_1\).
or with MATLAB

\[
F_{01} = m_1 \vec{a}_{C1} + F_{12s} - G_1;
\]

The vector sum of the moments that act on link 1 with respect to the
center of mass \( C_1 \) is equal to \( I_{C_1} \alpha_1 \)

\[
I_{C_1} \alpha_1 = \vec{r}_{C1A} \times \vec{F}_{01} - \vec{r}_{C1B} \times \vec{F}_{12} + \vec{M}_{\text{mot}},
\]

and the equilibrium moment (motor moment) is

\[
\vec{M}_{\text{mot}} = I_{C_1} \alpha_1 - \vec{r}_{C1A} \times \vec{F}_{01} + \vec{r}_{C1B} \times \vec{F}_{12}.
\]

In MATLAB

\[
\text{Mm} = \text{IC1} \ast \text{alpha1} - \text{cross(rA} - \text{rC1}, \text{F01}) + \text{cross(rB} - \text{rC1}, \text{F12s});
\]

Another way of calculating the equilibrium moment is to take the sum of
the moments that act on link 1 with respect \( A \)

\[
I_{C_1} \alpha_1 + \vec{r}_{C1} \times m_1 \vec{a}_{C1} = \vec{r}_{C1} \times \vec{G}_1 + \vec{r}_{B} \times (-\vec{F}_{12}) + \vec{M}_{\text{mot}},
\]

and the equilibrium moment is

\[
\vec{M}_{\text{mot}} = \vec{r}_{B} \times \vec{F}_{12} + \vec{r}_{C1} \times (m_1 \vec{a}_{C1} - \vec{G}_1) + I_{C_1} \alpha_1,
\]

or in MATLAB

\[
\text{Mm} = \text{cross(rB,F12s)} + \text{cross(rC1,m1*ac1-G1)} + \text{IC1*alpha1};
\]

The joint reaction force of the ground 0 on the link 1 is \( F_{01} = -11.42141 + 138.092 J \) N, and the equilibrium moment is \( \vec{M}_{\text{mot}} = 17.5356 \text{k N m} \).
4.7.2 Dyad Method

The dynamic force analysis starts with the last dyad (links 5 and 4) because the external moment $M_{5\text{ext}}$ on link 5 is known.

**$E_R D_T D_R$ dyad**

Figure 4.21 shows the forces and the moments that act on the dyad $E_R D_T D_R$. The unknown joint reaction forces are $F_{05} = F_{05x} \mathbf{i} + F_{05y} \mathbf{j}$, $F_{34} = F_{34x} \mathbf{i} + F_{34y} \mathbf{j}$, or in MATLAB

\[
F_{05x} = \text{sym('F05x', 'real')} \; ; \\
F_{05y} = \text{sym('F05y', 'real')} \; ; \\
F_{34x} = \text{sym('F34x', 'real')} \; ; \\
F_{34y} = \text{sym('F34y', 'real')} \; ; \\
F_{05} = [F_{05x}, F_{05y}, 0] \; ; \\
F_{34} = [F_{34x}, F_{34y}, 0] \; ;
\]

The Newton equation for links 5 and 4

\[
m_5 \mathbf{a}_{C5} + m_4 \mathbf{a}_{C4} = F_{05} + G_5 + G_4 + F_{34} \implies \\
\sum F^{(5\&4)} = F_{05} + G_5 + G_4 + F_{34} - m_5 \mathbf{a}_{C5} - m_4 \mathbf{a}_{C4} = 0. \quad (4.92)
\]

Equation (4.92) has a component on $x$-axis, $\sum F^{(5\&4)} \cdot \mathbf{i}$, a component on $y$-axis, $\sum F^{(5\&4)} \cdot \mathbf{j}$, and the MATLAB commands are

\[
eq F_{45} = F_{05} + G_5 + G_4 + F_{34} - m_5 \mathbf{a}_{C5} - m_4 \mathbf{a}_{C4} ; \\
eq F_{45x} = \text{eqF45}(1) ; % projection on x-axis \\
eq F_{45y} = \text{eqF45}(2) ; % projection on y-axis
\]

The Euler equation of moments for links 5 and 4 about $D_R$ gives

\[
I_{C5} \alpha_5 + r_{DC5} \times m_5 \mathbf{a}_{C5} + I_{C4} \alpha_4 = r_{DE} \times F_{05} + r_{DC5} \times G_5 + M_{5\text{ext}} \implies \\
\sum M^{(5\&4)}_D = (r_E - r_D) \times F_{05} + (r_{C5} - r_D) \times (G_5 - m_5 \mathbf{a}_{C5}) + M_{5\text{ext}} \\
-I_{C5} \alpha_5 - I_{C4} \alpha_4 = 0. \quad (4.93)
\]

The MATLAB commands for Eq. (4.93) are

\[
eq M_{D45} = \text{cross}(r_E - r_D, F_{05}) + \text{cross}(r_{C5} - r_D, G_5 - m_5 \mathbf{a}_{C5}) + \text{Me-IC5*Alpha5-IC4*Alpha4} ;
\]
\texttt{eqMD45z=} \texttt{eqMD45(3)}; \% \text{projection on z-axis}

The Newton equation for link 4 projected on the sliding direction \textit{ED} is

\begin{equation}
(m_4 \mathbf{a}_C) \cdot \mathbf{r}_{ED} = (\mathbf{F}_{34} + \mathbf{G}_4 + \mathbf{F}_{54}) \cdot \mathbf{r}_{ED} \implies \\
\sum \mathbf{F}^{(4)} \cdot \mathbf{r}_{ED} = (\mathbf{F}_{34} + \mathbf{G}_4 - m_4 \mathbf{a}_C) \cdot (\mathbf{r}_D - \mathbf{r}_E) = 0. \tag{4.94}
\end{equation}

The force of the link 5 on link 4 is \( \mathbf{F}_{54} \) and \( \mathbf{F}_{54} \cdot \mathbf{r}_{ED} = 0 \). The \texttt{Matlab} command for Eq. (4.94) is

\texttt{eqF4DE=} \texttt{dot(F34+G4-m4*aC4,rD-rE)};

There are four equations Eqs. (4.92)-(4.94) with four unknowns \( F_{05x}, F_{05y}, F_{34x}, F_{34y} \). The system is solved using \texttt{Matlab}

\begin{verbatim}
   solDI=solve(eqF45x, eqF45y , eqMD45z, eqF4DE);
   F05xs=eval(solDI.F05x);
   F05ys=eval(solDI.F05y);
   F34xs=eval(solDI.F34x);
   F34ys=eval(solDI.F34y);
   F05s=[ F05xs, F05ys, 0 ];
   F34s=[ F34xs, F34ys, 0 ];
\end{verbatim}

The force of the link 4 on link 5 is \( \mathbf{F}_{45} \) is calculated from Newton equation for link 5

\begin{equation}
m_5 \mathbf{a}_C = \mathbf{F}_{05} + \mathbf{G}_5 + \mathbf{F}_{45} \implies \\
\mathbf{F}_{45} = m_5 \mathbf{a}_C - \mathbf{G}_5 - \mathbf{F}_{05},
\end{equation}

and the \texttt{Matlab} command is

\texttt{F45=m5*aC5-G5-F05s};

The application point of the joint force \( \mathbf{F}_{45} \) is \( P(x_P, y_P) \). The point \( P \) is on the line \textit{ED} or

\[ \mathbf{r}_{ED} \times \mathbf{r}_{EP} = 0 \quad \text{or} \quad (\mathbf{r}_D - \mathbf{r}_E) \times (\mathbf{r}_P - \mathbf{r}_E) = 0, \]

and with \texttt{Matlab}
eqP=cross(rD-rE,rP-rE);
eqPz=eqP(3);

The second equation needed to calculate $x_P$ and $y_P$ is the moment equation on link 4 about $D = C_4$

$$I_{C_4} \alpha_4 = r_{C_4P} \times (-F_{45}),$$

and with MATLAB

$$eqM4=cross(rP-rC4,-F45)-IC4*Alpha4;$$
$$eqM4z=eqM4(3);$$

The coordinates $x_P$ and $y_P$ are calculated using the MATLAB commands

$$solP=solve(eqPz,eqM4z);$$
$$xPs=eval(solP.xP);$$
$$yPs=eval(solP.yP);$$
$$rPs=[xPs, yPs, 0];$$

$C_RB_TBR$ dyad

Figure 4.22 shows the forces and the moments that act on the dyad $C_RB_TBR$ (links 3 and 2). The unknown joint reaction forces are $F_{03} = F_{03x} \mathbf{i} + F_{03y} \mathbf{j}$, $F_{12} = F_{12x} \mathbf{i} + F_{12y} \mathbf{j}$, or in MATLAB

$$F03x=sym('F03x','real');$$
$$F03y=sym('F03y','real');$$
$$F12x=sym('F12x','real');$$
$$F12y=sym('F12y','real');$$
$$F03=[ F03x, F03y, 0 ];$$
$$F12=[ F12x, F12y, 0 ];$$

The joint force $F_{43} = -F_{34}$ was calculated from the previous dyad $EDD$

$$F43=-F34s;$$

The sum of all the forces that act on links 3 and 2 is

$$m_3 a_{C_3} + m_2 a_{C_2} = F_{43} + F_{03} + G_3 + G_2 + F_{12} \quad \implies$$
\[
\sum \mathbf{F}^{(3\&2)} = \mathbf{F}_{43} + \mathbf{F}_{03} + \mathbf{G}_3 + \mathbf{G}_2 + \mathbf{F}_{12} - m_3 \mathbf{a}_{C_3} - m_2 \mathbf{a}_{C_2} = \mathbf{0}. \tag{4.95}
\]

Equation (4.95) has a component on \(x\)-axis, \(\sum \mathbf{F}^{(3\&2)} \cdot \mathbf{i}\), a component on \(y\)-axis, \(\sum \mathbf{F}^{(3\&2)} \cdot \mathbf{j}\), and the MATLAB commands are

\[
\begin{align*}
\text{eqF23} &= \text{F}43+\text{F}03+\text{G}3-\text{m}3*\text{a}C3+\text{G}2-\text{m}2*\text{a}C2+\text{F}12; \\
\text{eqF23x} &= \text{eqF23}(1); \quad \% \text{projection on x-axis} \\
\text{eqF23y} &= \text{eqF23}(2); \quad \% \text{projection on y-axis}
\end{align*}
\]

The sum of moments of all the forces and moments on links 3 and 2 about \(B_R\) is zero

\[
I_{C_3} \alpha_3 + \mathbf{r}_{BC_3} \times m_3 \mathbf{a}_{C_3} + I_{C_2} \alpha_2 = \mathbf{r}_{BD} \times \mathbf{F}_{43} + \mathbf{r}_{BC} \times \mathbf{F}_{03} + \mathbf{r}_{BC_3} \times \mathbf{G}_3 \implies
\sum \mathbf{M}_{B}^{(3\&2)} = (\mathbf{r}_{D} - \mathbf{r}_{B}) \times \mathbf{F}_{43} + (\mathbf{r}_{C} - \mathbf{r}_{B}) \times \mathbf{F}_{03} + (\mathbf{r}_{C_3} - \mathbf{r}_{B}) \times (\mathbf{G}_3 - m_3 \mathbf{a}_{C_3})
\]

\[
-\sum \mathbf{M}_{B}^{(3\&2)} = I_{C_3} \alpha_3 - I_{C_2} \alpha_2 = \mathbf{0}. \tag{4.96}
\]

The MATLAB commands for Eq. (4.96) are

\[
\begin{align*}
\text{eqMB3} &= \text{cross}(\text{rD}-\text{rB}, \text{F}43)+\text{cross}(\text{rC}-\text{rB}, \text{F}03)+\text{cross}(\text{rC3}-\text{rB}, \text{G}3-\text{m}3*\text{a}C3); \\
\text{eqMB2} &= -\text{IC3}*\text{Alpha3}-\text{IC2}*\text{Alpha2}; \\
\text{eqMB23} &= \text{eqMB3}+\text{eqMB2}; \\
\text{eqMB23z} &= \text{eqMB23}(3);
\end{align*}
\]

The sum of all the forces on link 2 projected on the sliding direction \(BC\) is

\[
(m_2 \mathbf{a}_{C_2}) \cdot \mathbf{r}_{BC} = (\mathbf{F}_{12} + \mathbf{G}_2 + \mathbf{F}_{32}) \cdot \mathbf{r}_{BC} \implies
\sum \mathbf{F}^{(2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{12} + \mathbf{G}_2 - m_2 \mathbf{a}_{C_2}) \cdot (\mathbf{r}_C - \mathbf{r}_B) = \mathbf{0}. \tag{4.97}
\]

The force of the link 3 on link 2 is \(\mathbf{F}_{32}\) and \(\mathbf{F}_{32} \cdot \mathbf{r}_{BC} = \mathbf{0}\). The MATLAB command for Eq. (4.97) is

\[
\text{eqF2BC} = \text{dot(}\text{F12+G2-}m2*\text{a}C2, \text{rC-rB});
\]

There are four equations Eqs. (4.95)-(4.97) with four unknowns \(F_{03x}, F_{03y}, F_{12x}, F_{12y}\). The system is solved using MATLAB

\[
\text{solDII} = \text{solve(eqF23x, eqF23y, eqMB23z, eqF2BC)};
\]
The force of the link 3 on link 2 is $F_{32}$ is calculated from the sum of all the forces on link 2

$$m_2 \alpha_{C_2} = F_{32} + G_2 + F_{12} \quad \Rightarrow \quad F_{32} = m_2 \alpha_{C_2} - G_2 - F_{12},$$

and the MATLAB command is

$$F_{32} = m_2 \alpha_{C_2} - G_2 - F_{12};$$

The application point of the joint force $F_{32}$ is $Q(x_Q, y_Q)$. The point $Q$ is on the line $BC$ or

$$\mathbf{r}_{BC} \times \mathbf{r}_{QC} = 0 \quad \text{or} \quad (\mathbf{r}_C - \mathbf{r}_B) \times (\mathbf{r}_Q - \mathbf{r}_C) = 0,$$

and with MATLAB

$$\text{eqQ} = \text{cross}((\mathbf{r}_C - \mathbf{r}_B), (\mathbf{r}_Q - \mathbf{r}_C));$$
$$\text{eqQz} = \text{eqQ}(3);$$

The second equation needed to calculate $x_Q$ and $y_Q$ is the sum of all the moments on link 2 about $B = C_2$

$$I_{C_2} \alpha_2 = \mathbf{r}_{C_2Q} \times F_{32},$$

and with MATLAB

$$\text{eqM2} = \text{cross}((\mathbf{r}_Q - \mathbf{r}_{C2}), F_{32}) - I_{C2} \alpha_2;$$
$$\text{eqM2z} = \text{eqM2}(3);$$

The coordinates $x_Q$ and $y_Q$ are calculated using the MATLAB commands
The joint reaction force of the ground on the link 1 and the equilibrium moment (drive moment) shown in Figure 4.20 are calculated using the procedure presented in the previous subsection. The \textsc{Matlab} program using the dyad method and the results are given in Program 4.6.

```
solQ=solve(eqQz,epM2z);
xQs=eval(solQ.xQ);
yQs=eval(solQ.yQ);
rQs=[xQs, yQs, 0];
```
4.7.3 Contour Method

The contour diagram representing the mechanism is shown in Fig. 4.23. It has two contours 0-1-2-3-0 and 0-3-4-5-0.

Reaction force \( \mathbf{F}_{05} \)

The rotation joint \( E_R \) between the links 0 and 5 is replaced with the unknown reaction force \( \mathbf{F}_{05} \) (Fig. 4.24)

\[
\mathbf{F}_{05} = F_{05x}\mathbf{i} + F_{05y}\mathbf{j}.
\]

With MATLAB, the force \( \mathbf{F}_{05} \) is written as

\[
\mathbf{F}_{05} = \text{sym}('F05x','real'), \text{sym}('F05y','real'), 0\text{];}
\]

Following the path \( I \), as shown in Fig. 4.24, a force equation is written for the translation joint \( D_T \). The projection of all forces, that act on the link 5, onto the sliding direction \( \mathbf{r}_{DE} \) is zero

\[
\sum \mathbf{F}^{(5)} \cdot \mathbf{r}_{DE} = (\mathbf{F}_{05} + \mathbf{G}_5 + \mathbf{F}_{in5}) \cdot \mathbf{r}_{DE} = 0, \quad (4.98)
\]

where \( \mathbf{r}_{DE} = \mathbf{r}_E - \mathbf{r}_D \).

Equation (4.98) with MATLAB becomes

\[
\text{eqER1} = \text{dot} (\mathbf{F05} + \mathbf{G5} + \mathbf{Fin5}, \mathbf{r}_E - \mathbf{r}_D);
\]

where the command \( \text{dot}(a,b) \) gives the scalar product of the vectors \( a \) and \( b \). Continuing on the path \( I \), a moment equation is written for the rotation joint \( D_R \)

\[
\sum \mathbf{M}_{D}^{(4&5)} = \mathbf{r}_{DE} \times \mathbf{F}_{05} + \mathbf{r}_{DC5} \times (\mathbf{G}_5 + \mathbf{F}_{in5}) + \mathbf{M}_{in4} + \mathbf{M}_{in5} + \mathbf{M}_{ext} = \mathbf{0}, \quad (4.99)
\]

where \( \mathbf{r}_{DC5} = \mathbf{r}_{C5} - \mathbf{r}_D \).

Equation (4.99) with MATLAB gives

\[
\text{eqER2} = \text{cross} (\mathbf{r}_E - \mathbf{r}_D, \mathbf{F05}) + \text{cross} (\mathbf{r}_{C5} - \mathbf{r}_D, \mathbf{G}_5 + \mathbf{Fin5}) + \mathbf{Me} + \mathbf{Min4} + \mathbf{Min5};
\]

\[
\text{eqER2z} = \text{eqER2}(3);
\]
The system of two equations is solved using MATLAB commands

\[
\text{solF05} = \text{solve}(\text{eqER1}, \text{eqER2z});
\]
\[
\text{F05s} = [\text{eval(solF05.F05x)}, \text{eval(solF05.F05y)}, 0 ];
\]

The following numerical solution is obtained

\[
\text{F}_{05} = 268.165 \hat{i} + 135.057 \hat{j} \text{ N.}
\]

Reaction force \( \text{F}_{45} \)

The translation joint \( D_T \) between the links 4 and 5 is replaced with the unknown reaction force \( \text{F}_{45} \) (Fig. 4.25)

\[
\text{F}_{45} = -\text{F}_{54} = F_{45x} \hat{i} + F_{45y} \hat{j}.
\]

The position of the application point \( P \) of the force \( \text{F}_{45} \) is unknown

\[
\text{r}_P = x_P \hat{i} + y_P \hat{j},
\]

where \( x_P \) and \( y_P \) are the plane coordinates of the point \( P \).

The force \( \text{F}_{45} \) and its point of application \( P \) with MATLAB is written as

\[
\text{F}45=[ \text{sym}('F45x','\text{real}'), \text{sym}('F45y','\text{real}'), 0 ];
\]
\[
\text{F}54=-\text{F}45;
\]
\[
\text{r}P= [\text{sym}('xP','\text{real}'), \text{sym}('yP','\text{real}'), 0 ];
\]

Following the path \( I \) (Fig. 4.25), a moment equation is written for the rotation joint \( E_R \)

\[
\sum M_E^{(5)} = \text{r}_{EP} \times \text{F}_{45} + \text{r}_{EC_5} \times (G_5 + \text{F}_{in5}) + \text{M}_{in5} + \text{M}_{5ext} = 0, \quad (4.100)
\]

where \( \text{r}_{EP} = \text{r}_P - \text{r}_E \) and \( \text{r}_{EC_5} = \text{r}_{C_5} - \text{r}_E \).

One can write Eq. (4.100) using the MATLAB commands

\[
\text{eqDT1} = \text{cross(rP-rE,F45)} + \text{cross(rC5-rE,G5+Fin5)} + \text{Me+Min5};
\]
\[
\text{eqDT1z} = \text{eqDT1}(3);
\]

Following the path \( II \) (Fig. 4.25), a moment equation is written for the rotation joint \( D_R \)

\[
\sum M_D^{(4)} = \text{r}_{DP} \times \text{F}_{54} + \text{M}_{in4} = 0, \quad (4.101)
\]
where $\mathbf{r}_{DP} = \mathbf{r}_P - \mathbf{r}_D$ and $\mathbf{F}_{54} = -\mathbf{F}_{45}$.

Equation (4.101) with MATLAB is

$$
eq \mathbf{DT}_2 = \text{cross}(\mathbf{r}_P - \mathbf{r}_D, \mathbf{F}_{54}) + \text{Min}_4;$$
$$
eq \mathbf{DT}_2z = \mathbf{eqDT}_2(3);$$

The direction of the unknown joint force $\mathbf{F}_{45}$ is perpendicular to the sliding direction $\mathbf{r}_{ED}$

$$\mathbf{F}_{45} \cdot \mathbf{r}_{ED} = 0, \quad (4.102)$$

and using MATLAB command

$$
eq \text{eqF45DE} = \text{dot}(\mathbf{F}_{45}, \mathbf{r}_D - \mathbf{r}_E);$$

The application point $P$ of the force $\mathbf{F}_{45}$ is located on the direction $ED$, that is

$$(\mathbf{r}_D - \mathbf{r}_E) \times (\mathbf{r}_P - \mathbf{r}_E) = 0. \quad (4.103)$$

One can write Eq. (4.103) using the MATLAB commands

$$
eq \text{eqP} = \text{cross}(\mathbf{r}_D - \mathbf{r}_E, \mathbf{r}_P - \mathbf{r}_E);$$
$$
eq \text{eqPz} = \text{eqP}(3);$$

The system of four equations is solved using the MATLAB command

$$
\text{solF45} = \text{solve}(\text{eqDT}_1z, \text{eqDT}_2z, \text{F45DE}, \text{eqPz});
\mathbf{F}_{45} = [\text{eval(solF45.F45x)}, \text{eval(solF45.F45y)}, 0];
\mathbf{r}_{Ps} = [\text{eval(solF45.xP)}, \text{eval(solF45.yP)}, 0];
$$

The following numerical solutions are obtained

$$\mathbf{F}_{45} = -268.1091 \mathbf{i} - 134.646 \mathbf{j} \text{ N and } \mathbf{r}_P = -0.149492 \mathbf{i} + 0.0476701 \mathbf{j} \text{ m.}$$

Reaction force $\mathbf{F}_{34}$

The rotation joint $D_R$ between the links 3 and 4 is replaced with the unknown reaction force $\mathbf{F}_{34}$ (Fig. 4.26)

$$\mathbf{F}_{34} = -\mathbf{F}_{34} = F_{34x} \mathbf{i} + F_{34y} \mathbf{j},$$
and with MATLAB

\[
F34 = \left[ \text{sym}(\text{sym}(\text{sym}(\text{sym}(\text{sym}(\text{F34x},'\text{real}'), \text{sym}(\text{F34y},'\text{real}'), 0))) \right];
\]
\[
F43 = -F34;
\]

Following the path \(I\), a force equation can be written for the translation joint \(D_T\). The projection of all forces, that act on the link 4, onto the sliding direction \(ED\) is zero

\[
\sum F^{(4)} \cdot r_{ED} = (F_{34} + G_4 + F_{in4}) \cdot r_{ED} = 0, \quad (4.104)
\]

where \(r_{ED} = r_D - r_E\).

Equation (4.104) using MATLAB gives

\[
eq DR1 = \text{dot}(F34+G4+Fin4,rD-rE);
\]

Continuing on the path \(I\) (Fig. 4.26), a moment equation is written for the rotation joint \(E_R\)

\[
\sum M^{(4\&5)}_{E} = r_{EC4} \times (G_4 + F_{in4}) + r_{ED} \times F_{34} + M_{m4} + r_{EC5} \times (G_5 + F_{in5}) + M_{m5} + M_{5ext} = 0. \quad (4.105)
\]

where \(r_{EC4} = r_{C4} - r_E\), and \(r_{EC5} = r_{C5} - r_E\).

Equation (4.105) with MATLAB becomes

\[
eq DR24 = \text{cross}(rC4-rE,G4+Fin4)+\text{cross}(rD-rE,F34)+Min4;
\]
\[
eq DR25 = \text{cross}(rC5-rE,G5+Fin5)+Me+Min5;
\]
\[
eq DR2 = \text{eqDR24}+\text{eqDR25};
\]
\[
eq DR2z = \text{eqDR2}(3);
\]

The system of two equations is solved using the MATLAB commands

\[
\text{solF34} = \text{solve(eqDR1,eqDR2z)};
\]
\[
F34s = \left[ \text{eval(solF34.F34x)}, \text{eval(solF34.F34y)}, 0 \right];
\]

The following numerical solution is obtained

\[
F_{34} = -268.0721 - 134.583j \text{ N}.
\]
Reaction force $F_{03}$

The rotation joint $C_R$ between the links 0 and 3 is replaced with the unknown reaction force $F_{03}$ (Fig. 4.27)

$$F_{03} = F_{03x}\mathbf{i} + F_{03y}\mathbf{j}.$$ 

With MATLAB the force $F_{03}$ is written as

$$F03 = [\text{sym}('F03x','\text{real'}), \text{sym}('F03y','\text{real'}), 0];$$

Following the path $I$ (Fig. 4.27), a force equation is written for the translation joint $B_T$. The projection of all forces, that act on the link 3, onto the sliding direction $CD$ is zero

$$\sum F^{(3)} \cdot r_{CD} = (F_{03} + F_{43} + G_3 + F_{in3}) \cdot r_{CD} = 0, \quad (4.106)$$

where $r_{CD} = r_D - r_C$.

Equation (4.106) with MATLAB command is

$$\text{eqCR1} = \text{dot}(F03 - F34s + G3 + Fin3, rD - rC);$$

Continuing on the path $II$ (Fig. 4.27), a moment equation is written for the rotation joint $B_R$

$$\sum M^{(3k2)}_B = r_{BC_3} \times (G_3 + F_{in3}) + r_{BC} \times F_{03} + r_{BD} \times F_{43} + M_{in2} + M_{in3} = 0, \quad (4.107)$$

where $r_{BC_3} = r_{C_3} - r_B, r_{BC} = r_C - r_B,$ and $r_{BD} = r_D - r_B$.

With MATLAB Eq. (4.107) gives

$$\text{eqCR2} = \text{cross}(rC3-rB,G3+Fin3) + \text{cross}(rC-rB,F03) + \text{cross}(rD-rB,-F34s) + \text{Min2} + \text{Min3};$$

$$\text{eqCR2z} = \text{eqCR2}(3);$$

To solve the system of two equations the MATLAB commands are used

$$\text{solF03} = \text{solve(eqCR1,eqCR2z);}$$

$$F03s = [\text{eval(solF03.F03x)}, \text{eval(solF03.F03y)}, 0];$$

The following numerical solution is obtained

$$F_{03} = -256.7451 - 272.179 J\text{ N.}$$
Reaction force $F_{23}$

The translation joint $B_T$ between the links 2 and 3 is replaced with the unknown reaction force $F_{23}$ (Fig. 4.28)

$$F_{23} = -F_{32} = F_{23x} \mathbf{i} + F_{23y} \mathbf{j}.$$ 

The position of the application point $Q$ of the force $F_{23}$ is unknown

$$r_Q = x_Q \mathbf{i} + y_Q \mathbf{j},$$

where $x_Q$ and $y_Q$ are the plane coordinates of the point $Q$.

The force $F_{23}$ and its point of application $Q$ are written in MATLAB as

$$F_{23} = \left[ \text{sym('F23x','real')}, \text{sym('F23y','real')}, 0 \right];$$
$$F_{32} = -F_{23};$$
$$r_Q = \left[ \text{sym('xQ','real')}, \text{sym('yQ','real')}, 0 \right];$$

Following the path $I$ (Fig. 4.28), a moment equation is written for the rotation joint $C_R$

$$\sum M_{C}^{(3)} = r_{CQ} \times F_{23} + r_{CC_3} \times (G_3 + F_{in3}) + r_{CD} \times F_{43} + M_{in3} = 0, \quad (4.108)$$

where $r_{CQ} = r_Q - r_C$, $r_{CC_3} = r_{C_3} - r_C$, and $r_{CD} = r_D - r_C$.

Using MATLAB, Eq. (4.108) is written as

$$\text{eqBT1} = \text{cross}(r_Q - r_C, F_{23}) + \text{cross}(r_{C3} - r_C, G_3 + F_{in3}) + \text{cross}(r_D - r_C, -F_{34s}) + M_{in3};$$
$$\text{eqBT1z} = \text{eqBT1}(3);$$

Following the path $II$ (Fig. 4.28), a moment equation is written for the rotation joint $B_R$

$$\sum M_{B}^{(2)} = r_{BQ} \times F_{32} + M_{in2} = 0, \quad (4.109)$$

where $r_{BQ} = r_Q - r_B$.

Equation (4.109) with MATLAB becomes

$$\text{eqBT2} = \text{cross}(r_Q - r_B, F_{32}) + M_{in2};$$
$$\text{eqBT2z} = \text{eqBT2}(3);$$
The direction of the unknown joint force $F_{23}$ is perpendicular to the sliding direction $BC$. The following relation is written

$$F_{23} \cdot r_{BC} = 0,$$

or with MATLAB, it is

$$eqF23BC=dot(F23,rC-rB);$$

The application point $Q$ of the force $F_{23}$ is located on the direction $BC$, that is

$$(r_B - r_C) \times (r_Q - r_C) = 0.$$  

(4.110)

Equation (4.110) with MATLAB gives

$$eqQ=cross(rB-rC,rQ-rC);$$
$$eqQz=eqQ(3);$$

The system of four equations is solved using the MATLAB command

$$solF23=solve(eqBT1z,eqBT2z,F23BC,eqQz);$$
$$F23s=[ eval(solF23.F23x), eval(solF23.F23y), 0 ];$$
$$rQs=[ eval(solF23.xQ), eval(solF23.yQ), 0 ];$$

The following numerical solutions are obtained

$$F_{23} = -11.3762 \mathbf{i} + 137.93 \mathbf{j} \text{ N} \text{ and } r_Q = 0.121243 \mathbf{i} + 0.070 \mathbf{j} \text{ m}.$$  

Reaction force $F_{12}$

The rotation joint $B_R$ between the links 1 and 2 is replaced with the unknown reaction force $F_{12}$ (Fig. 4.29)

$$F_{12} = -F_{21} = F_{12x} \mathbf{i} + F_{12y} \mathbf{j}.$$  

With MATLAB it is written as

$$F12=[ \text{sym}('F12x','real'), \text{sym}('F12y','real'), 0 ];$$
$$F21=-F12;$$
Following the path $I$ (Fig. 4.29), a force equation is written for the translation joint $B_T$. The projection of all forces, that act on the link 2, onto the sliding direction $BC$ is zero

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{12} + \mathbf{G}_2 + \mathbf{F}_{in2}) \cdot \mathbf{r}_{BC} = 0. \quad (4.111)$$

Using MATLAB it is written as

```matlab
eqBR1=dot(F12+G2+Fin2,rC-rB);
```

Continuing on the path $I$, a moment equation is written for the rotation joint $C_R$

$$\sum \mathbf{M}_C^{(2\&3)} = \mathbf{r}_{CB} \times \mathbf{F}_{12} + \mathbf{r}_{CC_2} \times (\mathbf{G}_2 + \mathbf{F}_{in2}) + \mathbf{M}_{in2} + \mathbf{r}_{CC_3} \times (\mathbf{G}_3 + \mathbf{F}_{in3}) + \mathbf{r}_{CD} \times \mathbf{F}_{43} + \mathbf{M}_{in3} = 0, \quad (4.112)$$

where $\mathbf{r}_{CB} = \mathbf{r}_B - \mathbf{r}_C$, $\mathbf{r}_{CC_2} = \mathbf{r}_{C2} - \mathbf{r}_C$, $\mathbf{r}_{CC_3} = \mathbf{r}_{C3} - \mathbf{r}_C$, and $\mathbf{r}_{CD} = \mathbf{r}_D - \mathbf{r}_C$.

Using the MATLAB, commands Eq. (4.112) gives

```matlab
eqBR2=cross(rB-rC,F12)+cross(rC2-rC,G2+Fin2)+Min2...
+cross(rC3-rC,G3+Fin3)+cross(rD-rC,-F34s)+Min3;
eqBR2z=eqBR2(3);
```

The system of two equations is solved using the MATLAB commands

```matlab
solF12=solve(eqBR1,eqBR2z);
F12s=[ eval(solF12.F12x), eval(solF12.F12y), 0 ];
```

and the following numerical solution is obtained

$$\mathbf{F}_{12} = -11.4028 \mathbf{i} + 137.993 \mathbf{j} \text{ N.}$$

The motor moment $\mathbf{M}_m$

The motor moment needed for the dynamic equilibrium of the mechanism is $\mathbf{M}_m = M_m \mathbf{k}$ (Fig. 4.30). Following the path $I$ (Fig. 4.30), a moment equation is written for the rotation joint $A_R$

$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{r}_{AC_1} \times (\mathbf{G}_1 + \mathbf{F}_{in1}) + \mathbf{M}_{in1} + \mathbf{M}_m = 0. \quad (4.113)$$
Equation (4.113) is solved using the MATLAB command

\[ M_{1m} = -(\mathbf{cross}(r_B, -F_{12s}) + \mathbf{cross}(r_{C1}, G_1 + F_{in1}) + M_{in1}); \]

The numerical solution is

\[ M_m = 17.5356 \text{kN} \cdot \text{m}. \]

Reaction force \( F_{01} \)

The rotation joint \( A_R \) between the links 0 and 1 is replaced with the unknown reaction force \( F_{01} \) (Fig. 4.31)

\[ F_{01} = -F_{10} = F_{01x} \hat{\mathbf{i}} + F_{01y} \hat{\mathbf{j}}. \]

With MATLAB it is written as

\[ F_{01} = [ \text{sym}(\text{F01x}, \text{real}), \text{sym}(\text{F01y}, \text{real}), 0 ]; \]

Following the path \( I \) (Fig. 4.31), a moment equation is written for the rotation joint \( B_R \)

\[ \sum M_B^{(1)} = r_{BA} \times F_{01} + r_{BC1} \times (G_1 + F_{in1}) + M_{in1} + M_m = 0, \quad (4.114) \]

where \( r_{BA} = -r_B \), and \( r_{BC1} = r_{C1} - r_B \).

Equation (4.114) using the MATLAB commands gives

\[ \text{eqAR1} = \text{cross}(-r_B, F_{01}) + \text{cross}(r_{C1} - r_B, G_1 + F_{in1}) + M_{in1} + M_m; \]
\[ \text{eqAR1z} = \text{eqAR1}(3); \]

Continuing on the path \( I \) (Fig. 4.31), a force equation is written for the translation joint \( B_T \). The projection of all forces, that act on the links 1 and 2, onto the sliding direction \( BC \) is zero

\[ \sum F^{(1\&2)} \cdot r_{BC} = (F_{01} + G_1 + F_{in1} + G_2 + F_{in2}) \cdot r_{BC} = 0, \quad (4.115) \]

or with MATLAB it is

\[ \text{eqAR2} = \text{dot}(F_{01} + G_1 + F_{in1} + G_2 + F_{in2}, r_C - r_B); \]
The system of two equations is solved using the MATLAB commands

\[
\text{solF01} = \text{solve}(eqAR1z, eqAR2);
\]
\[
\text{F01s} = [\text{eval(solF01.F01x)}, \text{eval(solF01.F01y)}, 0];
\]

The following numerical solution is obtained

\[
\mathbf{F}_{01} = -11.4214 \mathbf{i} + 138.092 \mathbf{j} \text{ N.}
\]

The MATLAB program for the dynamic force analysis is presented in Program 4.7.